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# Three body force in peripheral $N\alpha$ scattering

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The exchange of a single pion does not contribute to $N\alpha$ scattering, due to the isoscalar nature of the target. Therefore peripheral $N\alpha$ scattering may be used to probe existing models for the two-pion exchange component of the $NN$ potential. In a recent work we have considered this possibility using an effective $N\alpha$ interaction based on two-body forces only. In the present communication we discuss the role played by the two-pion exchange three-nucleon force, since its range is comparable to the two-pion exchange potential (TPEP).

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As in [1], we assume the $\alpha$ ground state to be a S wave with a gaussian structure:

$$\big|\alpha\big\rangle = \left(\frac{4\alpha}{\pi}\right)^{9/4} \exp\!\left(-2\alpha\sum_{i=1}^{3}\boldsymbol{\rho}_i^2\right)\big|\chi_\alpha\big\rangle, \tag{0.1}$$

where $\rho_i$ are Jacobi coordinates, $\alpha$ is a parameter extracted from [2] and $\big|\chi_\alpha\big\rangle$ is the spin-isospin $\alpha$ wave function. The effective $N\alpha$ potential is given by

$$\begin{aligned}\bar{V} &= \bar{V}_{(2)} + \bar{V}_{(3)} \\ &= \big\langle\alpha\big|\sum_{i=1}^{4} V_{oi}\big|\alpha\big\rangle + \big\langle\alpha\big|\sum_{j>i=1}^{4}\Big[V_{oij} + V_{ijo} + V_{joi}\Big]\big|\alpha\big\rangle,\end{aligned} \tag{0.2}$$

where $\bar{V}_{(2)}$ and $\bar{V}_{(3)}$ are due to two and three body forces respectively. The former was considered in [1], and here we concentrate on the latter.

The two-pion exchange three-nucleon force involves the emission and absorption of virtual pions by nucleons $i$ and $j$, with an intermediate scattering by nucleon $k$. We use here the three body force derived in ref. [3], where the intermediate $\pi N$ scattering is described in the framework of chiral symmetry, and given by

$$\begin{aligned}V_{ijk}(\boldsymbol{r}_{ki},\boldsymbol{r}_{jk}) = \Bigg\{ &\left(\frac{C_s}{\mu^2}\right)\Big(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\Big)\Big(\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\nabla}_{ki}\Big)\Big(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{\nabla}_{jk}\Big) \\ &+\left(\frac{C_p}{\mu^4}\right)\Big(\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}\Big)\Big(\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\nabla}_{ki}\Big)\Big(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{\nabla}_{jk}\Big)\Big(\boldsymbol{\nabla}_{ki}\cdot\boldsymbol{\nabla}_{jk}\Big) \\ &-\left(\frac{C_p'}{\mu^4}\right)\Big(\boldsymbol{\tau}^{(i)}\times\boldsymbol{\tau}^{(j)}\cdot\boldsymbol{\tau}^{(k)}\Big)\Big(\boldsymbol{\sigma}^{(i)}\cdot\boldsymbol{\nabla}_{ki}\Big)\Big(\boldsymbol{\sigma}^{(j)}\cdot\boldsymbol{\nabla}_{jk}\Big)\Big(\boldsymbol{\sigma}^{(k)}\cdot\boldsymbol{\nabla}_{ki}\times\boldsymbol{\nabla}_{jk}\Big)\Bigg\} \\ &\times U(r_{ki})U(r_{jk}).\end{aligned} \tag{0.3}$$

In this result the function $U(x)$ is written as

$$U(x) = \frac{e^{-\mu x}}{\mu x} - \frac{\Lambda}{\mu}\,\frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2}\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2}-1\right)e^{-\Lambda x} \tag{0.4}$$

and the strenght coefficients have the numerical values $C_s = 0.9204$, $C_p = -2.0082$ and $C_p' = -0.6722$.

Due to the isoscalar nature of the $\alpha$ particle, the exchange of a single pion is not allowed and only $V_{ijo}$, which represents the external exchange of two pions, contributes to $\bar{V}_{(3)}$. It can be shown that the matrix element $\big\langle\chi_\alpha\big|\sigma_a^{(i)}\sigma_b^{(j)}\tau_c^{(i)}\tau_d^{(j)}\big|\chi_\alpha\big\rangle$ vanishes for $a \neq b$ or $c \neq d$, and is equal to -1/3 for $a = b$ and $c = d$. Therefore, the term in $\bar{V}_{(3)}$ proportional to $C_p'$ drops out and the remainder may be expressed as

$$\bar{V}_{(3)}(x) = \sum_{j>i=1}^{4} \left[ C_s I_{11} - \frac{C_p}{3}(I_{00} + 3I_{22} - I_{20}) \right], \tag{0.5}$$

where

$$I_{mn} = 16\sqrt{2}\left(\frac{2\alpha}{\pi}\right)^3 \int d\boldsymbol{u}\, d\boldsymbol{v}\ \frac{U_m(u)}{u^n}\frac{U_m(v)}{v^n}(\boldsymbol{u}\cdot\boldsymbol{v})^n$$
$$\times \exp\Big\{-2\alpha\big[3(\boldsymbol{x}-\boldsymbol{u})^2 + 3(\boldsymbol{x}-\ \boldsymbol{v})^2 + 2(\boldsymbol{x}-\boldsymbol{u})\cdot(\boldsymbol{x}-\boldsymbol{v})\big]\Big\}, \tag{0.6}$$

$$U_0(x) = \frac{e^{-\mu x}}{\mu x} - \frac{\Lambda}{\mu}\,\frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2}\frac{\Lambda}{\mu}\left(\frac{\Lambda^2}{\mu^2} - 1\right)e^{-\Lambda x}, \tag{0.7}$$

$$U_1(x) = -\frac{e^{-\mu x}}{\mu x}\left(1 + \frac{1}{\mu x}\right) + \frac{\Lambda^2}{\mu^2}\left(1 + \frac{1}{\Lambda x}\right)\frac{e^{-\Lambda x}}{\Lambda x} + \frac{1}{2}\left(\frac{\Lambda^2}{\mu^2} - 1\right)e^{-\Lambda x}, \tag{0.8}$$

$$U_2(x) = \frac{e^{-\mu x}}{\mu x}\left(1 + \frac{3}{\mu x} + \frac{3}{\mu^2 x^2}\right)$$
$$- \frac{\Lambda^3}{\mu^3}\left(1 + \frac{3}{\Lambda x} + \frac{3}{\Lambda^2 x^2}\right)\frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2}\frac{\Lambda}{\mu}\left(\frac{\Lambda^2}{\mu^2} - 1\right)e^{-\Lambda x}\left(1 + \frac{1}{\Lambda x}\right). \tag{0.9}$$

Using

$$(\boldsymbol{u}\cdot\boldsymbol{v})^n \exp\big[+4\alpha(4\boldsymbol{x} - \boldsymbol{u})\cdot\ \boldsymbol{v}\big] = \left(-\frac{1}{4\alpha}\frac{d}{d\xi}\right)^n\bigg|_{\xi=1} \exp\big[+4\alpha(4\boldsymbol{x} - \xi\ \boldsymbol{u})\cdot\boldsymbol{v}\big] \tag{0.10}$$

and a little more algebra, we get our final expression for the effective potential due to three-body interaction

$$\bar{V}_{(3)}(x) = 6\left(\frac{128\alpha^2\sqrt{2}}{\pi}\right)\int_0^\infty du \int_{-1}^{1} d(-\cos\theta)\int_0^\infty dv\ \exp\big[-2\alpha(8x^2 - 8xu\cos\theta + 3u^2 + 3v^2)\big]$$
$$\times \left\{ -\frac{C_s}{4\alpha}uv^2\big[uU_1(u)\big]\big[vU_1(v)\big]\left[\frac{1}{uv^3}\Big(A^{+\prime}(\xi) - A^{-\prime}(\xi)\Big)\right]_{\xi=1} - \frac{C_p}{3}(uv)^2 U_0(u)U_0(v)\right.$$
$$\times \left[\frac{1}{v}\Big(A^{+}(\xi) - A^{-}(\xi)\Big)\right]_{\xi=1} - \frac{C_p}{(4\alpha)^2}\big[u^2U_2(u)\big]\big[v^2U_2(v)\big]\left[\frac{1}{u^2v^3}\Big(A^{+\prime\prime}(\xi) - A^{-\prime\prime}(\xi)\Big)\right]_{\xi=1}$$
$$\left. + \frac{C_p}{3}\big[u^2U_2(u)\big]\big[v^2U_2(v)\big]\left[\frac{1}{v}\Big(A^{+}(\xi) - A^{-}(\xi)\Big)\right]_{\xi=1}\right\}, \tag{0.11}$$

where

$$A^{\pm}(\xi) = \frac{e^{\pm 4\alpha v\sqrt{16x^2 + \xi^2u^2 + 8xu(-\cos\theta)}}}{\sqrt{16x^2 + \xi^2u^2 + 8xu(-\cos\theta)}}. \tag{0.12}$$

The effective three body $N\alpha$ potential $\bar{V}_{(3)}$ is shown in fig. 1. It is repulsive, and we expect its inclusion to decrease the curves of the phase shifts in [1].

As the three body force employed in this work [3] and the chiral potential [4] are built inside the same theoretical framework, we compare their results in fig. 2, that shows the ratio of $\bar{V}_{(3)}$ and the central component of the effective chiral potential $\bar{V}_{(2)ch}$ [1]. Asymptotically, this ratio has a smooth variation, since both potentials have the same exponential pattern, differing only by a polynomial in $r$.

In fig. 3 we display the $N\alpha$ phase shifts, as functions of the laboratory energy, for the waves $D_{5/2}$ and $H_{11/2}$. The dashed lines represent our previous results [1], and the continuous lines, the inclusion of $\bar{V}_{(3)}$. Figure 4 exhibits the change $\Delta\delta$ due to the latter component. This quantity depends more on the $NN$ potential for waves with low angular momentum $L$, and this dependence tends to disappear for large values of $L$. In order to estimate the size of the three-body effect, we have replaced $\bar{V}_{(3)}$ by $\epsilon\,\bar{V}_{(2)ch}$. For $\epsilon \approx 0.0485$ the differences $\Delta\delta$ for waves with large $L$ are well reproduced [5], giving a rough idea of the size of this effect.

In this work we have shown that three body force in peripheral $N\alpha$ scattering gives rise to a small effect, about 5% of the two body interaction. Therefore the general features of the curves obtained previously are preserved. This shows that the three body force does not spoil the ability of peripheral $N\alpha$ scattering to discriminate the mid range part of $NN$ potential.


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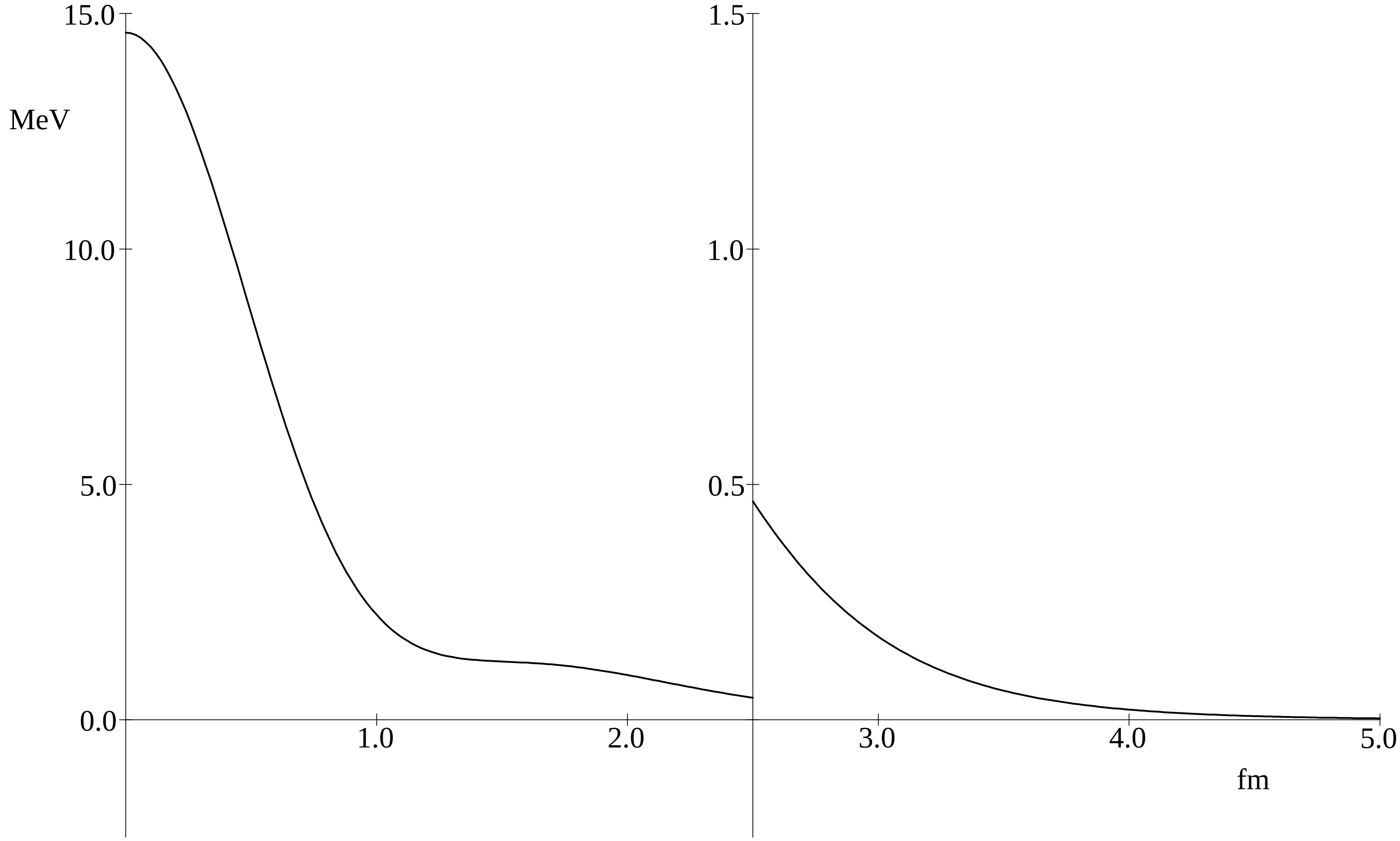
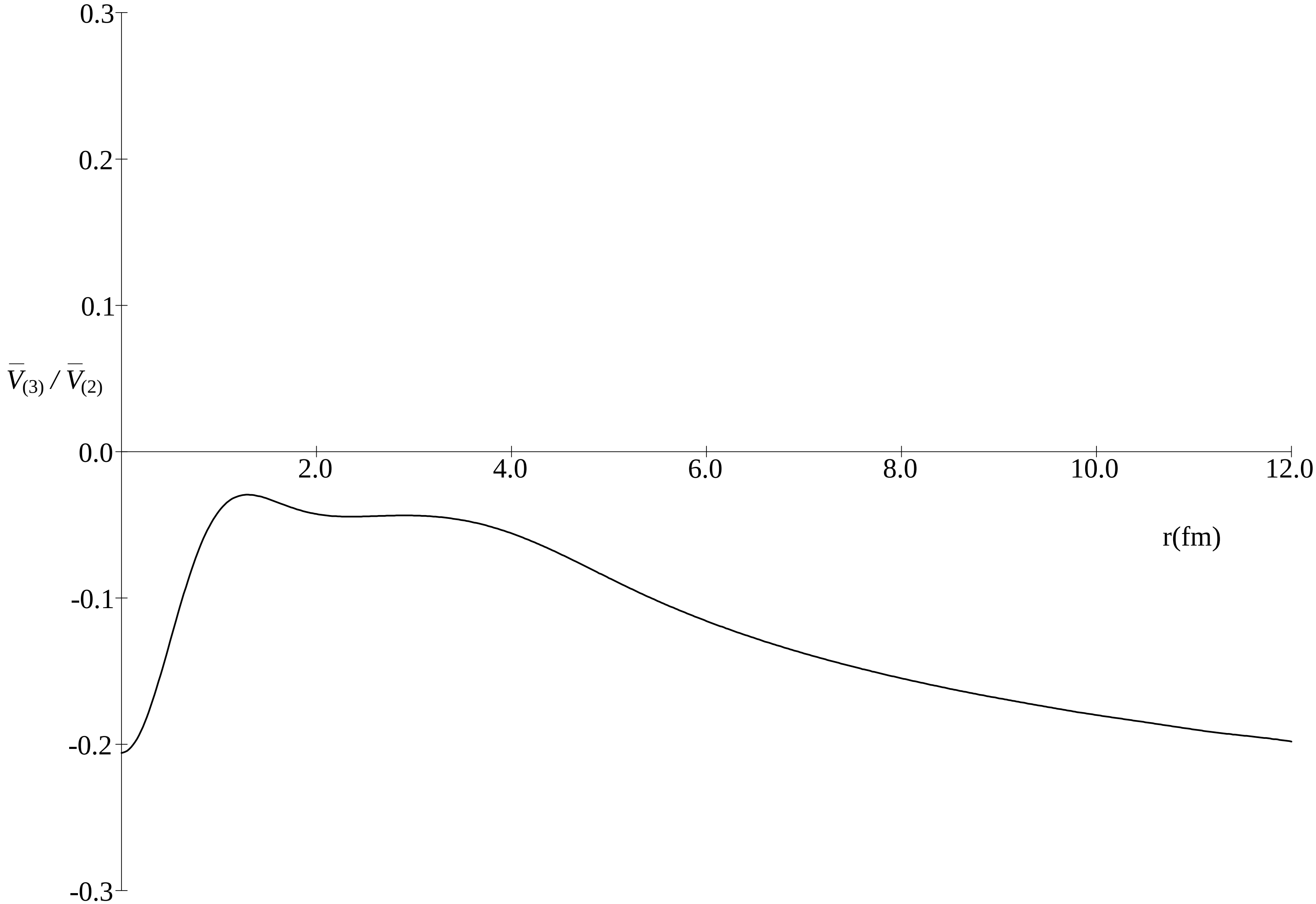
FIG. 1. The effective three body $N\alpha$ potential $\bar{V}_{(3)}(r)$.

FIG. 2. Ratio of $\bar{V}_{(3)}$ and the central component of the effective chiral potential $\bar{V}_{(2)ch}$.

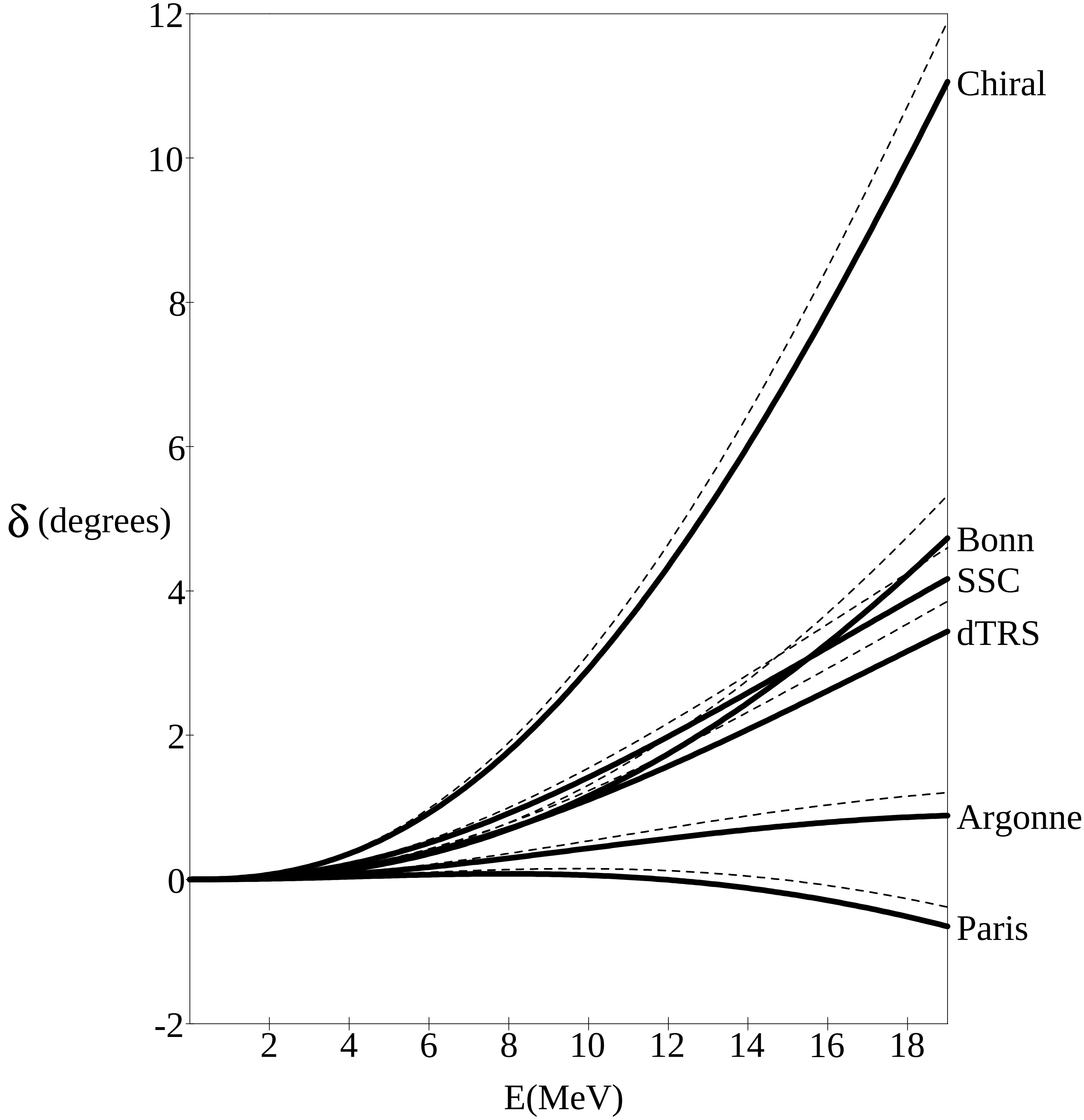
FIG. 3. $N\alpha$ phase shifts for the waves (a) $D_{5/2}$ and (b) $H_{11/2}$, as functions of the laboratory energy.

FIG. 4. Countribution of three body force to $N\alpha$ phase shifts, $\Delta\delta$, for the waves (a) $D_{5/2}$ and (b) $H_{11/2}$.


15.0
10.0
5.0
0.0
MeV
1.5
1.0
0.5
1.0
2.0
3.0
4.0
5.0
fm


0.3
0.2
0.1
$\bar{V}_{(3)} / \bar{V}_{(2)}$
0.0
-0.1
-0.2
-0.3
2.0
4.0
6.0
8.0
10.0
12.0
r(fm)

(L = 2 ; J = 5/2)



12
10
8
6
4
2
0
-2
δ (degrees)
2
4
6
8
10
12
14
16
18
E(MeV)
Chiral
Bonn
SSC
dTRS
Argonne
Paris


Fig.3a

(L = 5 ; J = 11/2)

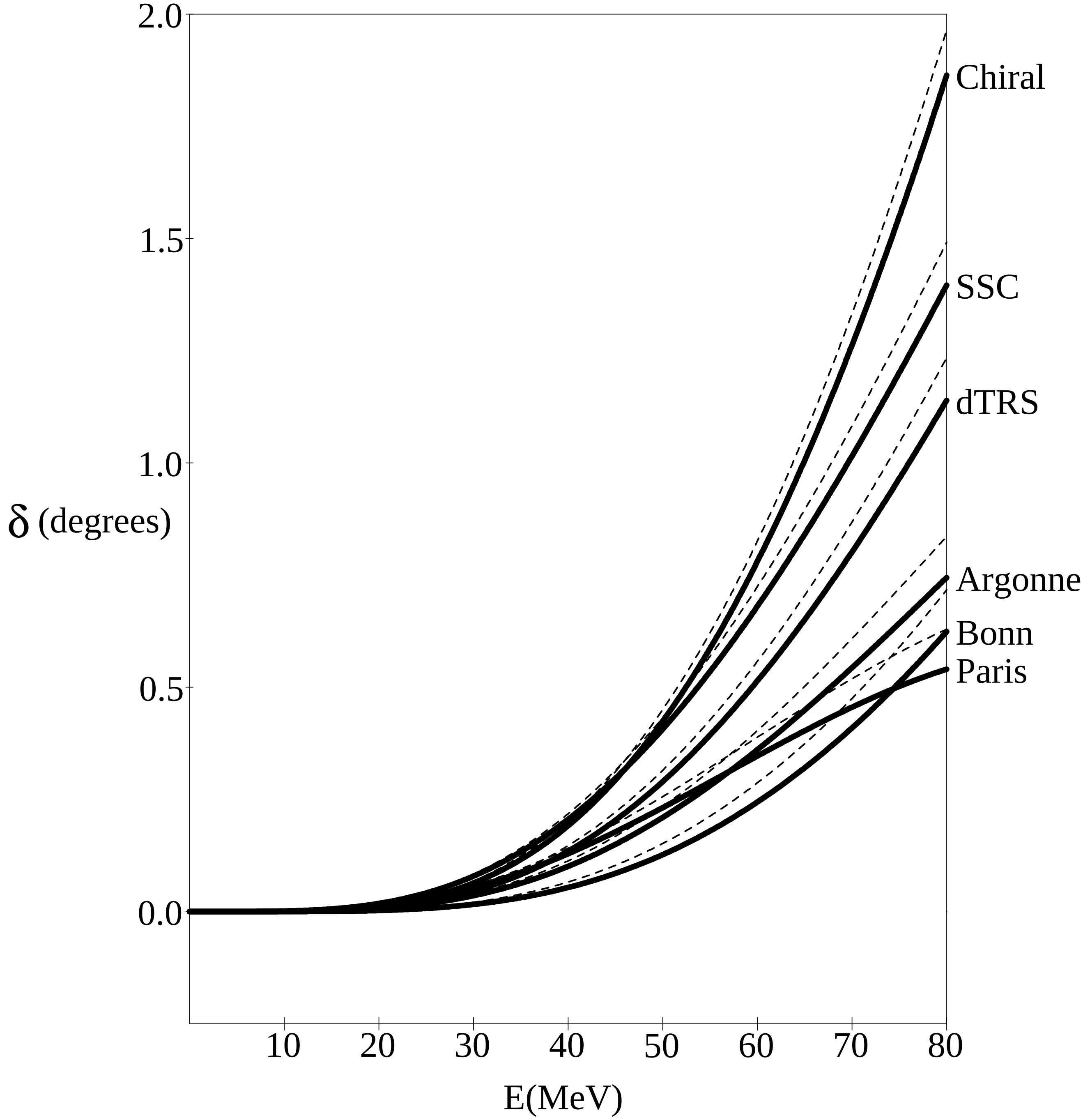



Fig.3b

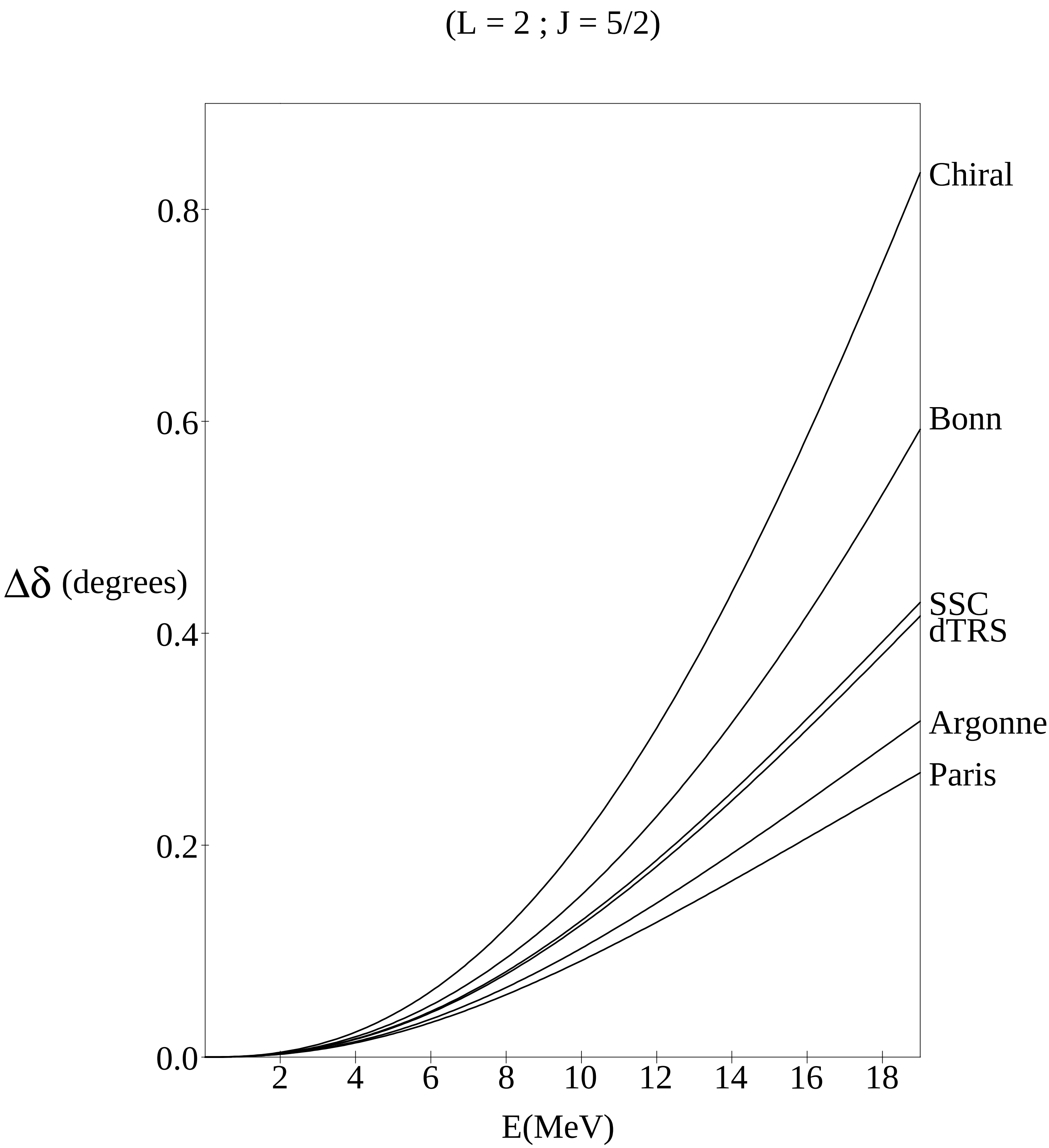

(L = 2 ; J = 5/2)
Chiral
Bonn
SSC
dTRS
Argonne
Paris
Δδ (degrees)
0.0
0.2
0.4
0.6
0.8
2
4
6
8
10
12
14
16
18
E(MeV)

Fig.4a

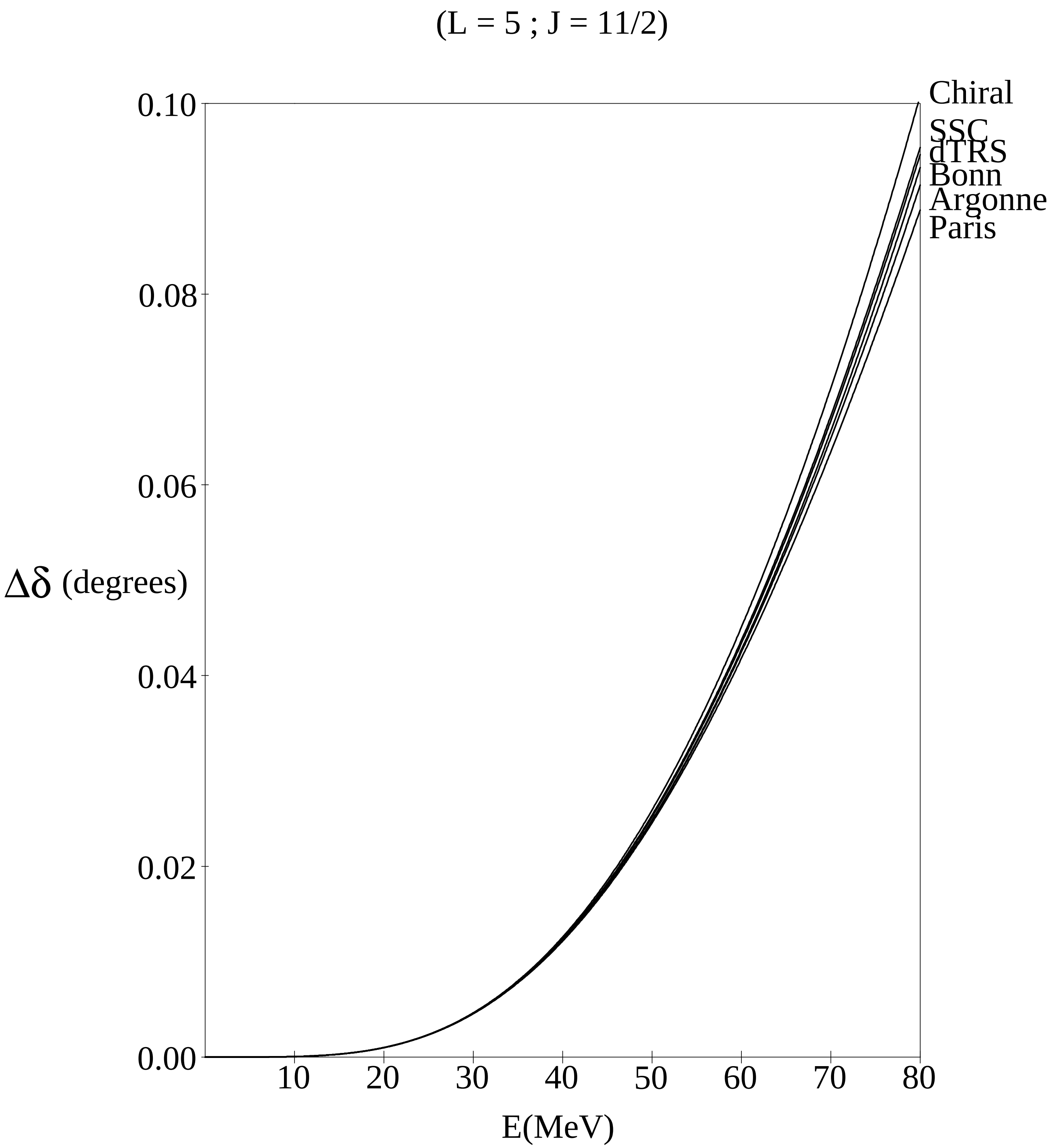

(L = 5 ; J = 11/2)
0.10
0.08
0.06
0.04
0.02
0.00
Δδ (degrees)
10
20
30
40
50
60
70
80
E(MeV)
Chiral
SSC
dTRS
Bonn
Argonne
Paris

Fig.4b