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A new microscopic model of pairing collective motion described mathematically by the group SU(2) x SU(3) is proposed. The concept of "gauge-spin" is introduced through which a set of heavy-ion reaction experiments is suggested.

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Theories have been advanced to treat pairing collective motion in medium and heavy nuclei[1-5]. For further understanding of this phenomenon we propose, in this letter, a new microscopic model described, mathematically, by the group SU(2)xSU(3). Starting with a general Hamiltonian the above scheme leads naturally to a description of pairing collective motion. This is analogous to Elliott's SU(3) treatment of quadrupole collective motion[6]. The members of each of our rotational bands are ground states and excited 0^+ states of neighbouring nuclei. This is to be differentiated from the conventional description[1-3] where members of a superconducting band are usually taken to be the ground states of neighbouring nuclei. The concept of "gauge-spin" is introduced through which we propose a set of experiments to study the different aspects of pairing collective motion. We also report some preliminary numerical results testing the goodness of pairing SU(6) scheme.

Starting with the pairing phonons that correspond to the solutions of the Tamm-Dancoff-Approximation of a general pairing Hamiltonian (i.e. pairing vibrational model[1]) we define the operators $A_{\alpha\beta}$ as:

$$A_{\alpha\beta} = \frac{1}{2}(B^+_\alpha B_\beta + B_\beta B^+_\alpha) \qquad (1)$$

where B^+_α , B_α are the creation and annihilation operators of the pairing quanta, α here refers to both the transfer quantum number ± 2, and the projection of isospin one. For identical nucleons it is easy to identify $A_{\alpha\beta}$ with the four shift operators of the group U_2. The three generators of SU_2 are then obtained from the $A_{\alpha\beta}$ after the removal of the phase factor transformation.

These are:

$$\Lambda_0 = \frac{1}{2}(A_{+2+2} - A_{-2-2})$$
$$\Lambda_1 = \frac{1}{2}(A_{-2+2} + A_{+2-2}) \qquad (2)$$
$$\Lambda_2 = \frac{i}{2}(A_{-2+2} - A_{+2-2})$$

The operators in (2) satisfy the well-known commutation relations of an angular momentum $\vec{\Lambda}$. We call this angular momentum "gauge spin" with eigenvalues given by $\Lambda(\Lambda+1)$.

In a system of n phonons the many-body operators which describe simultaneous SU_2 transformations in all phonons satisfy the same commutation relations. From now on Λ and Λ_o are summed over all phonons. The operators $B_{\alpha=2}, B_{\alpha=-2}$ can be regarded as elementary objects of gauge-spin 1/2, $C^+_{1/2\ 1/2}$ and $C^+_{1/2-1/2}$ respectively. Therefore irreducible gauge-spin tensors can be built from products of these elementary operators by standard vector coupling procedure. Taking the isospin structure of the pairing vibration into consideration we then write the generators of pairing $SU_2 x SU_3$ group in Racah form as follows:

$$U^{kk_T}_{qq_T} = -\frac{1}{\sqrt{6}} \sum_{\substack{m\tau \\ m'\tau'}} \left(\tfrac{1}{2}m\ \tfrac{1}{2}m' \middle| \tfrac{1}{2}\ \tfrac{1}{2}\ kq\right)(1\tau 1\tau' | 11k_T q_T) \times (-1)^{\frac{1}{2}+m'+\tau'} C^+_{1/2\ m\tau}\, C_{1/2-m'-\tau'} \qquad (3)$$

$U^{kk_T}_{qq_T}$ are the components of a double tensor of rank k in gauge-spin space and of rank k_T in isobaric-spin space. Accordingly the group chain for the $SU_2 x SU_3$ scheme is proposed as

$$\begin{array}{c} SU_6 \\ \downarrow \\ SU_2 x SU_3 \\ \downarrow \\ R_2 x R_3 \end{array} \qquad (4)$$

The pairing state is thus labeled in the following manner

$$(\Lambda\Lambda_o;[\lambda_1\lambda_2 0]KT) \tag{5}$$

where $[\lambda_1\lambda_2 0]$ are the representation label of U_3. The allowable values of T for $[\lambda_1\lambda_2 0]$ are given by

$$T = K,K+1,K+2,\ldots,K+\max\ (\lambda_1-\lambda_2,\lambda_2) \quad K\neq 0$$
$$= \max\ (\lambda_1-\lambda_2,\lambda_2),\ \max(\lambda_1-\lambda_2,\lambda_2)-2,\ldots,1 \text{ or } 0 \quad K=0$$

with the integral K taking the values

$$K = \min(\lambda_1-\lambda_2,\lambda_2),\ \min(\lambda_1-\lambda_2,\lambda_2)-2,\ldots,1 \text{ or } 0.$$

The states (5) classified as such look like a series of rotational bands cut off at $T=K+\max(\lambda_1-\lambda_2,\lambda_2)$ and indicates a relation between SU_2xSU_3 scheme and the pairing rotational model[2]. The parameter K in (5) corresponds to the projection of isospin on the symmetry axis of the pairing deformed system. Table 1 shows the comparison between the conventional labels[1,2] of a system of two phonons and ours. For the identical nucleon case Λ and Λ_o are related to the number of removal quanta n_1 and that of addition quanta n_2 as follows:

$$\Lambda = \frac{n}{2} = \frac{n_1+n_2}{2}$$
$$\Lambda_o = \frac{n_2-n_1}{2} \tag{6}$$

We have chosen to classify the states within a representation Λ by their Λ_o value for the obvious reason that Λ_o or its equivalence M ($|\Lambda_o|=M/4$) is a good quantum number, where M is the number of pairs of particles outside the double closed shell. We now turn to an equivalent basis, where Λ_o is not diagonal, that defines an intrinsic system from which the physical states are to

be obtained by projection. Using the relation

$$B^{+}_{\alpha=\pm 2} = B^{+}_{x} \pm i\, B^{+}_{y} \tag{7}$$

which defines the fictitious Cartesian coordinates in gauge spin space, we write the operator of U_2 in Cartesian form as

$$\begin{aligned} \Lambda_o &= \frac{i}{2}\,(A_{yx}-A_{xy}) \\ \Lambda_1 &= \frac{1}{2}\,(A_{xx}-A_{yy}) \\ \Lambda_2 &= \frac{1}{2}\,(A_{xy}+A_{yx}) \end{aligned} \tag{8}$$

The readers who are familiar with the nuclear SU_3 model[6] will recognize immediately that the gauge spin group has the same structure as that of the symmetry group of two-dimensional harmonic oscillators. The operator Λ_o can now be thought of as the angular momentum of the oscillators while Λ_1 as the operator which measures the pairing deformation in the x-y plane of the gauge-spin space. This x-y plane is nothing but the usual gauge space[1,3]

If we diagonalize Λ_1 instead of Λ_o and denote the eigenvalue of Λ_1 by Δ, then we may use a notation $|\Lambda\Delta\rangle$ for the Cartesian states of a representation Λ. The 'leading function' $|\Lambda\Delta_{max}\rangle$ of a representation Λ is defined in the same way as in Ref. 6. For brevity we denote it by φ. All functions $\psi(\Lambda\Lambda_o)$ in Racah basis may be easily expressed as a projection integral

$$\psi(\Lambda\Lambda_o) = \frac{1}{2\pi D^{\Lambda}_{\Lambda\Lambda_o}(o,\frac{\pi}{2},o)} \int e^{\,i\Lambda_o\phi}\, \varphi_{\phi}\, d\phi \tag{9}$$

Here ϕ is for the gauge angle and $\varphi_\phi = R_\phi \varphi$ with R being a two dimensional rotation operator. These functions with a definite gauge spin projection Λ_o in a representation Λ of SU_2 can be

written, in a way, very similar to states of the collective model with the intrinsic state being the state φ_ϕ.

For very strong pairing deformations, we were to suppose that the function φ_ϕ could be separated into a function of internal coordinates and a delta function of gauge angle ϕ' for strongly deformed distribution φ, $\varphi_\phi = \delta(\phi-\phi')\varphi_{int}$ (9) reduces to the simple rotational model form

$$e^{i\Lambda_o\phi'}\varphi_{int}$$

Thus a link with the pairing rotational model is established.

We are now arriving at the conclusion that the pairing SU_2 scheme for many pairing phonon states give rise to bands of states which correspond to pairing rotation. On the basis of this result one can interpret the state φ as being the intrinsic state and Δ as the intrinsic pairing moment.

In what follows, we will prove that, for any Hamiltonian, the energy of states within a pairing SU_2 representation can always have the collective feature which is shown through the observed Λ_o^2 or M^2 dependence of the 0^+ energy spectra. Let H be the Hamiltonian. Then

$$H\psi(\Lambda\Lambda_o) = \frac{1}{2\pi D^{\Lambda}_{\Lambda\Lambda_o}(o,\frac{\pi}{2},o)} \int e^{i\Lambda_o\phi} H_\phi \varphi_\phi d\phi \qquad (10)$$

since $[H,\Lambda_o] = 0$

Using Elliott's technique described in Ref. 6, we obtain

$$H\psi(\Lambda\Lambda_o) = \frac{1}{D^{\Lambda}_{\Lambda\Lambda_o}(o,\frac{\pi}{2},o)} \sum_{\Lambda'} D^{\Lambda'}_{\Lambda'\Lambda_o}(o,\frac{\pi}{2},o)\, g(\Lambda\Lambda'\Lambda_o)\psi(\Lambda'\Lambda_o)$$

where g is similar to the one defined in Ref 6. In the single representation limit this can be written in the following form:

$$E(\Lambda,\Lambda_o) = \sum_{p=o}^{2\Lambda} h(\Lambda,p)\Lambda_o^p \qquad (11)$$

Here we see for $h(\Lambda,p=\text{odd})=0$ we are led to a description of our nuclear system in terms of a rotation-vibration picture in which $h(\Lambda,p=2)$ in (11) is related to the pairing moment of inertia and the $h(\Lambda,p=\text{even}(\neq 2))$ as the rotation-vibration interaction terms. It should be noted that the rotation referred to here is taking place in the gauge space, the x-y plane of gauge-spin space.

It is interesting to note that one can generate exactly the same energy (11) by considering many-phonon forces among pairs, three, etc. The k-phonon force is expressed by a gauge-spin tensor of rank k. It is easy to see that all of these forces favor the maximum gauge-spin configuration. This is due to the nature of Bose statistics. Denoting the strengths of these many-body forces by $G^{(2)}$, $G^{(3)}$, $G^{(4)}$ etc... the energy can then be written down:

$$E(\Lambda,\Lambda_o) = \sum_{m=2}^{2\Lambda} \sum_{\substack{m_o=-m/2 \\ o}}^{m/2} \frac{\Gamma(\Lambda+\Lambda_o+1)\Gamma(\Lambda-\Lambda_o+1)}{\Gamma(\Lambda+\Lambda_o-\frac{m}{2}-m_o+1)\Gamma(\Lambda-\Lambda_o-\frac{m}{2}+m_o+1)} G^{(m)}_{(m_o)} \qquad (12)$$

where $\Gamma(x)$ is the Γ-function and the subscript m_o refers to the fact that one has different force stregths for members of the m-multiplet.

The rotation-vibration spectrum can be obtained from (12) if one assumes "gauge symmetry" i.e. $G^m_{m_o} = G^m_{-m_o}$. Within the single representation limit the rotation-vibration interaction must be due to the many phonon forces. However gauge-spin violating part of the general Hamiltonian also contributes to the rotation-vibration interaction. As a matter of fact by a straight forward use of Wigner-Eckart theorem in perturbation theory one can generate the same kind of expression for the energy (12). Equation (11) or equivalently (12) indicates how pairing collective spectrum arises from our simple model. As for the isospin degree

of freedom one again uses the analogy with Elliott's SU_3 model to generate the corresponding collective motion in isospin space. Thus our SU_6 model is similar to the macroscopic model[3] where the pairing deformation is considered to be taking place in a four dimensional isospin and gauge space.

Our model has been applied for calculating the 0^+ collective states of Pb isotopes using a pairing plus single particle Hamiltonian. We have used the boson expansion of such a Hamiltonian as given by B.Sorensen[8]. The pairing strength has been set to .086 and the Fermi level adjusted in order to have the two-particles and two-holes excitations equal. For low-lying levels (below 10 MeV) it has been found that the single representation limit is a good aproximation, whereas going to higher energies the mixing of states with $\Lambda' = \Lambda_o \pm 1$ is necessary in order to improve the results (see figure 1). It has been found that this is always the case near the critical value of the pairing strength g. The trend of the results shown in figure 1 reproduces that of experimental data.

More refined calculation would include realistic Hamiltonian, finite boson expansion[4] and isospin degree of freedom. This calculation is now in progress.

A way of checking the model proposed in this letter is through heavy-ion four-nucleon transfer reactions. We define members of a gauge-spin multiplet differing by one unit of Λ_o as gauge-analog of each other. The gauge-analog states (GAS) are highly excited 0^+ states of the final nucleus in a heavy-ion reaction where the target used is the member of the same gauge-spin multiplet with $|\Lambda_o|=\Lambda$. To make full use of the concept of gauge spin we use as a projectile a gauge-spin doublet i.e. nuclei with gauge-spin $\frac{1}{2}$. Then the GAS are reached by means of an optical potential having the form of a Lane potential[7] namely proportional to $\vec{\Lambda}(1).\vec{\Lambda}(2)$

The idea behind using such an optical potential stems from the observation that since the GAS are simple "excitations" of the target the corresponding two channels are rather strongly coupled and one may then feel justified in using an optical potential of the above form taking into account the influence of the other open channels through the introduction of an appropriate imaginary part. The cross-section one derives contains, among other things, the elastic scattering in both initial and final channels of the nuclei involved. The energy of the projectile should not exceed by much the Coulomb barrier so that one could have appreciable transfer. Coulomb excitation can be included easily in different ways since they appear in the cross section as distorting effects on the channel wave functions.

Possible projectiles are Pb^{206} and/or Pb^{210} used to probe targets in the lead region. One may also use the gauge-spin doublet Fe^{54} and Ni^{58} as projectiles on target nuclei in the Ni^{56} region.

Mixing states with different Λ, s makes it possible to estimate certain branching ratios for the above reactions. To first order in the dominant mixing parameter A_1 we obtain*.

$$\frac{\sigma(\rightarrow \text{GROUND})}{\sigma(\rightarrow \text{GAS})} \cong (2\Lambda-1)\frac{|A_1|^2}{|E_{GAS}-E_{g.s.}|^2}$$

and (13)

$$\frac{\sigma(\rightarrow \text{2nd } 0^+)}{\sigma(\rightarrow \text{GAS})} \cong 4\Lambda\ \frac{|A_1|^2}{|E_{GAS}-E_{2nd\ 0^+}|^2}$$

where Λ is the gauge-spin of the target as shown in figure 1. Taking as an example the reaction $^{212}Pb(^{206}Pb,^{210}Pb)^{208}Pb$ with a typical value of A_1 i.e. ~ 1 MeV we obtain $\frac{\sigma(\rightarrow \text{GROUND})}{\sigma(\rightarrow \text{GAS})} \approx 0.04$ and $\frac{\sigma(\rightarrow \text{2nd } 0^+)}{\sigma(\rightarrow \text{GAS})} \simeq 0.16$

* This rough estimate was obtained by ignoring the differences among the distorted channels.

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FIGURE CAPTION

FIG. 1: Pb isotopes collective 0^+ states. Calculated using g=.086. Levels are labeled with $(2\Lambda, 2\Lambda_o)$. Solid lines refer to single representation limit and dotted lines to the mixing of states with $\Lambda=\Lambda_o\pm1$.

TABLE CAPTION

TABLE 1: Classification of pairing states for a system of two phonons.

TABLE 1

$(n_r T_r, n_a T_a)_T$	$(A, \eta_\Delta, \eta_T, K, T)$	$(\Lambda\Lambda_o; [\lambda_1 \lambda_2 o] KT)$
$(00,20)_0$	(60,0,0,0,0)	(11; [200],00)
$(00,22)_2$	(60,0,0,0,2)	(11; [200],02)
$(20,00)_0$	(52,0,0,0,0)	(1-1; [200],00)
$(22,00)_2$	(52,0,0,0,2)	(1-1; [200],02)
$(11,11)_0$	(56,1,0,0,0)	(10; [200],00)
$(11,11)_1$	(56,0,1,1,1)	(00; [110],11)
$(11,11)_2$	(56,1,0,0,2)	(10; [200],02)

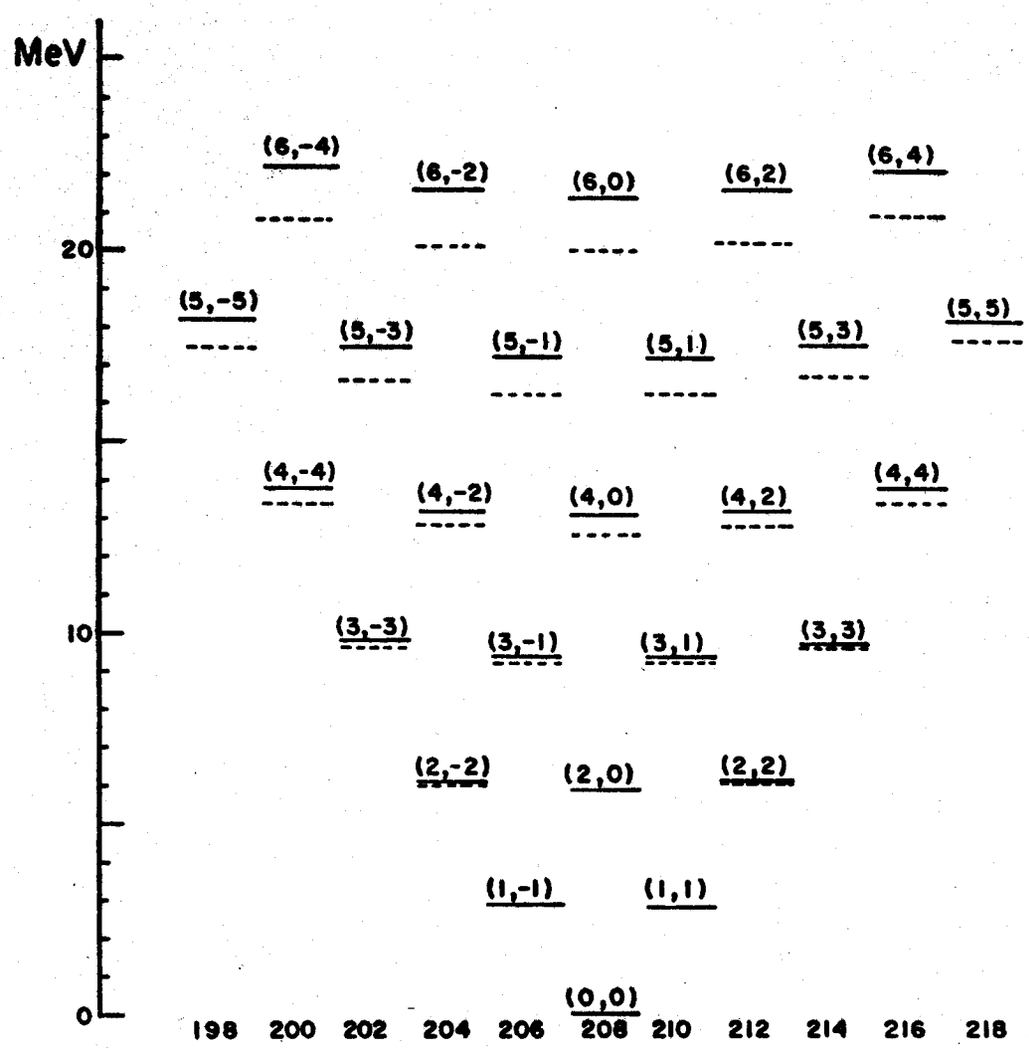
MeV
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Fig. 1