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# Symmetry energy coefficients of hot asymmetric nuclear matter

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## Symmetry energy coefficients of hot asymmetric nuclear matter

### Abstract

Generalized symmetry energy coefficients of nuclear matter at finite temperature, variable densities and in asymmetric nuclear matter are studied in detail. The use of different Skyrme-type effective forces allows us to obtain analytical expressions for these parameters and reproduce alternative fittings for nuclear observables. The dependence of the generalized symmetry energy coefficients on the density, only from .5 up to  $2 \rho_0$ , temperature and neutron-proton (n-p) -explicit-asymmetry are investigated and it is compared to other results obtained for the n-p symmetry energy coefficient in the isovector channel. Some consequences for the nuclear and neutron matter structure and dynamics are discussed, among them the possibility of diverse states of nucleonic matter are found for certain Skyrme forces.

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# 1 Introduction

The symmetry energy coefficients and their density dependence are of great importance for the studies of exotic nuclei structure [1, 2], for the giant resonances, for example in [3] for the breathing mode, and nuclear heavy ion collisions at intermediary and high energies [4, 5]. We argue, as done in [6] that there is a sounder physical basis for several nuclear phenomena based on other parametrization. The generalized symmetry energy coefficients (or screening functions) are some of these parameters. For nuclei, proto-neutron and neutron stars it is important to extract the symmetry coefficients at different temperatures, densities and n-p asymmetries as it will be discussed latter. Preliminary results concerning the variation of some symmetry energy coefficients are presented in [6, 7]. In the present article we make a quite complete analysis of them.

Among the main parameters of the macroscopic mass formulae is the the symmetry energy coefficient. The neutron-proton (n-p) symmetry energy coefficient of nuclear matter ( $a_\tau$ ) represents the tendency of nuclear forces to have greater binding energies  $E$  for a symmetric system (equal number of protons and neutrons). It contributes as a coefficient for the squared neutron-proton asymmetry:

$$E/A = H_0(A, Z) + a_\tau(N - Z)^2/A^2, \quad (1)$$

where  $H_0$  does not depend on the asymmetry,  $Z$ ,  $N$  and  $A$  are the proton, neutron and mass numbers respectively. Expression (1) is obtained from the Fermi gas model [8] as well as from microscopic calculations [9] but it is well known that there are important corrections due to nucleon-nucleon interactions and correlations. Higher orders effects of the asymmetry (proportional to  $(N - Z)^n$  for  $n \neq 2$ ,  $n=1, 3$  for the breaking of isospin symmetry term and other corrections [10]) are expected, in principle, to be less important for the equation of state (EOS) of nuclear matter [11, 12] based on such parametrizations.

The parameter which measures the response of the system to a perturbation which tends to separate protons from neutrons is given by the static polarizability of the system (the isospin screening function) which “coincides” with the symmetry energy coefficient at zero temperature symmetric nuclear matter at the saturation density. However this cost in energy should depend on the asymmetry of the medium. This is reasonable since the nuclear potential and dynamics should be self consistent. Other (symmetry coefficients) are also defined in nuclear matter, for instance the spin ( $a_\sigma$ ) and spin-isovector ( $a_{\sigma\tau}$ ). The former corresponds to the energy difference between the unpolarized and polarized nuclear matter while the latter to the completely symmetric unpolarized and asymmetric polarized neutron matter.

The use of Skyrme forces allows us to derive analytical expressions for the generalized symmetry energy coefficients in the isovector, spin, spin-isovector and scalar channels of the particle-hole (p-h) interaction. In this work the Coulomb interaction between protons is not considered explicitly.

For the purpose of comparison we quote one work in which the density dependence of the isospin symmetry energy coefficient worked out [13]. In this reference the relevance of  $a_\tau$  for the equation of state (EOS) of dense neutron stars was studied with an effective form for the nuclear interaction. In that work the following parametrization was obtained:

$$a_\tau(\rho) = S(\rho) = (2^{\frac{2}{3}} - 1) \frac{3}{5} E_F^0 (u^{\frac{2}{3}} - F(u)) + S_0 F(u), \quad (2)$$

where  $u = \rho/\rho_0$ ,  $\rho_0$  being the saturation density of nuclear matter,  $E_F^0$  is the Fermi energy of nuclear matter,  $S_0$  may be taken as 30 MeV and  $F(u)$  is a generic function which satisfies the condition  $F(1) = 1$ . Three cases of  $F(u)$  will be compared to our results later. With microscopic non relativistic approaches,  $a_\tau$  is found to vary linearly with the density until  $\rho \simeq 2.5\rho_0$  and to saturate at higher densities. However, this behaviour may depend on the nucleon-nucleon potential [9, 14, 15]

In this paper and in [6] we suggest that the nuclear matter screening functions are more suitable than, for example,  $a_\tau$  (or the nuclear incompressibility  $K$ ) for the description of nuclear processes. Several Skyrme functions are used in order to assess the possible behaviour of these functions. These functions take into account corrections to the self consistent mean field. These correlations seem to originate the Spontaneously Symmetry Breakdowns (SSB) which produce Goldstone bosons. They are likely to be the Isovector (and Spin-Isovector) Giant Dipole Resonances [16, 17, 18, 19, 6]. The SSB may be related to the Chiral SSB which is present in Nature [20]. Besides that, they provide a different conceptual frame for the construction of macroscopic models of nuclei which are located along and beyond of the drip line and nuclear matter. Another behaviour occurs in the scalar channel: a symmetry coefficient (related to the nuclear matter incompressibility) will be associated to the static polarizability and may be directly related to the Isoscalar Dipole Giant Resonance [21]. It provides a direct check of the stability of nuclear matter with respect to scalar dipolar density fluctuations. Another aim of this article is to show that different (realistic) Skyrme-type forces provide different results depending on the phenomena under investigation. Therefore we are led to conclude that, diverse Skyrme interactions may be valid for different ranges of the system parameters. The parameters of one of them (SLyb) were fitted from results of neutron matter properties obtained from microscopic calculations in [22]. Other forces (SkSC4, SkSC6 and SkSC10), which have slightly different density

dependencies, had their parameters fixed by adjusting a large amount of nuclear masses yielding the same results of the Extended Thomas Fermi method and shell corrections calculated by the Strutinsky-integral method [23, 24].

## 2 Generalized Symmetry Energy Coefficients

The static nuclear matter screening functions will be investigated in the following. It is interesting to review and to extend a qualitative argument from [6, 18] for exploring them.

### 2.1 General Remarks

Let us consider a small amplitude ( $\epsilon$ ) external perturbation which acts, through the third Pauli isospin matrix  $\tau_3$ , in nuclear matter separating nucleons with isospin up and down <sup>1</sup>. This originates a fluctuation  $\delta\rho = \rho_n - \rho_p$  of the nucleon density. The energy of the system can be written as:

$$H = H_0 + a_\tau \frac{\delta\rho^2}{\rho} + \epsilon\delta\rho, \quad (3)$$

where  $a_\tau$  is the isospin symmetry coefficient. In the equilibrium:  $\delta H/\delta\rho = 0$ . The resulting static polarizability (now generalizing for any channel as done in [19, 7, 16] with (s,t) for spin, isospin numbers) yields the static screening function. In the static limit ( $\omega \rightarrow 0$ ) for zero momentum transfer, i.e., in an homogeneous system, it is given by:

$$\Pi^{s,t}(\omega \rightarrow 0, q \rightarrow 0, T) \equiv \frac{\delta\rho_{s,t}}{\epsilon_{s,t}} = -\frac{\rho_0}{2A_{s,t}}. \quad (4)$$

This expression corresponds to the static limit of the response function of symmetric nuclear matter. For the isospin channel  $A_{0,1} = a_\tau$ . It is very interesting to note that the spin asymmetry in nuclei may also generate a variation on the binding energy, though it is a small contribution.

Now let us consider asymmetric hot nuclear matter. In the usual parametrizations, there are several effects depend on the induced asymmetry  $\delta\rho = \rho_n - \rho_p \equiv \beta$ . One can thus extend expression (4) to include a more general dependence on the asymmetry: our approach consists in considering a general parameter  $\mathcal{A}(\beta)$  instead of the energy symmetry coefficient as usually defined in mass formulas of the type of (3). The same external perturbation (in the desired channel) is introduced as was shown

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<sup>1</sup>This argument is valid for all the four channels. It is enough to consider other external perturbations:  $\tau_3$ ,  $\sigma_3\tau_3$  and  $\mathbf{1}$  for the isospin, spin-isospin and scalar channels respectively.

before:

$$H = H_0 + \mathcal{A}^{0,1}(\beta) \frac{\beta^2}{\rho} + \epsilon\beta. \quad (5)$$

In equilibrium, the binding energy is minimized with respect to  $\beta$ . The corresponding fluctuation is obtained from a quadratic equation for  $\beta$  whose solutions are given by:

$$2\beta \frac{\delta \mathcal{A}^{0,1}}{\delta \beta} = -2\mathcal{A}^{0,1} \pm 2\mathcal{A}^{0,1} \sqrt{1 - \frac{\epsilon\rho}{\mathcal{A}^{(0,1)2}} \frac{\delta \mathcal{A}^{0,1}}{\delta \beta}} \quad (6)$$

In the limit of small amplitude  $\epsilon$  there are two analytical solutions for this equation. For the positive sign it is:

$$\Pi^{s=0,t=1} \equiv \frac{\beta}{\epsilon} = -\frac{\rho}{2\mathcal{A}^{0,1}}, \quad (7)$$

While the following differential equation appears for the negative sign:

$$\frac{\partial \mathcal{A}^{0,1}}{\partial \beta} \left( 2\beta - \frac{\epsilon\rho}{\mathcal{A}^{0,1}} \right) = -4\mathcal{A}^{0,1}. \quad (8)$$

This equation has the same solution of the first case as well as of symmetric nuclear matter:

$$\Pi^{s=0,t=1} \equiv \frac{\beta}{\epsilon} = -\frac{\rho}{2\mathcal{A}^{0,1}}. \quad (9)$$

These expressions (7,9) are defined as the static screening functions. These arguments are valid for any perturbation of the other channels for asymmetric nuclear matter in isospin as well as in spin yielding the functions  $\mathcal{A}^{s,t}$ .

A nearly exact expression for the dynamical polarizability of a non relativistic hot “dense” asymmetric nuclear matter was derived with Skyrme interactions in [7, 19]. The approximation consisted in (i) equating the asymmetry coefficient defined for the momentum density to the density asymmetry coefficient (the variation of the corresponding term yields differences in the response function of less than 1%), (ii) choosing a particular (well discussed and justified [19, 7]) prescription for the asymmetry density fluctuations. In the isovector-related channels collective modes were found which seem to be manifestation of SSB in nuclear systems, in finite nuclei they may be identified as the Isovector Dipole Giant Resonances. Observation of these phenomena in celestial bodies and in Relativistic Heavy Ions Collisions can be expected. In the present work the static limits are investigated in an intermediate range of the n-p asymmetry. The asymmetry in the n-p densities can be parametrized in terms of a coefficient  $b = \rho_n/\rho_p - 1$ . At zero temperature in the symmetric nuclear matter limit the generalized symmetry energy coefficients reduce to the conventional symmetry energy coefficients as:

$$a_\tau = A_{0,1}(T \rightarrow 0, b \rightarrow 0),$$

the n-p symmetry energy coefficient;

$$a_\sigma = A_{1,0}(T \rightarrow 0, b \rightarrow 0)$$

the spin symmetry energy coefficient;

$$a_{\sigma\tau} = A_{1,1}(T \rightarrow 0, b \rightarrow 0)$$

the spin-isospin symmetry energy coefficient;

$$K_D = A_{0,0}(T \rightarrow 0, b \rightarrow 0),$$

a ‘‘dipolar incompressibility’’ of nuclear matter [19, 7].

## 2.2 Screening functions with Skyrme forces

The general static screening functions  $A_{s,t}$  can be written as:

$$A_{s,t} = \frac{\rho}{N} \left\{ 1 + 2\overline{V_0^{s,t}} N_c + 6V_1^{s,t} M_p^* (\rho_c + \rho_d) + 12M_p^* V_1^{s,t} \overline{V_0^{s,t}} (N_c \rho_d - \rho_c N_d) + (V_1^{s,t})^2 (36(M_p^*)^2 \rho_c \rho_d - 16M_p^* M_c N_d) \right\}. \quad (10)$$

This expression is the main concern of the present article. The densities  $\rho_v$ ,  $N_v$  and  $M_v$  are given by:

$$\rho_v = v\rho_n + (1-v)\rho_p,$$

$$M_v = vM_n + (1-v)M_p,$$

$$N_v = vN_n + (1-v)N_p,$$

where  $v$  stands for n-p asymmetry coefficients ( $c, d$ ) defined below (a measure of the fraction of neutron density). The above densities are defined by:

$$(N_q, \rho_q, M_q) = \frac{2M_p^*}{\pi^2} \int df_q(k)(k, k^3, k^5).$$

In these expressions  $f_q(k)$  are the fermion occupation numbers for neutrons ( $q = n$ ) and protons ( $q = p$ ),  $\overline{V_0}$  and  $V_1$  are functions of the Skyrme forces parameters (see in [7]) and  $M_p^* = m_p^*/(1+a/2)$  is an effective mass for the proton. Besides that, the four asymmetry coefficients are:

$$a = \frac{m_p^*}{m_n^*} - 1, \quad b = \frac{\rho_{0n}}{\rho_{0p}} - 1, \quad c = \frac{1+b}{2+b}, \quad d = \frac{1}{1+(1+b)^{\frac{2}{3}}}. \quad (11)$$



(The coefficient  $b$  is related to a frequently used asymmetry coefficient:  $\alpha = (2\rho_{0n} - \rho_0)/\rho_0$ , by the expression:  $b = 2\alpha/(1 - \alpha)$ .)

As discussed in other works [7, 25] the parameters  $V_0$  are related to  $\bar{V}_0$  which are given in reference [7]. Together with  $V_1$ , they can be written in terms of the Landau parameters of the Fermi liquid theory for nuclear matter. The usual stability condition in each channel of the interaction is given by

$$J_0^{s,t} > -1. \quad (12)$$

where  $J_0^{s,t}$  stands for  $F_0, F'_0, G_0, G'_0$  respectively for the scalar ( $s = 0, t = 0$ ), isovector ( $s = 0, t = 1$ ), spin ( $s = 1, t = 0$ ) and spin-isovector ( $s = 1, t = 1$ ) channels [26]. These conditions are just the denominators of the response function of symmetric nuclear matter at zero temperature and at saturation density.

The symmetric nuclear matter limit yields, as discussed in [19, 7, 6], the symmetry energy coefficients of nuclear matter at finite temperature (T) and density:

$$2A_{s,t} = \frac{\rho}{N} \left\{ 1 + (J_0^{s,t} + J_1^{s,t})N + 3 \frac{J_1^{s,t} 2^{2/3}}{(3\pi^2 \rho)^{2/3}} m^* \rho - \frac{(J_1^{s,t})^2 2^{4/3}}{(3\pi^2 \rho)^{4/3}} \left( m^* N M - \frac{9}{4} (m^*)^2 \rho^2 \right) \right\}. \quad (13)$$

Therefore we obtained extended conditions for (asymmetric) (hot) nuclear matter stability with relation to the respective perturbation [6]:

$$A_{s,t} \geq 0. \quad (14)$$

The scalar screening function will be called as the dipolar incompressibility,  $K_D$ , and is related to the usual nuclear matter incompressibility,  $K_\infty$  by the following expression in terms of the SLy Skyrme force parameters:

$$K_D = K_\infty + \frac{4}{5} T_F - 2V_1 k_F^2 \rho_0 + \frac{3}{4} t_3 \rho_0^{\alpha+1}. \quad (15)$$

This relationship is slightly different for the SkSC interactions. At different densities, temperatures and n-p asymmetries the static screening functions have different values.

In [24] the EOS of a collapsing star is studied and the results for neutron matter favors forces for which  $a_\tau$  is reasonable greater than  $27MeV$ . Besides that the authors fit neutron star properties with the following parametrizations for the n-p energy symmetry at saturation density: SkSC4 ( $27MeV$ ), SkSC6 ( $30MeV$ ) and SkSC10 ( $32MeV$ ).

### 3 Results and discussion

In figure 1 we show the generalized isovector symmetry energy coefficient  $A_{0,1}$  as a function of the ratio of the density to the saturation density for Skyrme interactions SLyb [22], SkSC4, SkSC6 and SkSC10 [23, 24]. One strong characteristic of these last three forces, besides the discussed above, is the value of the effective mass which is kept to be equal to the free nucleon mass. This may be interpreted as a result of the coupling between particle modes and surface vibration modes [24]. Since Skyrme forces are not necessarily expected to describe physics at high densities we decided to investigate the dependence of the polarizabilities up to  $2\rho_0$ . Most of the isovector screening functions of figure 1 reach a maximum value when  $\rho_0 < \rho \leq 1.7\rho_0$  and then decrease. This last behaviour is typical from non relativistic calculations (as discussed in [6]).

There are two forces which yield “anomalous” behaviour: SkSC4 makes the slope of  $A_{0,1}$  much less negative reaching an instability point for smaller densities than the other forces. The point in which the symmetry energy is zero would correspond to a phase transition of nuclear matter to new state(s) of “nucleonic matter”. In this phase (at high densities, but low n-p asymmetries, as seen in Figures 2 and 3) the collective modes would eventually cease to exist and propagate and the corresponding “isospin-related” is restored. In [27, 6] we argued that these collective modes correspond to the Dipole Giant Resonances in Nuclei, in particular, in the isovector dipolar mode the resonances are observed in nuclei with high excitation energies. Their widths show sign of continuous increasing until disappear in spite of the saturation of its values at finite values have been claimed (when  $T \simeq 5 - 7MeV$ ) for values of the excitation energy (usually parametrized with a Temperature) below the fragmentation point. This range of temperature- which is also approximately that in which the collective modes would disappear in nuclear matter [16, 17, 18, 19, 7]- copes with the observed first order transition liquid-gas phase transition in nuclei [28].

The two above possibilities are contemplated with the Skyrme interactions which were used. In the case that the width saturates the behaviour of the symmetry energy for forces SLyb and SkSC6 (but probably not SkSC10) are more appropriate, but in the case the phase transition described occurs, the collective modes disappears. This is described, for example, with force SkSC4 at not very high densities or excitation energies. This last Skyrme force (SkSC4), by the way, leads to a collapse of neutron stars <sup>2</sup> [24]. On the other hand (and secondly) SkSC10 presents a behaviour more compatible

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<sup>2</sup>This indicates the necessity of taking into account other degrees of freedom than those used in the non relativistic

with the results of the parametrizations of expression (2) which was taken from [13] (circles, squares, rotated squares). Although the usual Skyrme forces are not well suited for high densities there is a trend of a decrease of this screening function (symmetry energy coefficient) until a phase transition in a dense nuclear system. In this figure it is also shown the n-p symmetry energy dependence on the density for force SLy<sub>b</sub> corresponding to the expression (13) without the last term proportional to  $V_1^2$  (thin dotted-dashed curve  $-a_\tau$  SLy). This term increases the attractive character of nuclear forces at higher densities (seen with  $A_{0,1}$  SLy thick dotted-dashed line).

In figure 2 the same parameter  $A_{0,1}$  is presented but for non zero n-p asymmetries, i.e., for  $b=.25$  and  $b=.54$ , this last corresponding to the asymmetry of the nucleus of  $^{208}Pb$ . The values of the generalized isovector symmetry energy coefficient increase with relation to the symmetric nuclear matter and the tendency of decreasing (with higher  $b$ ) for higher densities remains for several forces. Nevertheless for the forces SkSC6 (shown in figure 2) and SkSC10 (not shown) the isovector screening function has a higher slope and may eventually decrease for much higher densities (however in such range one can not expect these forces to reproduce necessarily the correct physics). The curves for increasing n-p asymmetry  $b$  is shown in figure 3. The n-p asymmetry corresponds to an explicit symmetry breaking of "Nuclear-Isospin" symmetry [6]. Again we note extremely different behaviours depending on the force. For the forces SLy<sub>b</sub> and SkSC4 the screening functions, as noted above, start increasing for higher densities and n-p asymmetries but for  $\rho = 2\rho_0$  the values are smaller and  $A_{0,1}$  seems to have the tendency to reach a maximum value for  $b \simeq 2$ . For the forces SkSC6 and, still more, for SkSC10 (not shown) the higher the density the higher the slope with  $b$ . The trend is to obtain more repulsive isovector forces.

The spin screening functions are plotted in figures 4 to 6. For increasing densities  $A_{1,0}$  may decrease until a minimum and then increase for  $\rho/\rho_0 > 1.6$  in the case of (SLy<sub>b</sub>) or decrease smoothly almost continuously for the force SGII (not shown) eventually reaching negative values for higher densities, which would make the matter undergoes a phase transition to a spin polarized state. In this figure it is also shown the spin symmetry energy dependence on the nuclear density for force SLy<sub>b</sub> corresponding to the expression (13) without the last term proportional to  $V_1^2$  ( $a_\sigma$  SLy). The issues of figure 4 are clear in figure 5. For the interactions SkSC4, SkSC6 and SkSC10 the spin- symmetry energy coefficients are already negative at low density and decrease still more for higher densities. This feat is clear in figure present description.

5 where the same screening function is shown for non zero n-p asymmetries. However one can see that the higher the asymmetry the less attractive is the interaction (less negative screening function), i.e., for neutron stars the spin channel seems to be repulsive in this channel. Figure 6 may be seen as a summary for these results. It is interesting to recall that, in spite of small at the saturation density, the spin-symmetry energy may be non zero for nuclear matter and finite nuclei. The way Nature functions concerning polarized nuclear matter also seems to be contemplated with Skyrme forces, either in the case the generalized spin-symmetry energy coefficient increases or decreases for higher densities. New analysis of experimental information is needed.

The spin-isovector screening function of symmetric matter with increasing density is shown in figure 7. All the forces which are shown exhibit  $A_{1,1}$  (and  $a_{\sigma\tau}$ ) as decreasing functions of the density becoming negative at  $\rho \simeq 1.8$  or  $2.1$ . This behaviour is expected to be associated with the pion condensation phase transition. In this figure it is also shown the spin-isospin symmetry energy dependence on the nuclear density for force SLy $\tau$  corresponding to the expression (13) without the last term proportional to  $V_1^2$  ( $a_{\sigma,\tau}$  SLy). The dynamical aspects are similar and analogous to the isovector channel, including the consequences for the finite nuclei. In figure 8 the same analysis holds for asymmetric matter (also  $b = .25$  and  $.54$ ). However with increasing asymmetry the instability tends to occur at higher densities for the force SLy $\tau$  while the opposite behaviour is seen for SkSC4 and SkSC6. This feature can be stressed by observing figure 9. In this figure the dependence of the spin-isospin screening function on the n-p asymmetry parameter  $b$  is shown for different nuclear matter densities (below, equal to and above the saturation density). It can be noted that for the force SLy $\tau$  the increasing n-p asymmetry makes the spin-isovector interaction continuously more repulsive instead of allow for attractiveness. Once again Skyrme interactions are able to reproduce roughly well almost any kind of realistic dependence of the (generalized) symmetry energy coefficients.

For the generalized scalar dipolar symmetry energy coefficient, figures 10 and 11 show that, roughly speaking, all the interactions are attractive for lower densities associated to instabilities which seems to be related to those found, for example, in [29]<sup>3</sup>. There is an exception for the force SkSC4. The interaction in this channel becomes more and more repulsive for higher densities. The increase of the n-p asymmetry make the interaction continuously more attractive (for lower densities) for most of the forces. In figure 10 it is also shown the dipole scalar symmetry energy dependence on the nuclear

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<sup>3</sup>These instabilities should be related to the problem of the saturation of strong nuclear interactions: it seems to exist a "critical density" under which nucleons in nuclear systems behave as free (or almost free) fermions.

density for force SLyb corresponding to the expression (13) without the last term proportional to  $V_1^2$  ( $K_D$  SLy). The features of figures 10 and 11 are stressed in figure 12. In this figure the parametrization for the n-p asymmetry dependence of the incompressibility of nuclear matter of reference [30] -from the equation of state- (not the incompressibility  $K_\infty$  itself, but its asymmetry dependence- for the case of SLyb) is compared with results for the scalar screening function ("dipolar incompressibility  $K_D$ "). This was also done in [6] to compare the qualitative behaviour as a function of the n-p asymmetry because otherwise we would be comparing different parameters. Their parametrization (using the n-p asymmetry parameter of our work) is given by:

$$K(b) = K_0(b=0)(1 - a'(b/(b+2))^2) \quad (16)$$

where  $a'$  (in [30]) is of the order of 1.28 up to 1.99 for several Skyrme interactions (zero temperature) and  $K(b)$  stands for  $K_\infty$  and  $K_{0,0}$ . On the other hand our curve (for  $\rho = \rho_0$  of SLyb) is well fitted (for  $b$  up to 8) by:  $K_D = 17.4 - 4.3b - 1.6b^2 + .07b^3$ . It is seen in figure 12 that this case (from [30]) present nearly the same behaviour (dependence on the n-p asymmetry  $b$  with  $\rho = \rho_0$ ) as our calculation for forces SLyb and SGII (not shown), a remarkable feature. In another work [24] it is found the same kind of dependence on the n-p asymmetry for the incompressibility of nuclear matter but with other numerical coefficients:

$$K(b) = K_v + K_s(b/(b+2))^2 \quad (17)$$

These parameters assume the following values for the forces used in that article:

$$\begin{aligned} SkSC4(K_v = 234.7MeV, K_s = -334.9MeV); & \quad SkSC10(K_v = 235.4MeV, K_s = -203.5MeV); \\ SkSC6(K_v = 235.8MeV, K_s = -136.6MeV). & \end{aligned} \quad (18)$$

These values are comparable to or a little smaller than those obtained in the figure 12 and expression (16). We expect that this dipolar incompressibility also could be of relevance for the study concerning fragmentation - besides the usual (monopolar) incompressibilities (volume and surface).  $K_{0,0}$  could also intervene for the description of the different behaviours of the free-nucleon like systems and the correlated nuclei and nuclear matter at different densities.

The dependence of the screening functions on the temperature (as traditionally defined: with Fermi-Dirac distribution for nucleons) was checked for fixed densities (which are not considered to depend on the temperature). The variations are usually very small: there is an increase of the order of  $\Delta A_{s,t} \simeq 0.1$  to 1 MeV when T increases from 0 up to 6 MeV for almost all the forces in the

isovector channel. This is consistent with the finite nuclei analysis of the symmetry energy coefficient of reference [31]. In the spin-isospin channel, the dependence of  $A_{1,1}$  on temperature varies according to the density, n-p asymmetry and effective force. At  $\rho = \rho_0$  and  $2\rho_0$  the variation is negative, very close to zero when  $b = 0$  but becomes of the order of  $\Delta A_{1,1} \approx -8 \rightarrow -14$  when  $b \simeq .54$ . This variation may be smaller or bigger when  $b = 2$ : it is of the order of  $-5 \rightarrow -15$ . depending on the force (respectively for SkSC6, SkSC10 and SkSC4). For the spin and scalar channels the screening functions also have specific and similar density, n-p asymmetry and force dependences. For instance, there is a meaningful variation mainly for the forces SkSC4 and SkSC10, which are positive for  $\rho = \rho_0$  and negative for  $\rho = 2\rho_0$  at  $b = 0$  and also when  $\rho = .5\rho_0$  at  $b = 2$ . To sum up, no systematic and unique behaviour was found for the analysed Skyrme forces.

## 4 Conclusions

Summarizing, generalized symmetry energy coefficients of hot asymmetric (non relativistic) nuclear matter at variable densities were investigated. Their density, temperature and n-p asymmetry dependences were analysed for different Skyrme forces which may yield very different behaviours including the possibility (or not) of nuclear matter to undergo phase transitions to spin/isospin (un)polarized states as well as free (or almost free) nucleonic states in the scalar fluctuations analysis. The use of Skyrme-type interactions allowed to obtain analytical expressions for the symmetry energy coefficients. These forces can describe different behaviours of the symmetry energy coefficients which could be hopefully measured (provided a suitable modelization) in laboratories and astrophysical observations. Although one should not believe that only one Skyrme force parametrization could account for the description of all nuclear observables at different densities, temperatures and n-p asymmetries it is acceptable the idea that several parametrizations could hopefully describe different ranges of the dependence of nuclear observables (as the symmetry energy coefficients) with these three variables.

In fact, this paper and the previous one [6] also are suggesting another way of parametrizing the usual mass formulae in macroscopic model for nuclear system using a conceptually different frame. This is furnished by the screening functions of the nuclear system which is a measure of how the system reacts to external probes yield pieces of information of the system itself. In the isovector channel the symmetry energy coefficient is found to receive important contribution from the nuclear correlations which yield the Goldstone mode. This was found not only from the study of variable densities (as

expected) but also from the n-p (explicit) asymmetry <sup>4</sup>.

We also think that the dipole screening functions may be the relevant restoring force for the Isoscalar Dipole Giant Resonance, being object of intense experimental and theoretical studies nowadays. In our view, this work shows that a realistic Skyrme-type force which fits well several nuclear observables for a large range of nuclei (as for example SkSC6) will probably not be suitable to describe nuclear systems at different densities, temperatures and (eventually different n-p asymmetries). It is still possible to use one specific Skyrme force parametrization for a particular range of the above variables. We also hope that the present work may contribute to a different view on how to parametrize mass formulae from clear physical basis specially in the case of nuclei far from the drip line (in spite of shell and finite size effects) - very asymmetric nuclear systems. Nevertheless, it is important to stress that different Skyrme forces provide very different results for the present calculations which, in most part, are difficult to test due to the difficulties of extracting experimental data at different densities, temperatures and n-p asymmetries. Furthermore, the presence of the above mentioned correlations in ground and excited states should be taken into account for the description of nuclear properties. For instance, the presence of the Goldstone bosons for the isospin-related channels can be considered in a lagrangian description with derivative couplings of (pseudo-)scalar and iso-vector fields. This is a very basic property of these modes [32, 33].

Some remarks concerning the supernovae mechanism can be summarized now. The higher the n-p symmetry energy coefficient the smaller is the deleptonization in the quasi-static phase of the supernovae mechanism yielding a larger final proton fraction and faster cooling (via neutrino emission). This picture yields a stronger shock wave since the energy loss is smaller [34, 35]. The increase of symmetry energy coefficient (or rather the isovector screening function), in principle, helps a successful explosion of the (contracting) star. Contrary to the isovector channel for which  $A^{0,1}$  have values close to  $30\text{MeV}$  due to mass formulae parametrizations, the other channels present rather different values for the respective screening functions, mainly those related to the spin channel. The presence (and eventually the disappearance) of the Goldstone bosons found in [16, 17, 18, 7, 6] (and more generally, the collective modes) may alter significantly the usual Supernovae and neutron star pictures [36]. This dynamical effect have reflexes in the screening functions discussed above since the Goldstone bosons

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<sup>4</sup>This explicit symmetry breaking actually seems to be possible due to the Spontaneous Symmetry Breaking which is related to the fact that the nuclear potential for neutrons be different for the protons. This discussion, to be complete, certainly has to take into account Electromagnetic interactions in a consistent way [20].

are zero-energy modes and imply correlations also in the ground state. This was seen for the limit of  $q \rightarrow 0$  (and  $\omega \rightarrow 0$ ) of the dynamical polarizabilities. Relevant consequences are also found for the spin dependent channels (possibility of formation of spin-polarized matter). The incompressibility coefficient ( $K$ ) also changes with the value of the n-p asymmetry acquiring lower values [9], i.e., for high enough asymmetries the incompressibility disappears since there is no more minimum for the EOS. The strong dependence of the incompressibilities on the n-p (varying) asymmetry is also of great relevance for the shock wave formation in the Supernovae mechanism [35]. We actually will find very intricate phase diagrams for nuclear matter and finite nuclei.

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## Figure caption

**Figure 1** Isovector screening function  $A_{0,1} = \rho/(2\Pi_R^{0,1})$  of symmetric nuclear matter as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for interactions SLy<sub>b</sub> (thick dotted-dashed line), SkSC4 (solid), SkSC6 (dotted), SkSC10 (dashed). The simplified expression ( $a_\tau$ ), i.e. without terms of order of  $V_1^2$ , for the force SLy (thin dotted-dashed). Circles, squares and diamonds (P.A.L. 1,2,3) for the expression of  $a_\tau$  as a function of  $u$  with three different functions  $F(u)$  from reference [13].

**Figure 2** Isovector screening function  $A_{0,1} = \rho/(2\Pi_R^{0,1})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for interactions SLy with  $b=0.25$  (thick dotted-dashed line),  $b=.54$  (thin dotted-dashed line), SkSC6 with  $b=.25$  (thick dotted),  $b=.54$  (thin dotted), SkSC4 with  $b=.25$  (thick solid) and  $b=.54$  (thin solid).

**Figure 3** Isovector screening function  $A_{0,1} = \rho/(2\Pi_R^{0,1})$  as a function of the asymmetry coefficient  $b$  at different densities: solid lines for SkSC4 (very thick:  $\rho = .5\rho_0$ , thick:  $\rho = \rho_0$ , thin:  $\rho = 2\rho_0$ ), dotted lines for SkSC6 (same conventions as SkSC4) and dotted-dashed lines for SLy.

**Figure 4** Spin screening function  $A_{1,0} = \rho/(2\Pi_R^{1,0})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of figure 1.

**Figure 5** Spin screening function  $A_{1,0} = \rho/(2\Pi_R^{1,0})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of figure 2.

**Figure 6** Spin screening function  $A_{1,0} = \rho/(2\Pi_R^{1,0})$  as a function of the asymmetry coefficient  $b$  at different densities and for the different forces: with the same conventions of figure 3.

**Figure 7** Spin-isospin screening function  $A_{1,1} = \rho/(2\Pi_R^{1,1})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for interactions SLy, SkSC4, SkSC6 and SkSC10 with the same conventions of figure 1.

**Figure 8** Spin-isospin screening function  $A_{1,1} = \rho/(2\Pi_R^{1,1})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for the different interactions: with the same conventions of figure 2.

**Figure 9** Spin-isospin screening function  $A_{1,1} = \rho/(2\Pi_R^{1,1})$  as a function of the asymmetry coefficient  $b$  at different densities and with the different forces: with the same conventions of figure 3.

**Figure 10** Scalar (dipole) screening function  $A_{0,0} = \rho/(2\Pi_R^{0,0})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for the different interactions with the conventions of figure 1.

**Figure 11** Scalar (dipole) screening function  $A_{0,0} = \rho/(2\Pi_R^{0,0})$  as a function of the ratio of density to density at saturation ( $u = \rho/\rho_0$ ) for the different interactions: with the conventions of figure 2.

**Figure 12** Scalar (dipole) screening function  $A_{0,0} = \rho/(2\Pi_R^{0,0})$  as a function of the asymmetry coefficient  $b$  at different densities and with the different forces (the conventions of figure 3) and also the n-p asymmetry dependence of the compressibility  $K_\infty$  of nuclear matter of reference [30] using  $K_D(b=0)$  from the SLy force (star).



Figure 1

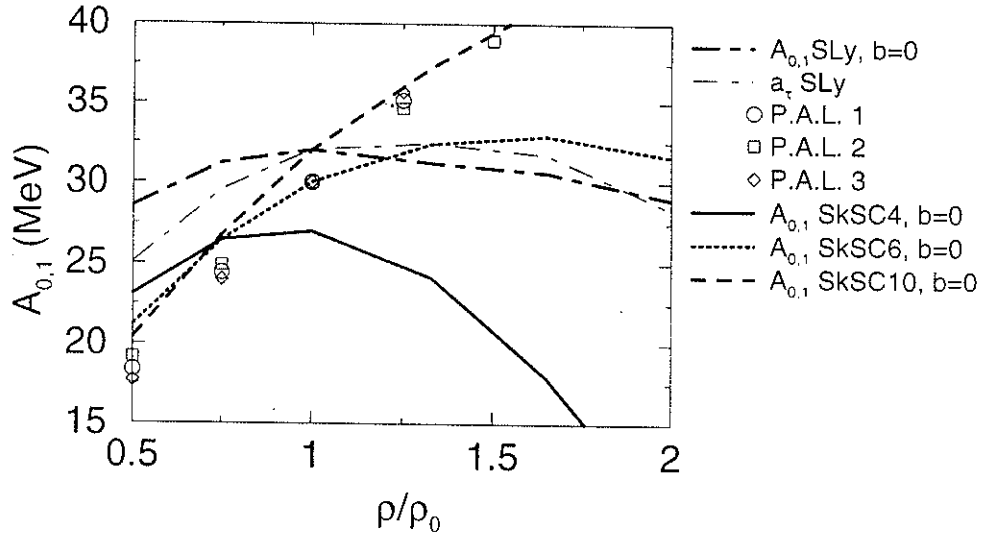


Figure 2

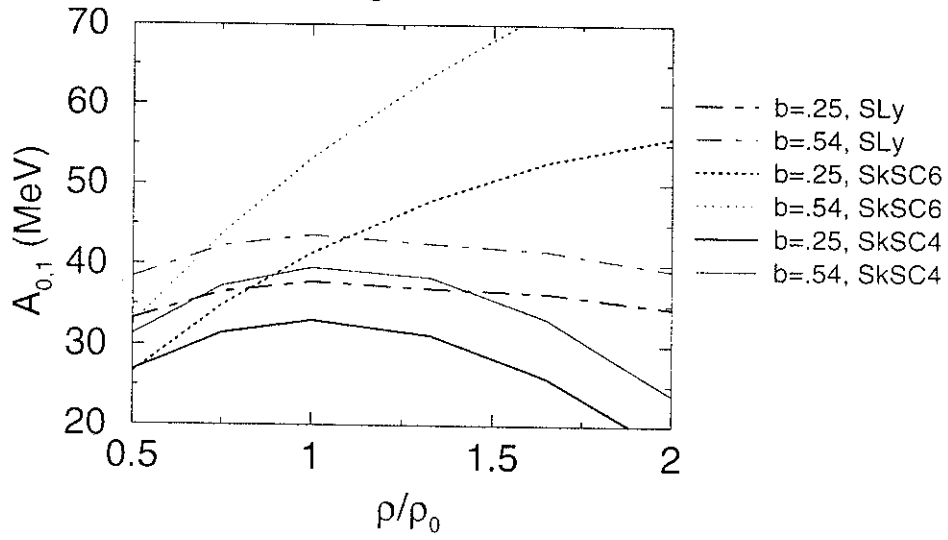


Figure 3

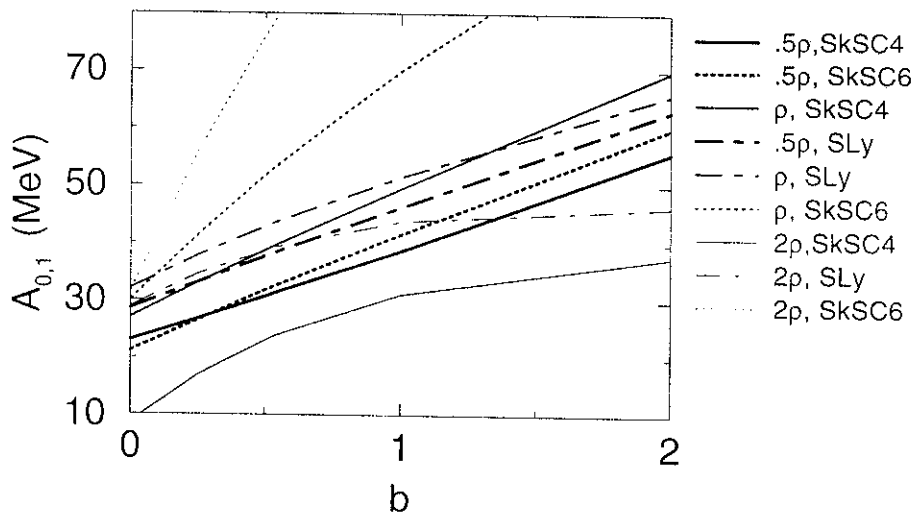




Figure 4

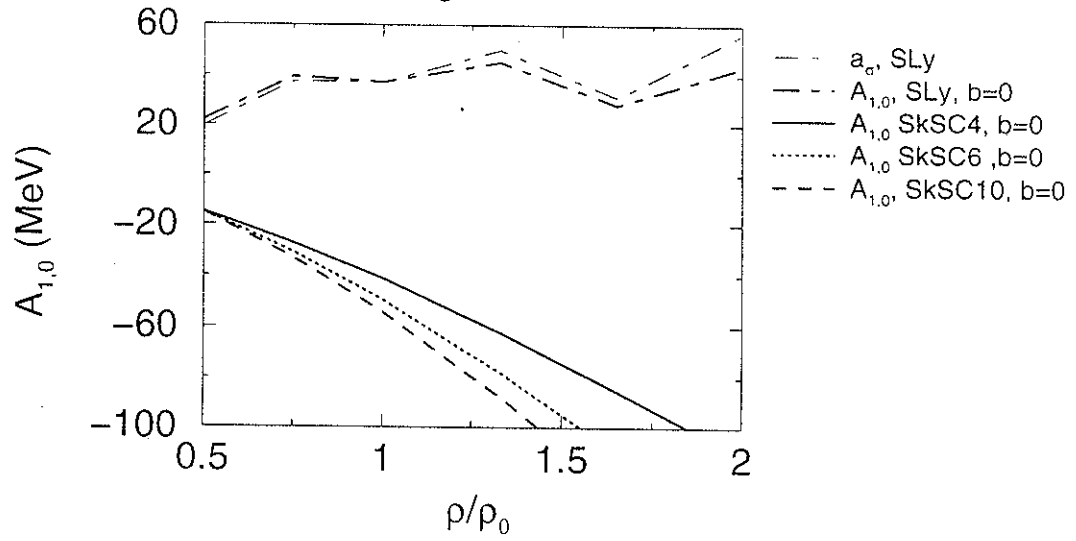


Figure 5

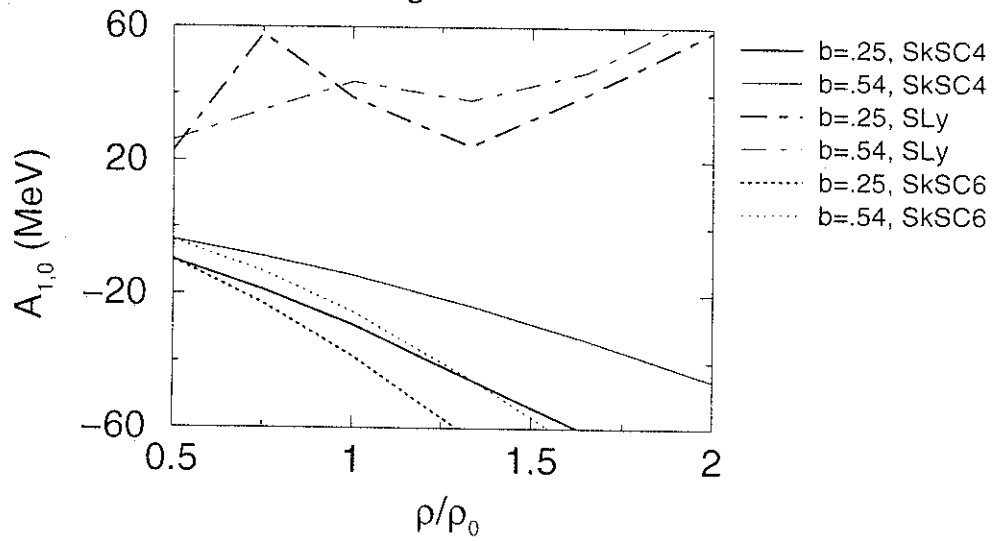
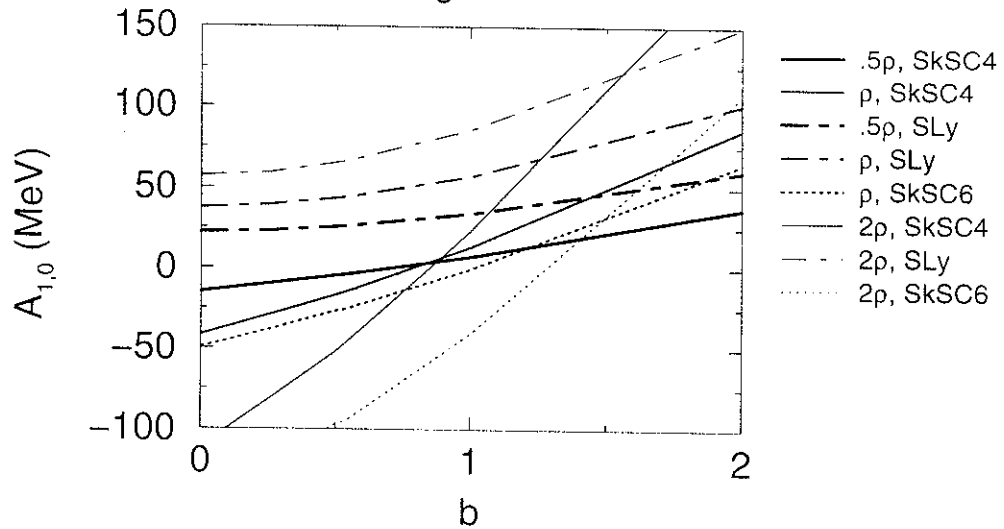


Figure 6



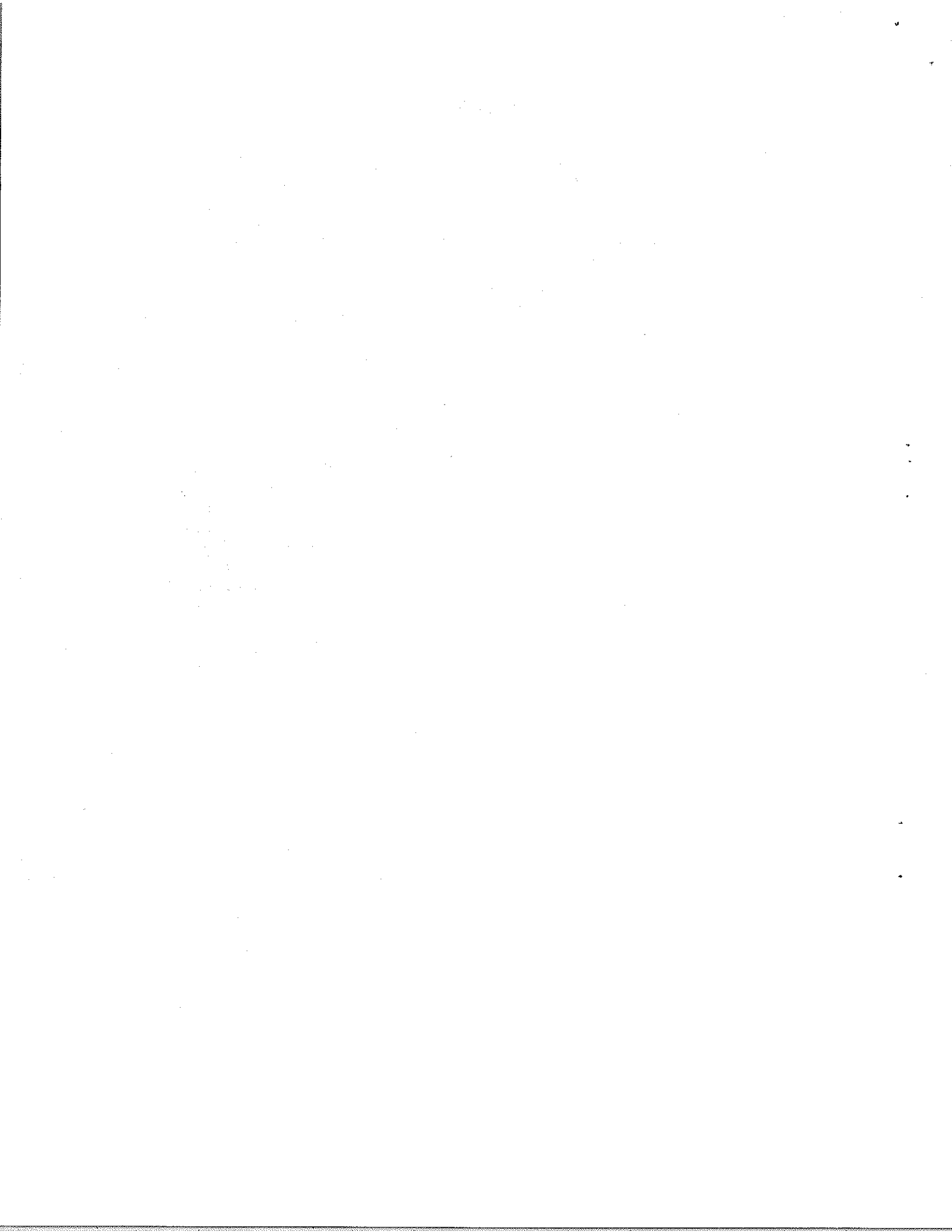




Figure 7

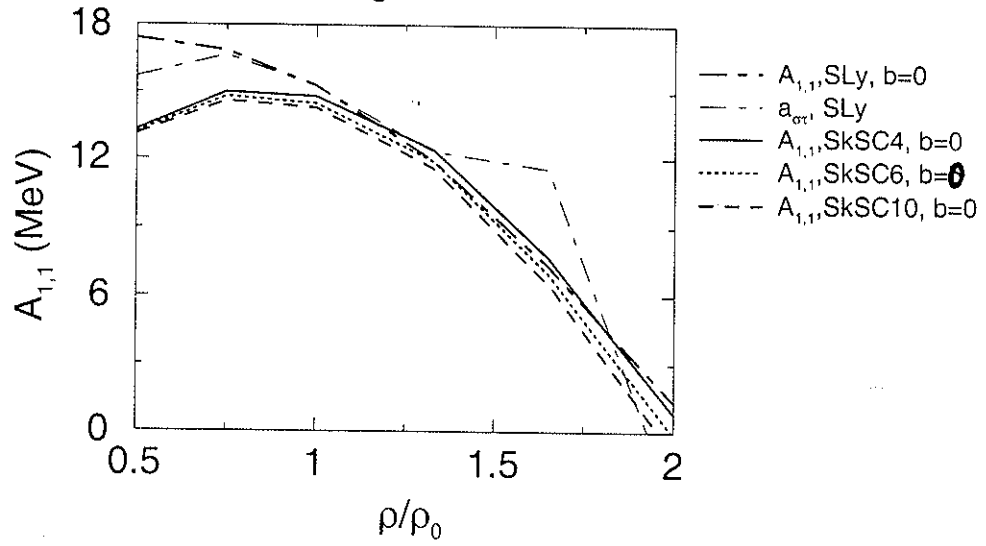


Figure 8

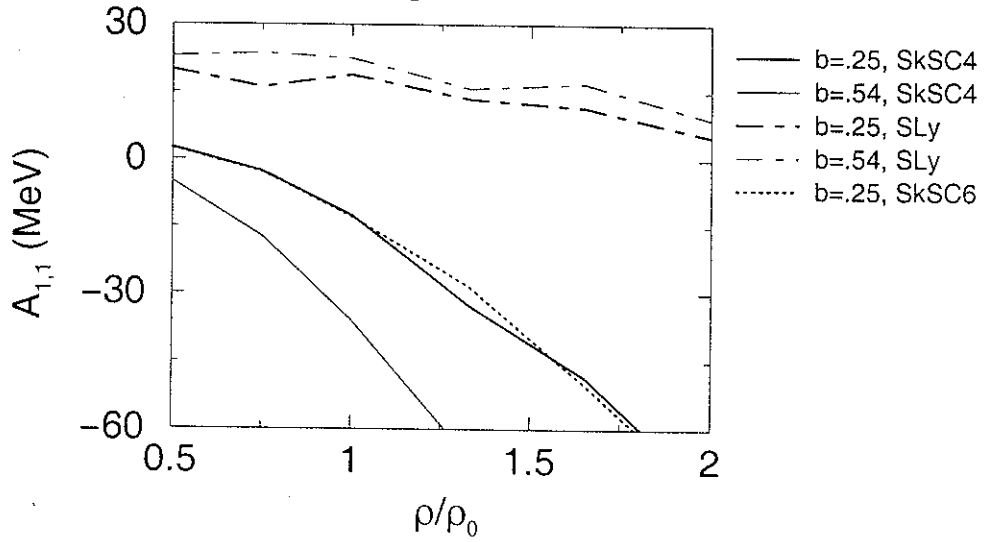
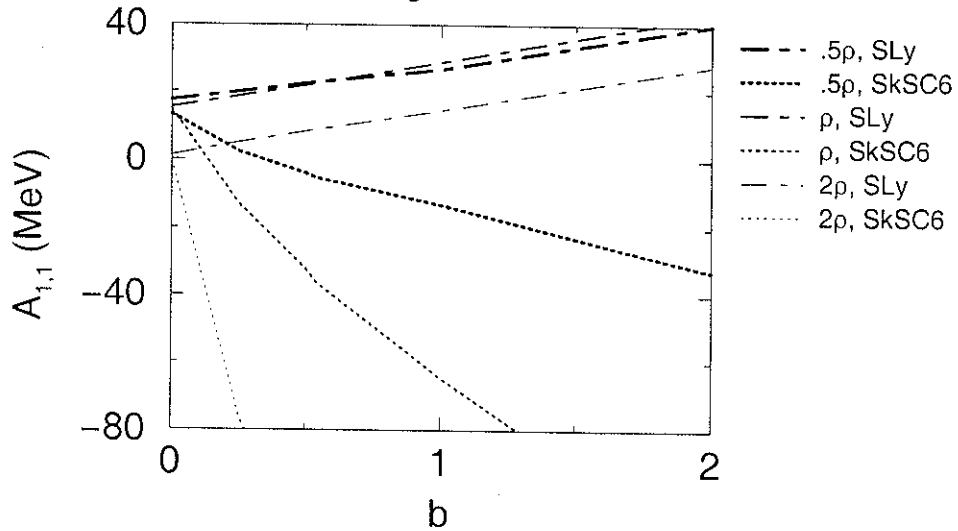


Figure 9



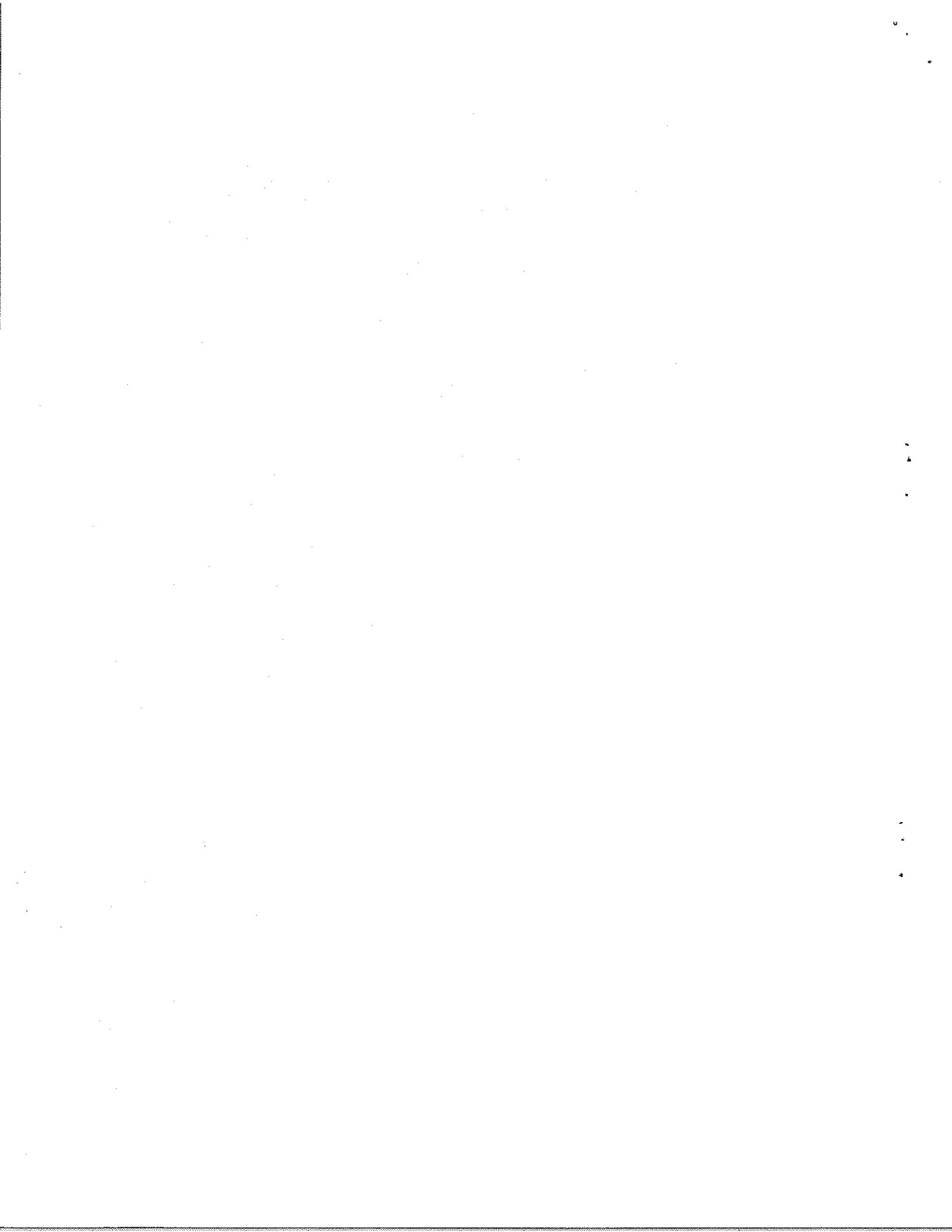


Figure 10

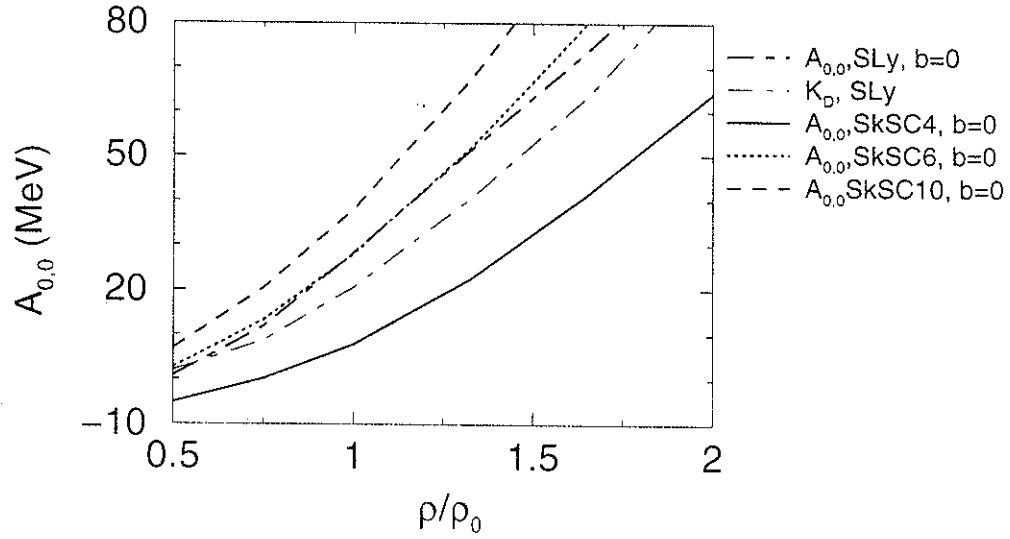


Figure 11

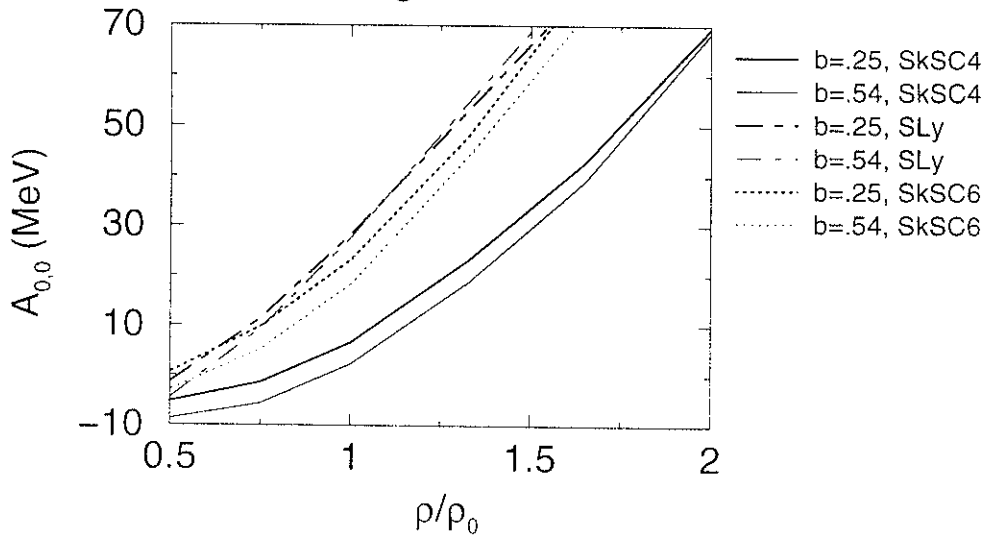


Figure 12

