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Quantum Topology Change in $(2+1)d$

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Abstract

The topology of orientable $(2+1)$ spacetimes can be captured by certain lumps of non-trivial topology called topological geons. They are the topological analogues of conventional solitons. We give a description of topological geons where the degrees of freedom related to topology are separated from the complete theory that contain metric (dynamical) degrees of freedom. The formalism also allows us to investigate processes of quantum topology change. They correspond to creation and annihilation of quantum geons. Selection rules for such processes are derived.

1 Introduction

It is very common to make the reasonable assumption that the topology of space-time is fixed. We assume that space-time is a manifold of the form $\Sigma \times \mathbb{R}$, and that for each time t, we have a space-like surface that is always homeomorphic to a given Σ. However, when (quantum) gravity is taken into account, the very geometry of space becomes a degree of freedom, and one can conceive the possibility that Σ changes in the course of time [1]. Such a process is called topology change. Creation of baby universes, production of topological defects (cosmic strings, domain walls), and changes in genus (production of wormholes and topological geons) are examples of topology change. Each of them have received some attention in the literature. Several authors have investigated topology change within the context of both classical and quantum gravity [2]. It is interesting to notice that in the usual canonical approach to gravity, only the metric of the spatial manifold Σ appears as a degree of freedom and receives a quantum treatment. The topology of Σ in its turn is implicitly treated as a classical entity. There are, of course, other approaches to quantum gravity such as string theory [7] and Euclidean quantum gravity [8] where topology may appear as an entity of a quantum nature via a sum over topologies.

It would be desirable to have a formalism where topology can in a certain sense be canonically quantized and if possible separated from degrees of freedom coming from metric and other fields. In spite of the fact that topology change has been inspired by quantum gravity, it has been demonstrated in [9] that it can happen in ordinary quantum mechanics. In this approach, metric is not dynamical, but degrees of freedom related to topology are quantized. The notion of a space with a well defined topology appears only as a classical limit. (See also [10] for related ideas). The views we would like to present in this paper are similar, to a certain extent, to the ones in [9]. In our approach, variables related to topology are separated from other degrees of freedom and then quantized.

The topology of space is well captured by soliton-like excitations of Σ called topological geons. They can be thought of as lumps of nontrivial topology. For example, in $(2+1)d$, the topology of an orientable, closed surface Σ is determined by the number of connected components of Σ and by the number of of handles on each connected component. Each handle corresponds to a topological geon, i.e., a localized lump of nontrivial topology. It is well known that these solitons have particle like properties such as spin and statistics. However unlike ordinary particles they can violate the spin-statistics relation [4, 11]. It has been suggested [11, 13, 12] that the standard spin-statistics relation can be recovered if one considers processes where geons are (possibly pairwise) created and annihilated, but this necessarily implies a change of the topology of Σ. In other words, one may have to consider topology change in order to have a spin-statistics theorem for geons [13, 12].

The Euclidean path integral approach can in some sense be carried out in low dimensions [14], but it represents a formidable task in the case of a $(3+1)d$ theory. It would be nice to stay closer to a "canonical" quantization, even though topology change and the canonical approach appear to be incompatible. One may search for alternative descrip-