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Three body force in peripheral $N\alpha$ scattering

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The exchange of a single pion does not contribute to $N\alpha$ scattering, due to the isoscalar nature of the target. Therefore peripheral $N\alpha$ scattering may be used to probe existing models for the two-pion exchange component of the NN potential. In a recent work we have considered this possibility using an effective $N\alpha$ interaction based on two-body forces only. In the present communication we discuss the role played by the two-pion exchange three-nucleon force, since its range is comparable to the two-pion exchange potential (TPEP).

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As in [1], we assume the α ground state to be a S wave with a gaussian structure:

$$|\alpha\rangle = \left(\frac{4\alpha}{\pi}\right)^{9/4} \exp\left(-2\alpha \sum_{i=1}^3 \rho_i^2\right) |\chi_\alpha\rangle, \quad (0.1)$$

where ρ_i are Jacobi coordinates, α is a parameter extracted from [2] and $|\chi_\alpha\rangle$ is the spin-isospin α wave function. The effective $N\alpha$ potential is given by

$$\begin{aligned} \bar{V} &= \bar{V}_{(2)} + \bar{V}_{(3)} \\ &= \langle\alpha| \sum_{i=1}^4 V_{oi} |\alpha\rangle + \langle\alpha| \sum_{j>i=1}^4 \left[V_{oij} + V_{ijo} + V_{joi} \right] |\alpha\rangle, \end{aligned} \quad (0.2)$$

where $\bar{V}_{(2)}$ and $\bar{V}_{(3)}$ are due to two and three body forces respectively. The former was considered in [1], and here we concentrate on the latter.

The two-pion exchange three-nucleon force involves the emission and absorption of virtual pions by nucleons i and j , with an intermediate scattering by nucleon k . We use here the three body force derived in ref. [3], where the intermediate πN scattering is described in the framework of chiral symmetry, and given by

$$\begin{aligned} V_{ijk}(\mathbf{r}_{ki}, \mathbf{r}_{jk}) &= \left\{ \left(\frac{C_s}{\mu^2}\right) (\boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)}) (\boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\nabla}_{ki}) (\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\nabla}_{jk}) \right. \\ &+ \left(\frac{C_p}{\mu^4}\right) (\boldsymbol{\tau}^{(i)} \cdot \boldsymbol{\tau}^{(j)}) (\boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\nabla}_{ki}) (\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\nabla}_{jk}) (\boldsymbol{\nabla}_{ki} \cdot \boldsymbol{\nabla}_{jk}) \\ &\left. - \left(\frac{C'_p}{\mu^4}\right) (\boldsymbol{\tau}^{(i)} \times \boldsymbol{\tau}^{(j)} \cdot \boldsymbol{\tau}^{(k)}) (\boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\nabla}_{ki}) (\boldsymbol{\sigma}^{(j)} \cdot \boldsymbol{\nabla}_{jk}) (\boldsymbol{\sigma}^{(k)} \cdot \boldsymbol{\nabla}_{ki} \times \boldsymbol{\nabla}_{jk}) \right\} \\ &\times U(r_{ki})U(r_{jk}). \end{aligned} \quad (0.3)$$

In this result the function $U(x)$ is written as

$$U(x) = \frac{e^{-\mu x}}{\mu x} - \frac{\Lambda}{\mu} \frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2} \frac{\mu}{\Lambda} \left(\frac{\Lambda^2}{\mu^2} - 1\right) e^{-\Lambda x} \quad (0.4)$$

and the strenght coefficients have the numerical values $C_s = 0.9204$, $C_p = -2.0082$ and $C'_p = -0.6722$.

Due to the isoscalar nature of the α particle, the exchange of a single pion is not allowed and only V_{ijo} , which represents the external exchange of two pions, contributes to $\bar{V}_{(3)}$. It can be shown that the matrix element $\langle\chi_\alpha| \sigma_a^{(i)} \sigma_b^{(j)} \tau_c^{(i)} \tau_d^{(j)} |\chi_\alpha\rangle$ vanishes for $a \neq b$ or $c \neq d$, and is equal to $-1/3$ for $a = b$ and $c = d$. Therefore, the term in $\bar{V}_{(3)}$ proportional to C'_p drops out and the remainder may be expressed as

$$\bar{V}_{(3)}(x) = \sum_{j>i=1}^4 \left[C_s I_{11} - \frac{C_p}{3} (I_{00} + 3I_{22} - I_{20}) \right], \quad (0.5)$$

where

$$I_{mn} = 16\sqrt{2} \left(\frac{2\alpha}{\pi} \right)^3 \int d\mathbf{u} d\mathbf{v} \frac{U_m(u) U_m(v)}{u^n v^n} (\mathbf{u} \cdot \mathbf{v})^n \times \exp \left\{ -2\alpha [3(\mathbf{x} - \mathbf{u})^2 + 3(\mathbf{x} - \mathbf{v})^2 + 2(\mathbf{x} - \mathbf{u}) \cdot (\mathbf{x} - \mathbf{v})] \right\}, \quad (0.6)$$

$$U_0(x) = \frac{e^{-\mu x}}{\mu x} - \frac{\Lambda}{\mu} \frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2} \frac{\Lambda}{\mu} \left(\frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda x}, \quad (0.7)$$

$$U_1(x) = -\frac{e^{-\mu x}}{\mu x} \left(1 + \frac{1}{\mu x} \right) + \frac{\Lambda^2}{\mu^2} \left(1 + \frac{1}{\Lambda x} \right) \frac{e^{-\Lambda x}}{\Lambda x} + \frac{1}{2} \left(\frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda x}, \quad (0.8)$$

$$U_2(x) = \frac{e^{-\mu x}}{\mu x} \left(1 + \frac{3}{\mu x} + \frac{3}{\mu^2 x^2} \right) - \frac{\Lambda^3}{\mu^3} \left(1 + \frac{3}{\Lambda x} + \frac{3}{\Lambda^2 x^2} \right) \frac{e^{-\Lambda x}}{\Lambda x} - \frac{1}{2} \frac{\Lambda}{\mu} \left(\frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda x} \left(1 + \frac{1}{\Lambda x} \right). \quad (0.9)$$

Using

$$(\mathbf{u} \cdot \mathbf{v})^n \exp[+4\alpha(4\mathbf{x} - \mathbf{u}) \cdot \mathbf{v}] = \left(-\frac{1}{4\alpha} \frac{d}{d\xi} \right)^n \Big|_{\xi=1} \exp[+4\alpha(4\mathbf{x} - \xi \mathbf{u}) \cdot \mathbf{v}] \quad (0.10)$$

and a little more algebra, we get our final expression for the effective potential due to three-body interaction

$$\begin{aligned} \bar{V}_{(3)}(x) = & 6 \left(\frac{128\alpha^2\sqrt{2}}{\pi} \right) \int_0^\infty du \int_{-1}^1 d(-\cos\theta) \int_0^\infty dv \exp[-2\alpha(8x^2 - 8xu\cos\theta + 3u^2 + 3v^2)] \\ & \times \left\{ -\frac{C_s}{4\alpha} uv^2 [uU_1(u)] [vU_1(v)] \left[\frac{1}{uv^3} (A^{+'}(\xi) - A^{-'}(\xi)) \right]_{\xi=1} - \frac{C_p}{3} (uv)^2 U_0(u)U_0(v) \right. \\ & \times \left[\frac{1}{v} (A^+(\xi) - A^-(\xi)) \right]_{\xi=1} - \frac{C_p}{(4\alpha)^2} [u^2U_2(u)] [v^2U_2(v)] \left[\frac{1}{u^2v^3} (A^{+''}(\xi) - A^{-''}(\xi)) \right]_{\xi=1} \\ & \left. + \frac{C_p}{3} [u^2U_2(u)] [v^2U_2(v)] \left[\frac{1}{v} (A^+(\xi) - A^-(\xi)) \right]_{\xi=1} \right\}, \quad (0.11) \end{aligned}$$

where

$$A^\pm(\xi) = \frac{e^{\pm 4\alpha v \sqrt{16x^2 + \xi^2 u^2 + 8xu(-\cos\theta)}}}{\sqrt{16x^2 + \xi^2 u^2 + 8xu(-\cos\theta)}}. \quad (0.12)$$

The effective three body $N\alpha$ potential $\bar{V}_{(3)}$ is shown in fig. 1. It is repulsive, and we expect its inclusion to decrease the curves of the phase shifts in [1].

As the three body force employed in this work [3] and the chiral potential [4] are built inside the same theoretical framework, we compare their results in fig. 2, that shows the ratio of $\bar{V}_{(3)}$ and the central component of the effective chiral potential $\bar{V}_{(2)ch}$ [1]. Asymptotically, this ratio has a smooth variation, since both potentials have the same exponential pattern, differing only by a polynomial in r .

In fig. 3 we display the $N\alpha$ phase shifts, as functions of the laboratory energy, for the waves $D_{5/2}$ and $H_{11/2}$. The dashed lines represent our previous results [1], and the continuous lines, the inclusion of $\bar{V}_{(3)}$. Figure 4 exhibits the change $\Delta\delta$ due to the latter component. This quantity depends more on the NN potential for waves with low angular momentum L , and this dependence tends to disappear for large values of L . In order to estimate the size of the three-body effect, we have replaced $\bar{V}_{(3)}$ by $\epsilon \bar{V}_{(2)ch}$. For $\epsilon \approx 0.0485$ the differences $\Delta\delta$ for waves with large L are well reproduced [5], giving a rough idea of the size of this effect.

In this work we have shown that three body force in peripheral $N\alpha$ scattering gives rise to a small effect, about 5% of the two body interaction. Therefore the general features of the curves obtained previously are preserved. This shows that the three body force does not spoil the ability of peripheral $N\alpha$ scattering to discriminate the mid range part of NN potential.

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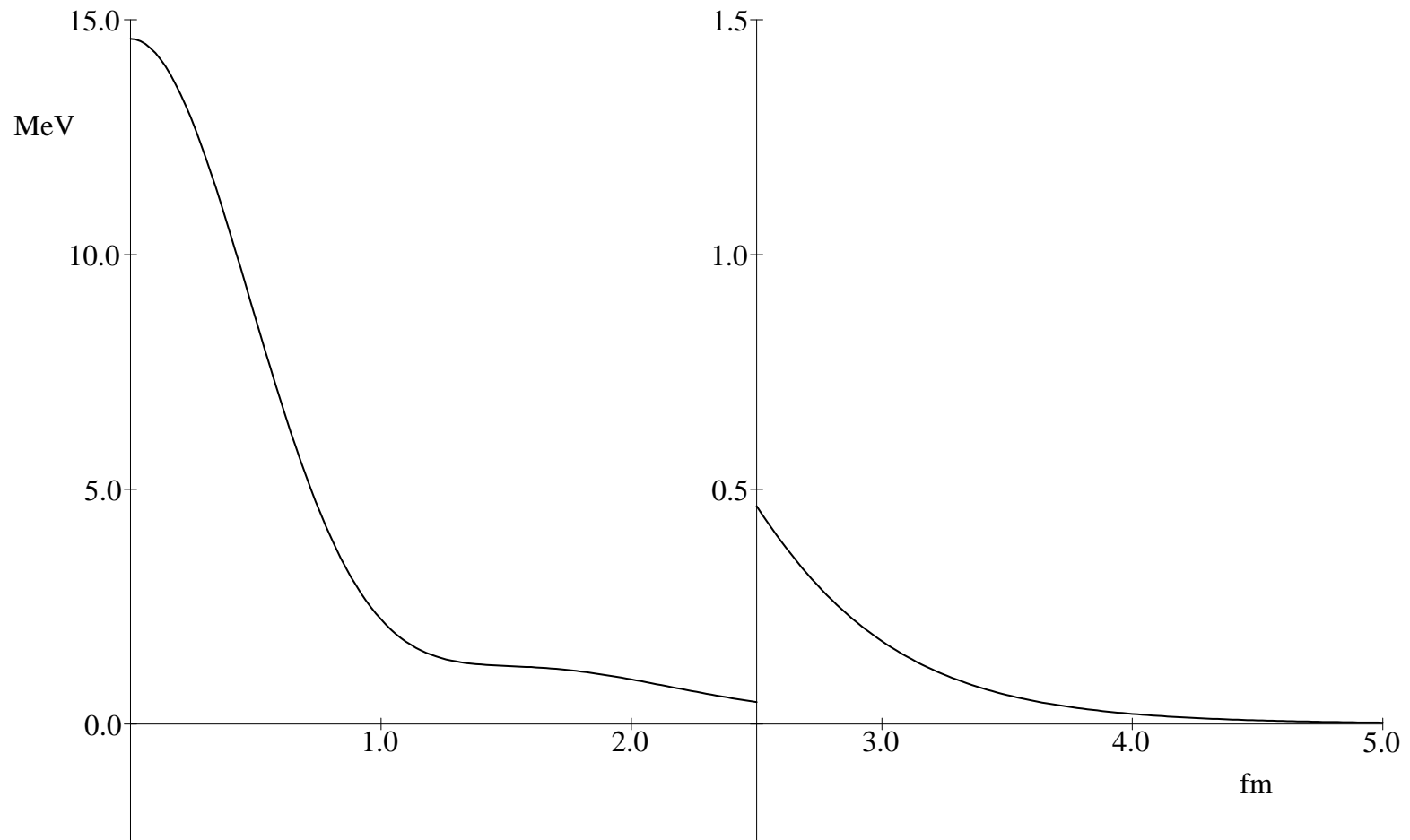
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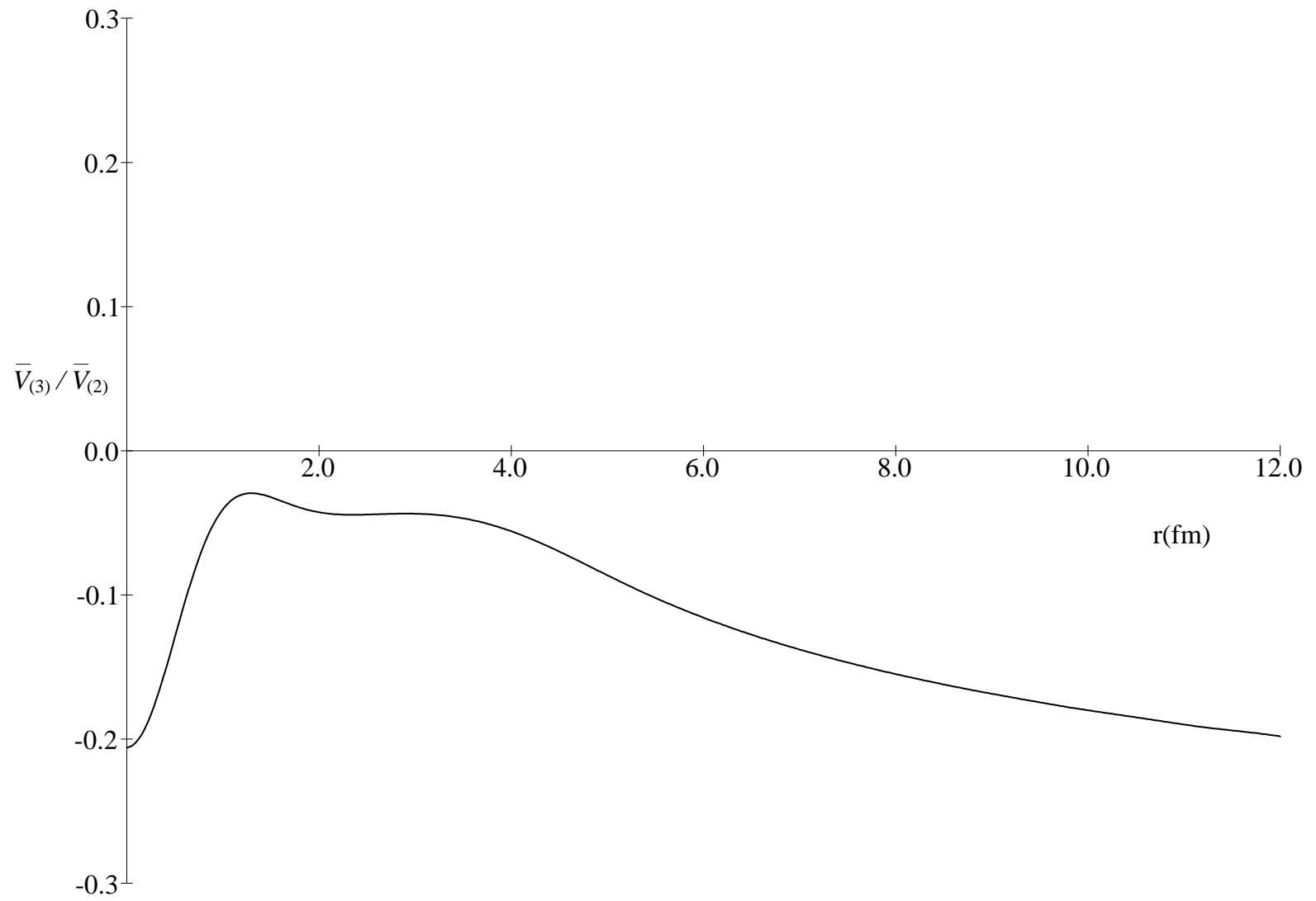
FIG. 1. The effective three body $N\alpha$ potential $\bar{V}_{(3)}(r)$.

FIG. 2. Ratio of $\bar{V}_{(3)}$ and the central component of the effective chiral potential $\bar{V}_{(2)ch}$.

FIG. 3. $N\alpha$ phase shifts for the waves (a) $D_{5/2}$ and (b) $H_{11/2}$, as functions of the laboratory energy.

FIG. 4. Contribution of three body force to $N\alpha$ phase shifts, $\Delta\delta$, for the waves (a) $D_{5/2}$ and (b) $H_{11/2}$.





(L = 2 ; J = 5/2)

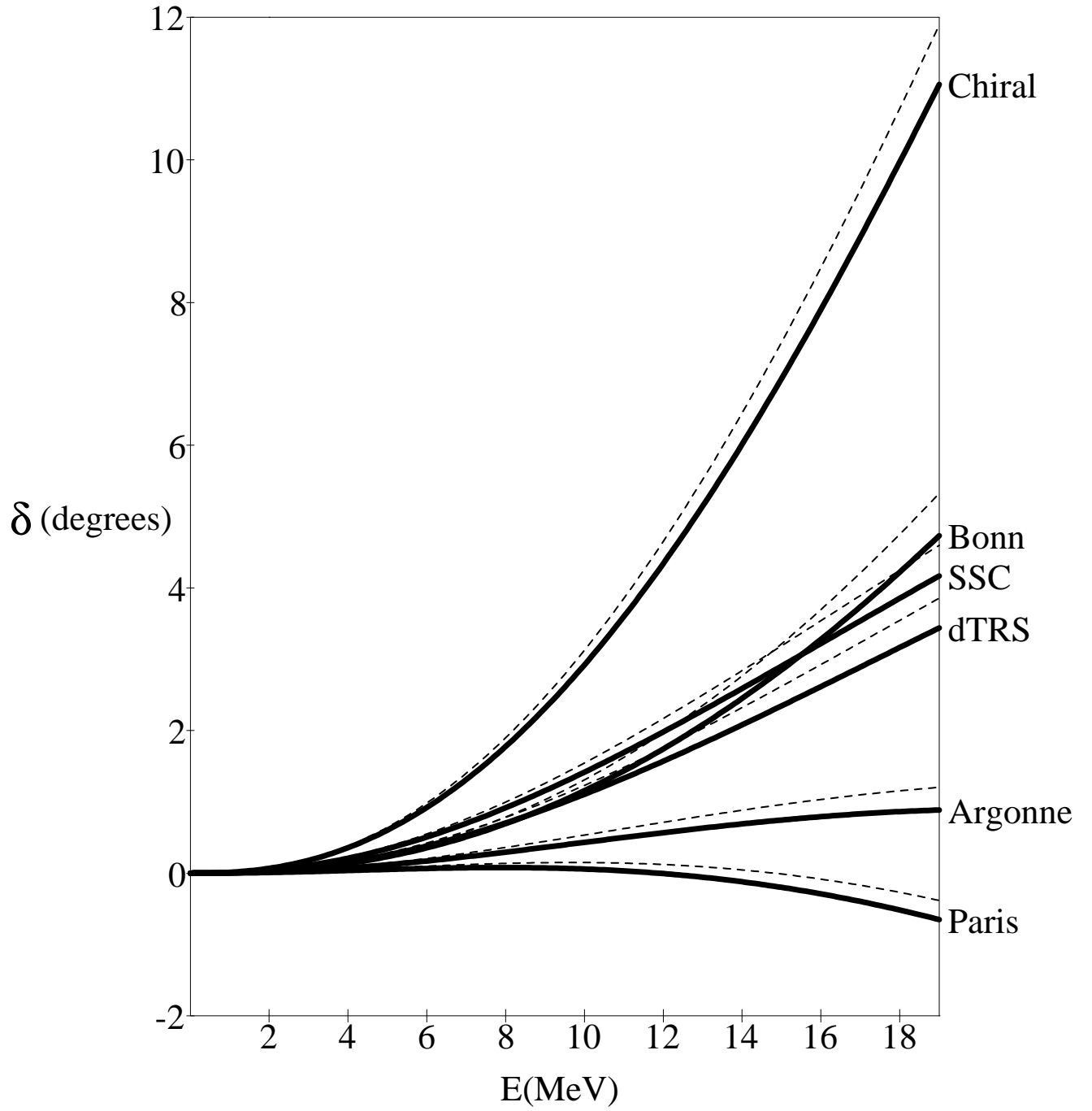


Fig.3a

(L = 5 ; J = 11/2)

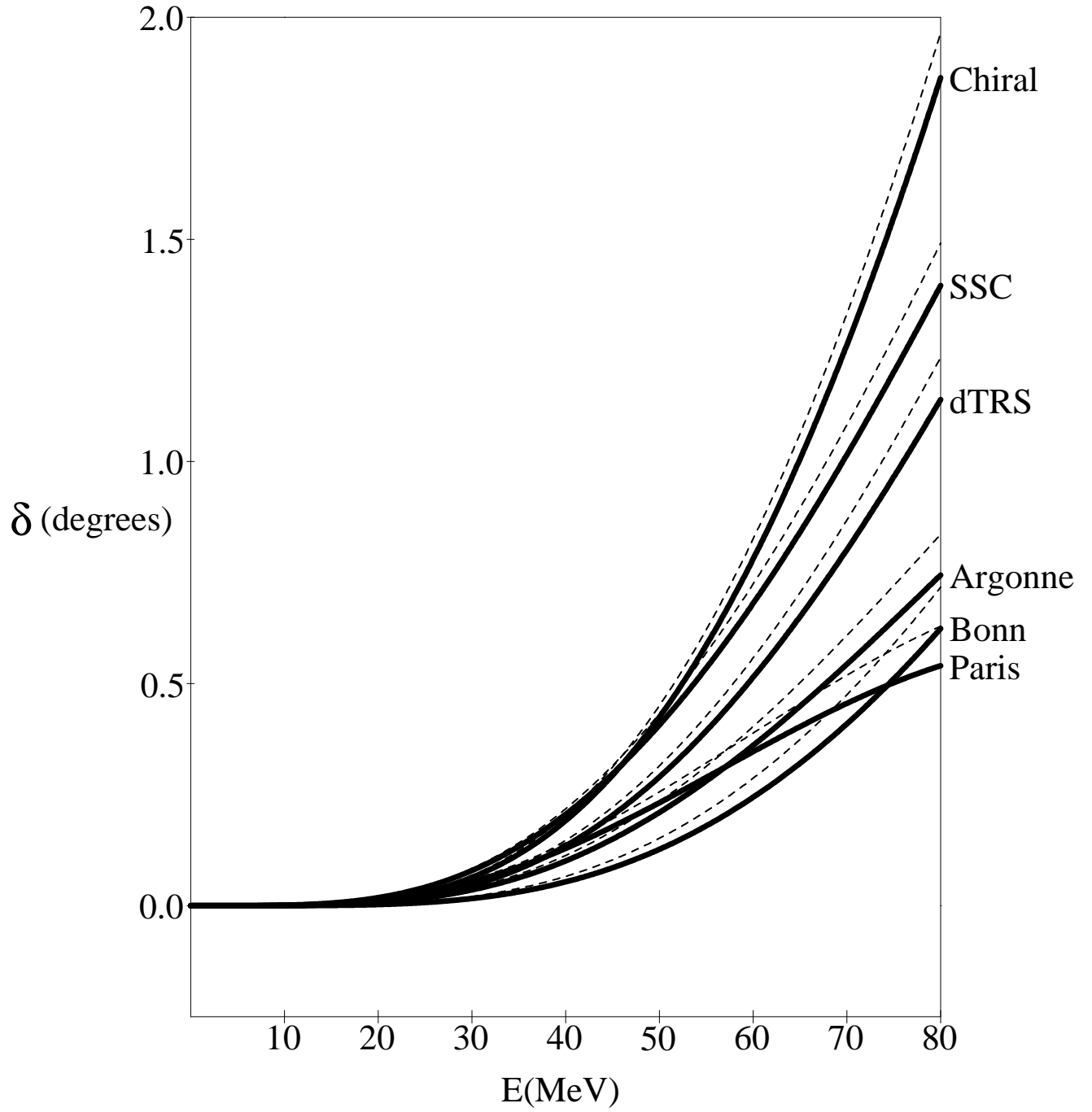


Fig.3b

(L = 2 ; J = 5/2)

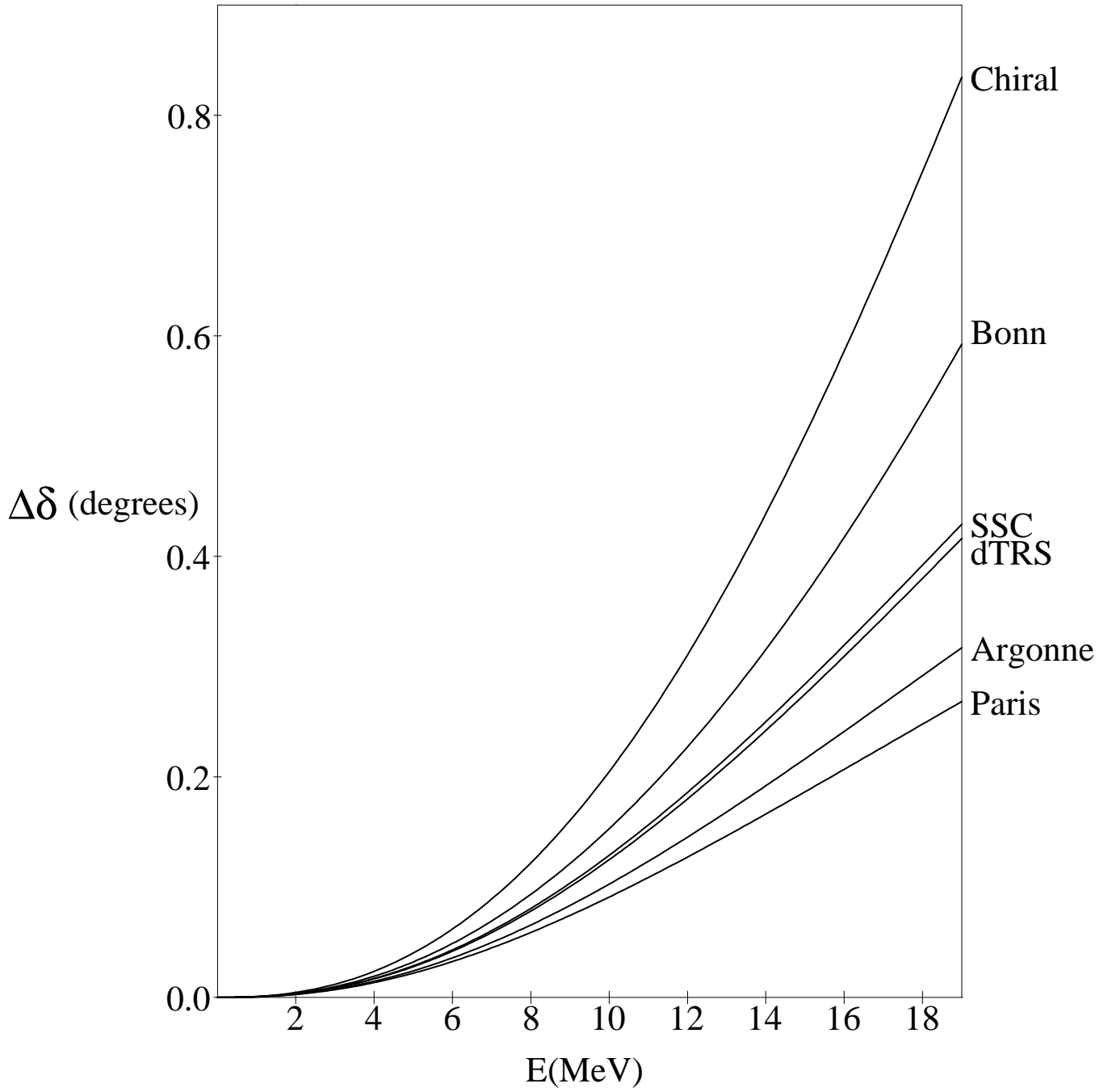


Fig.4a

(L = 5 ; J = 11/2)

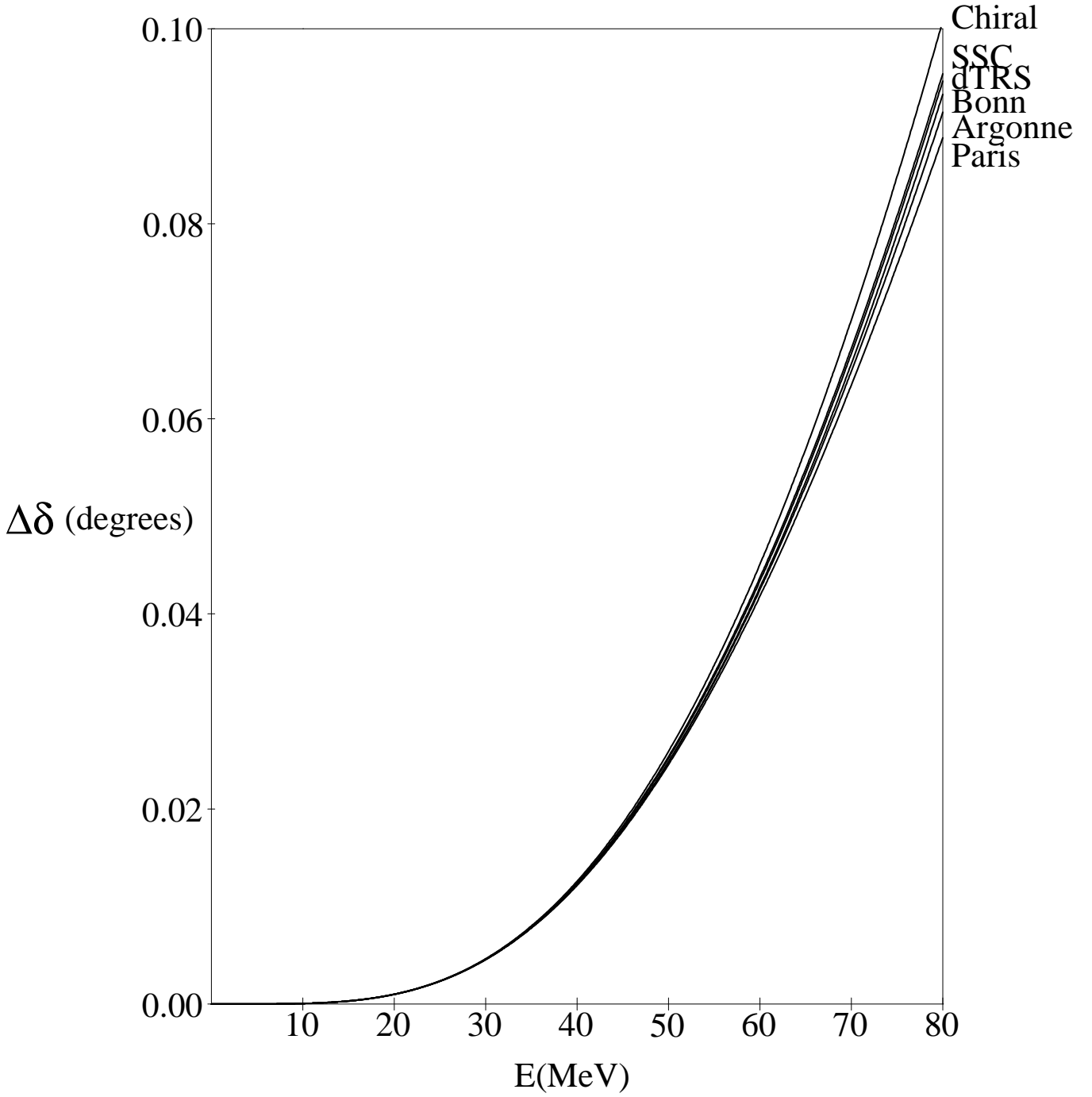


Fig.4b