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MASS IN THE GROSS - NEVEU MODEL

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A B S T R A C T

A reinterpretation of the Nambu - Jona - Lasinio self-consistent technique allows, in the Gross-Neveu model , for interpreting some recent works on the spontaneous mass generation in a unified way and also for the extension of the computations to a higher order.

1 - INTRODUCTION

The study of spontaneous mass generation, that is,of the appearance of massive particles in theories described by lagrangians with only dimensionless parameters, initiated by the Landau school[1], has, as a landmark, the work of Nambu and Jona-Lasinio[2], who introduced a technique of self-consistent computation of the mass as the effect of a spontaneous breakdown of symmetry. This is made possible because renormalization introduces parameters with the dimension of mass. A well-known example is the work of Coleman and Weinberg[3]. An important contribution to the methodology of the problem was done in the classical paper by Gross and Neveu[4],where a model involving a quartic fermionic interaction was solved in two dimensions and in the limit of large number of components N, and shown to exhibit the phenomenon of spontaneously generated mass. A paper by Frishman, Römer and Yankielowicz[5] added to the understanding of the problem by stating some more restrictive conditions for the existence of a generated mass. Recently Abarbanel[6], with a different approach, suceeded into extending the range of validity of the solutions to higher values of the coupling constant, and, in so doing, discovered a second zero of the β-function (the Callan-Symanzik function[7]),a fact that,connected to the analysis of ref.(5), is of great interest, as it raises suspicion as to the physical meaning of the computed mass. Finally, Muta[8] has shown how to obtain Abarbanel's results by applying the classical Nambu-Jona-Lasinio self-consistent method to the Gross-Neveu model, a nice, if formal, completion of the original program, which was marred by the fact that ref.(2) used the Fermi lagrangian, known to be unrenormalizable.By the end of last year the situation was the following: the Gross-Neveu model, with N two-component fermions,was believed to present spontaneously generated mass in the limit $N\to\infty$. Two classes of computation existed: one, essentially perturbative, perfomed by Gross and Neveu, and other, self-consistent, performed by Abarbanel and by Muta. It was not clear whether they were computing the same mass, as the β-function of Gross and Neveu had only one zero(at the origin), whereas Abarbanel's one vanished for still another value of the coupling constant. They coincide for small values of the coupling constant but are very different for larger ones: there, Abarbanel's results have been considered to be better on account of its being "less perturbative", relying as they do on ideas of self-consistency.

In this paper we intend to clarify some points of this problem, as well as to investigate whether the generated mass survives the addition of corrections of order N^{-1}. This is done by reinterpreting the Nambu-Jona-Lasinio self-consistent method in terms of a condition on the tadpoles of the Gross-Neveu model with a symmetry-breaking linear term, made to vanish in the end(9), and reveals a simple way to compute higher order corrections. We find out that the results of Gross and Neveu and those of Abarbanel are just consequences of different renormalization procedures, and so describe the same physical effect. Introducing the 1/N corrections, the spontaneously generated mass still appears, as well as the second zero of β. The dimension of the composite boson operator $\bar{\Psi}\Psi$ (see below) at this second zero is canonical, thus satisfying the requisites of ref. (5). Of some interest is the fact that the Nambu-Jona-Lasinio method, in our version, is essentially perturbative in all but the last step, which is the solution of a transcendental equation. We consider this to be an improvement.

In section 2 we describe the Gross-Neveu model, their results, and the new results of Abarbanel. Section 3 presents our method of computation and shows how to obtain either Gross-Neveu's or Abarbanel's results. In section 4 the next order calculation is performed, and comments are made.

2 - THE MODEL

4.

It is a sort of Fermi interaction in 2-dimensional space-time.The lagrangian is (imaginary metric)

$$\mathcal{L} = -\bar{\Psi}\gamma\cdot\partial\Psi + \frac{1}{2} g_o^2 (\bar{\Psi}\Psi)^2 \tag{2.1}$$

where g_o is a dimensionless (in 2 dimensions) coupling constant. The fermion field Ψ is, in fact, a multiplet of N fields, being represented by a column of 2N elements. It happens to be renormalizable and, in the case N=1 , is equivalent to the Thirring model[10]. A very interesting feature of this model is that the identity

$$Z[\eta,\bar{\eta}] = \text{const} \int [d\Psi][d\bar{\Psi}] \exp i\left[-\bar{\Psi}\gamma\cdot\partial\Psi + \frac{1}{2} g_o^2(\bar{\Psi}\Psi)^2 + \bar{\eta}\Psi + \bar{\Psi}\eta\right]$$

$$= \text{const}' \int [d\Psi][d\bar{\Psi}][d\sigma] \exp i\left[-\bar{\Psi}\gamma\cdot\partial\Psi - \frac{1}{2}\sigma^2 + g_o\bar{\Psi}\Psi\sigma + \bar{\eta}\Psi + \bar{\Psi}\eta\right]$$

allows one to study its Green functions using the lagrangian

$$\mathcal{L}' = -\bar{\Psi}\gamma\cdot\partial\Psi - \frac{1}{2}\sigma^2 + g_o\bar{\Psi}\Psi\sigma \tag{2.2}$$

where the composite boson operator σ, equal to $g_o\bar{\Psi}\Psi$ at the "classical level", appears. It is this operator that, by developing a nonvanishing vaccum expectation value, will provide the parameter in terms of which the spontaneous breaking of symmetry will be measured. Gross and Neveu[4] compute the effective potential[11] of (2.2) as a function of σ, detect a minimum a $\sigma \neq o$ and, proceeding in the usual way, observe that the corrected fermion propagator has a pole at $p^2 = -m^2$, with

$$m^2 = \mu^2 e^{-\frac{2\pi}{\lambda^2}} \tag{2.3}$$

μ being a parameter introduced by the renormalization and λ being the renormalized coupling constant, quantities to be described in all detail in our unifying approach of next Section. The β-function of Gross and Neveu is given by

$$\beta(\lambda) = -\frac{\lambda^3}{2\pi} \tag{2.4}$$

showing that the model is asymptotically free (as it should[4]).

The corresponding quantities in Abarbanel's computation are[16]

$$\sqrt{1+4\frac{m^2}{\mu^2}}\ \ln \frac{\sqrt{1+4\frac{m^2}{\mu^2}}-1}{\sqrt{1+4\frac{m^2}{\mu^2}}+1} = -\frac{2\pi}{\lambda^2}\ , \tag{2.5}$$

a mass-gap equation that replaces eq. (2.3), and a β-function that, though agreeing with (2.4) for small λ , is markedly different for large values. In fact,

$$\beta(\lambda) \sim -\frac{\pi-\lambda^2}{2\lambda} \tag{2.6}$$

for $\lambda^2 \sim \pi$, exhibiting a new zero for β, absent from (2.4). Another importante feature of Abarbanel's solution is that the anomalous dimension of σ vanishes at the second zero of β. If this were not the case, no mass generation would be possible, according to Frishman et al.[5] We will now show how to get either eq. (2.3) or eq.(2.5) , depending on how we renormalize our theory: the results of Gross and Neveu will be obtained when the counterterms of the symmetric theory are used[12], whereas Abarbanel's come out in a renormalization that, in a sense to be made precise later,"follows" the breakdown of symmetry.

3 - LARGE N (one-loop) COMPUTATION

We will be working with the lagrangian of eq.(2.2) . To study the symmetry breaking of the theory, we add to it a driving term which breaks the symmetry and, simultaneously, shift the σ field by a constant v , getting

$$\mathcal{L} = -Z\bar{\Psi}\gamma\cdot\partial\Psi - \frac{Z_\sigma}{2}(\sigma-v)^2 + Z_\lambda \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}\bar{\Psi}\Psi(\sigma-v) + c\sigma \tag{3.1}$$

where the renormalization constants are defined by the following relations with the unrenormalized fields and coupling constants :

$$\begin{aligned} \Psi_0 &= Z^{1/2}\Psi \\ \sigma_0 - v_0 &= Z_\sigma^{1/2}(\sigma - v) \\ Z_\lambda^2 \lambda^2 &= Z^2 Z_\sigma N g_0^2 \mu^{n-2} \end{aligned} \tag{3.2}$$

where μ is a parameter with the dimension of mass and n is the continuous dimension in the sense of dimensional regularization[13]. Isolating the counterterms, one writes

$$\begin{aligned} \mathcal{L} = &-\bar{\Psi}\gamma\cdot\partial\Psi - \frac{1}{2}(\sigma-v)^2 + \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}\bar{\Psi}\Psi(\sigma-v) + c\sigma \\ &- z\bar{\Psi}\gamma\cdot\partial\Psi - \frac{z_\sigma}{2}(\sigma-v)^2 + z_\lambda\frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}\bar{\Psi}\Psi(\sigma-v) \end{aligned} \tag{3.3}$$

where $Z=1+z$ and so on. We will look for solutions with spontaneously broken symmetries by examining the possibility of having a nonvanishing vacuum expectation value of σ when c is put equal to zero after the computations are done. This means we must put the sum of all tadpoles with a σ-leg, equal to zero, thus getting an equation for v ; if this equation has nonvanishing solutions for $c=o$, then we have spontaneous breakdown of symmetry[9].

The Feynman rules of (3.3) are (see Fig. 1)

7,

Fig. 1

a) $\frac{1}{(2\pi)^2 i}\,\frac{-i\gamma\cdot k + a}{k^2 + a^2}$ e) $(2\pi)^2 z\, \gamma\cdot k - (2\pi)^2 i\, z_\lambda a$ (3.4)

b) $\frac{1}{(2\pi)^2 i}$ f) $\frac{(2\pi)^2}{i} z_\sigma$

c) $(2\pi)^2 i\, \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}$ g) $(2\pi)^2 i\, z_\sigma v$

d) $(2\pi)^2 i\,(c+v)$ h) $(2\pi)^2 i\, z_\lambda \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}$

where we introduced

$$a = \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}} v \tag{3.5}$$

Let us work at the one loop level, what corresponds to the limit $N\to\infty$ .Then the tadpole equation, given in Fig. 2, is

Fig. 2

$$c + v + v z_\sigma - \frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}} N \,\mathrm{Tr}\int \frac{dq}{(2\pi)^2 i}\,\frac{-i\gamma\cdot q + a}{q^2 + a^2} = 0 \,. \tag{3.6}$$

Performing the integration, one has

$$c = -v - v z_\sigma + \frac{\lambda^2}{2\pi} v \frac{\Gamma(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} \tag{3.7}$$

which gives, for $c=o$ and $v\neq o$, the equation

$$1 + z_\sigma - \frac{\lambda^2}{2\pi}\,\frac{\Gamma(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} = 0 \tag{3.8}$$

As soon as z_σ is determined by some renormalization prescription, (3.8) will give <u>a</u> as a function of the coupling constant λ .This will be our mass-gap equation.

8.

Before doing that, let us analyze eq.(3.6) in a different way. Putting c=o and multiplying by $\frac{\lambda}{\sqrt{N}}\mu^{\frac{2-n}{2}}$, one gets

$$-a = \lambda^2 \mu^{2-n}\,\mathrm{Tr}\int \frac{dq}{(2\pi)^2 i}\,\frac{-i\gamma\cdot q + a}{q^2+a^2} \quad + \quad a\, z_\sigma$$

and, omitting the counterterm,

$$-a = \frac{N g_0^2}{(2\pi)^2 i}\,\mathrm{Tr}\int dq\,\frac{1}{i\gamma\cdot q + a} \qquad (3.9)$$

This is equivalent to the equation depicted in Fig.3, valid for $N\to\infty$, which reads

Fig. 3

$$\Sigma(p) = \frac{N g_0^2}{(2\pi)^2 i}\,\mathrm{Tr}\int dq\,\frac{1}{i\gamma\cdot q - \Sigma(q)} \qquad (3.10)$$

In fact, as $\Sigma(p)$ is a constant ($\Sigma(p)=-m$) , (3.10) goes into (3.9) for $a=m$ .Equation (3.10) is the Nambu-Jona-Lasinio self--consistent condition(2) ,and is also the starting equation of Muta(8). We learn in this way that <u>a</u> is equal to the electron mass(13).

After this digression,let us go back to eq.(3.8). It is necessary to determine Z_σ .We do that in two different ways. Consider the one-loop approximation for $\Gamma^{(2)}_\sigma$, the two-point proper Green function of the field σ . It is represented in Fig.4, and is given by

Fig. 4

$$\Gamma^{(2)}_\sigma(k) = -\lambda^2\mu^{2-n}\int dq\,\frac{n\, q\cdot(k-q) + n a^2}{[q^2+a^2][(k-q)^2+a^2]}$$
$$= -i\,\pi^{\frac{n}{2}}\, n(1-n)\,\lambda^2\,\Gamma\!\left(1-\frac{n}{2}\right)\int_0^1 dx\,\frac{1}{\left[\frac{k^2}{\mu^2}x(1-x)+\frac{a^2}{\mu^2}\right]^{1-\frac{n}{2}}} \qquad (3.11)$$

First, we determined z_σ by the condition

$$\Gamma_\sigma^{(2)}(k^2=\mu^2) + \frac{(2\pi)^2}{i} z_\sigma = 0 \tag{3.12}$$

which gives

$$z_\sigma = -\frac{\lambda^2}{2\pi}(1-n)\Gamma(1-\tfrac{n}{2})\int_0^1 dx \frac{1}{\left[x(1-x)+\frac{a^2}{\mu^2}\right]^{1-\frac{n}{2}}} \tag{3.13}$$

Putting this into (3.8) and using

$$\int_0^1 dx \ln(1+\alpha x-\alpha x^2) = -2 - \sqrt{1+\frac{4}{\alpha}}\ln\frac{\sqrt{1+\frac{4}{\alpha}}-1}{\sqrt{1+\frac{4}{\alpha}}+1} \tag{3.14}$$

one gets

$$\sqrt{1+4\frac{a^2}{\mu^2}}\ln\frac{\sqrt{1+4\frac{a^2}{\mu^2}}-1}{\sqrt{1+4\frac{a^2}{\mu^2}}+1} = -\frac{2\pi}{\lambda^2} \tag{3.15}$$

which is Abarbanel's mass formula.

If, instead, we use the renormalization prescription

$$-i\pi^{\frac{n}{2}}n(1-n)\lambda^2\Gamma(1-\tfrac{n}{2})\int_0^1 dx \frac{1}{[x(1-x)]^{1-\frac{n}{2}}} + \frac{(2\pi)^2}{i}z_\sigma = 0, \tag{3.16}$$

that is, determine z_σ by the condition analogous to (3.12) but for the symmetric theory (a=o) , then

$$z_\sigma = -\frac{\lambda^2}{2\pi}(1-n)\int_0^1 dx \frac{\Gamma(1-\frac{n}{2})}{[x(1-x)]^{1-\frac{n}{2}}} \tag{3.17}$$

Putting this into (3.8), one gets

$$a^2 = \mu^2 \exp(-2\pi/\lambda^2) \tag{3.18}$$

which is Gross-Neveu's mass formula.

It seems plausible to us that the renormalization prescription (3.12) is more adequate to describe the theory when the symmetry breaking is large. As this is the only difference between the two approaches,we think that Abarbanel's result is the correct one for finite λ values.

4 - CORRECTIONS OF ORDER 1/N (two loops).

No special difficulties remain to extend our computation to higher orders. This is not so if we look at the problem from the point of view of ref.(6), (8) or, for that matter, (2). No obvious scheme of sucessive approximations is apparent, there. Within our strategy, however, things are quite clear: it is just a matter of adding, to the tadpole equation, tadpoles of higher and higher order in the loop expansion. We perform here, in detail, the two-loop computation.

The tadpole equation is then given in Fig. 5,

Fig. 5

that is,

$$A + B + C + D + E + F = o \tag{4.1}$$

where

$$A = -n \lambda^2 i \pi^{\frac{n}{2}} v \frac{\Gamma(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} \tag{4.2}$$

$$B = (2\pi)^2 i v \tag{4.3}$$

$$C = (2\pi)^2 i z_\sigma v \tag{4.4}$$

$$D = \frac{n(n-1) i \pi^n \lambda^4 v}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{2-n}} \tag{4.5}$$

$$E = \frac{\lambda \sqrt{N} z_\lambda \mu^{1-\frac{n}{2}} i \pi^{\frac{n}{2}} a\, n(1-n)}{(a^2)^{1-\frac{n}{2}}} \Gamma(1-\tfrac{n}{2}) \tag{4.6}$$

$$F = \lambda \sqrt{N} z_\lambda \mu^{1-\frac{n}{2}} i \pi^{\frac{n}{2}} a\, n \frac{\Gamma(1-\frac{n}{2})}{(a^2)^{1-\frac{n}{2}}} \tag{4.7}$$

To fully write the tadpole equation one needs, therefore, the values of Z_σ to second order, as well as Z_λ to first order.

11. The counterterm z is not yet necessary. z_λ is determined as explained in Fig. 6,

Fig. 6

that is,

$$\frac{\lambda^2 \mu^{2-n} a}{N} \, \frac{i\pi^{\frac{n}{2}}}{(a^2)^{1-\frac{n}{2}}} \Gamma(1-\tfrac{n}{2}) + (2\pi)^2 z \, \gamma \cdot k - (2\pi)^2 i z_\lambda a = 0$$

giving

$$z_\lambda = \frac{\lambda^2}{(2\pi)^2 N} \pi^{n/2} \frac{\Gamma(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} \tag{4.8}$$

The counterterm z_σ is determined as explained in Fig. 7 .

Fig. 7

This gives

$$(2\pi)^2 i z_\sigma = n(n-1) i \pi^{\frac{n}{2}} \lambda^2 \Gamma(1-\tfrac{n}{2}) I + \frac{n(n-1)^2 \lambda^4}{(2\pi)^2 i N} \Gamma^2(1-\tfrac{n}{2}) I^2 + \frac{2i\pi^n n(n-1)\lambda^4}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} I \tag{4.9}$$

where

$$I = \int_0^1 dx \, \frac{1}{\left[\frac{a^2}{\mu^2} + x(1-x)\right]^{1-\frac{n}{2}}} \; . \tag{4.10}$$

Inserting (4.8) and (4.9) in the tadpole equation (4.1), one gets, for c=o,

$$n\lambda^2 i \pi^{n/2} \frac{\Gamma(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} - (2\pi)^2 i - n(n-1) i \pi^{\frac{n}{2}} \lambda^2 \Gamma(1-\tfrac{n}{2}) I - \frac{n(n-1)^2\lambda^4}{(2\pi)^2 i N} \Gamma^2(1-\tfrac{n}{2}) I^2$$

$$- \frac{2i\pi^n n(n-1)\lambda^4}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} - \frac{n(n-1) i\pi^n \lambda^4}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{2-n}} + \tag{4.11}$$

$$+ \frac{n(n-1) i\pi^n \lambda^4}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{2-n}} + \frac{n i \pi^n \lambda^4}{(2\pi)^2 N} \frac{\Gamma^2(1-\frac{n}{2})}{\left(\frac{a^2}{\mu^2}\right)^{2-n}} = 0 \; .$$

This can be put in the form

$$n\lambda^2 i\pi^{\frac{n}{2}}\Gamma(1-\tfrac{n}{2})\left[\frac{1}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} - (n-1)\,I\right] - \frac{n\pi^n\lambda^4\Gamma^2(1-\frac{n}{2})}{(2\pi)^2 i\,N}\left[\frac{1}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}}-(n-1)I\right]^2 = (2\pi)^2 i \quad . \tag{4.12}$$

Now,

$$\frac{1}{\left(\frac{a^2}{\mu^2}\right)^{1-\frac{n}{2}}} - (n-1)\,I = 1-(1-\tfrac{n}{2})\ln\frac{a^2}{\mu^2} - (n-1)\left[1-(1-\tfrac{n}{2})\int_0^1 dx\,\ln\left(\frac{a^2}{\mu^2}+x(1-x)\right)\right] =$$

$$= (1-\tfrac{n}{2})\left[2+\int_0^1 dx\,\ln\left(\frac{a^2}{\mu^2}+x(1-x)\right)-\ln\frac{a^2}{\mu^2}\right] = -(1-\tfrac{n}{2})X$$

where

$$X = \sqrt{1+4\frac{a^2}{\mu^2}}\;\ln\frac{\sqrt{1+4\frac{a^2}{\mu^2}}-1}{\sqrt{1+4\frac{a^2}{\mu^2}}+1} \tag{4.13}$$

The tadpole equation, eq.(4.12), is, therefore,

$$\frac{\lambda^4}{2N}X^2 - 2\pi\lambda^2 X - (2\pi)^2 = 0 \tag{4.14}$$

whose solution is

$$X = \frac{2\pi N}{\lambda^2}\left(1-\sqrt{1+\frac{2}{N}}\right) \tag{4.15}$$

This is the choice of sign that reproduces, for $N\to\infty$,the one-loop result. Therefore, our mass equation is

$$\sqrt{1+4\frac{a^2}{\mu^2}}\;\ln\frac{\sqrt{1+4\frac{a^2}{\mu^2}}-1}{\sqrt{1+4\frac{a^2}{\mu^2}}+1} = \frac{2\pi N}{\lambda^2}\left(1-\sqrt{1+\frac{2}{N}}\right) \tag{4.16}$$

Notice that for small a one has

$$a^2 = \mu^2\exp\left[-\frac{2\pi N}{\lambda^2}\left(\sqrt{1+\frac{2}{N}}-1\right)\right] \tag{4.17}$$

satisfying the condition that the generated mass should vanish for vanishing λ. On the other hand, a become very large for λ^2 near π. In fact, $a^2/\mu^2 \to \infty$ for

$$\lambda^2 = \pi N\left(\sqrt{1+\frac{2}{N}} - 1\right) \tag{4.18}$$

which is close to π for large N.

The β-function can be computed from (4.16). One finds

$$\beta(\lambda) = -\frac{\lambda^3}{2\pi}\left[1 + \frac{2a^2}{\mu^2\sqrt{1+4\frac{a^2}{\mu^2}}}\ln\frac{\sqrt{1+4\frac{a^2}{\mu^2}}-1}{\sqrt{1+4\frac{a^2}{\mu^2}}+1}\right] \tag{4.19}$$

wherefrom it is clear that, besides the ultraviolet-stable zero at the origin, a second zero exists at precisely the value of λ^2 given by (4.18). This second zero is, for growing λ , an infrared stable one. In order that these results make sense, it is then necessary , as shown by Frishman et al.[15], that the anomalous dimension vanish. That of the fermion field is trivially zero, as Z equals one,to this order. The anomalous dimension of σ is given by

$$\gamma_\sigma(\lambda^2) = \mu^2\frac{\partial}{\partial\mu^2}\ln(1+z_\sigma) \tag{4.20}$$

which, taking (3.13) into account,is shown to be

$$\gamma_\sigma(\lambda^2) = -\frac{\lambda^2}{2\pi}\left[1-\frac{\lambda^2}{2\pi N}\sqrt{1+4\frac{a^2}{\mu^2}}\ \ln\frac{\sqrt{1+4\frac{a^2}{\mu^2}}-1}{\sqrt{1+4\frac{a^2}{\mu^2}}+1}\right]\left[1+\frac{2a^2}{\mu^2\sqrt{1+4\frac{a^2}{\mu^2}}}\ln\frac{\sqrt{1+\frac{4a^2}{\mu^2}}-1}{\sqrt{1+\frac{4a^2}{\mu^2}}+1}\right] \tag{4.21}$$

and, in fact, vanishes at the second zero of β , as so does the last factor of its expression.

Some comments on these results are in order. First , the remarkable simplicity of equations (4.12) and (4.16), which are results of a two-loop calculation, hints possibly at some other , wiser, method of analysis. Second, the introduction of the 1/N corrections arose no new complications, as compared to the one-loop result. However, something must happen, because as we proceed towards smaller values of N we will reach $N=1$,where no mass generation occurs, the model being then equivalent to the Thirring model. One possibility is that something new could appear when Z(the fermion wave-function renormalization constant) become nontrivial. We are investigating this now.

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FIGURE CAPTIONS

Fig.1 - Feynman diagrams.

Fig.2 - The tadpole equation:one loop

Fig.3 - Diagrammatic equivalent of eq.(3.10).

Fig.4 - The σ-propagator: one loop.

Fig.5 - The tadpole equation: two loops.

Fig.6 - Determination of Z_λ .

Fig.7 - Determination of Z_σ

Fig. 1

18.

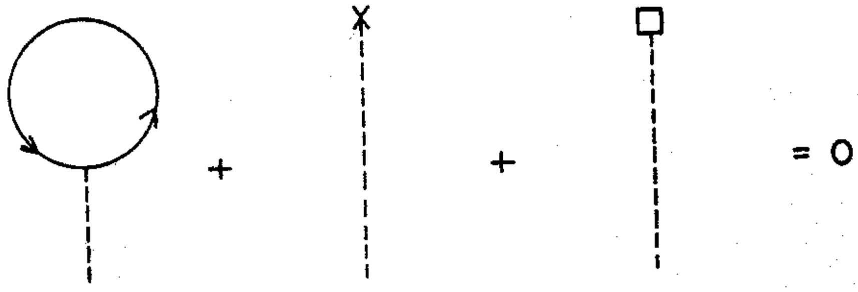


Fig. 2

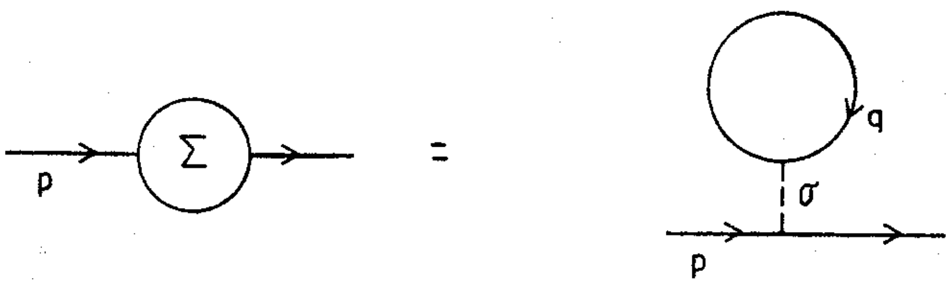


Fig. 3

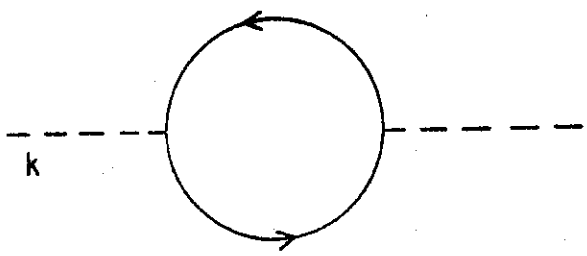


Fig. 4

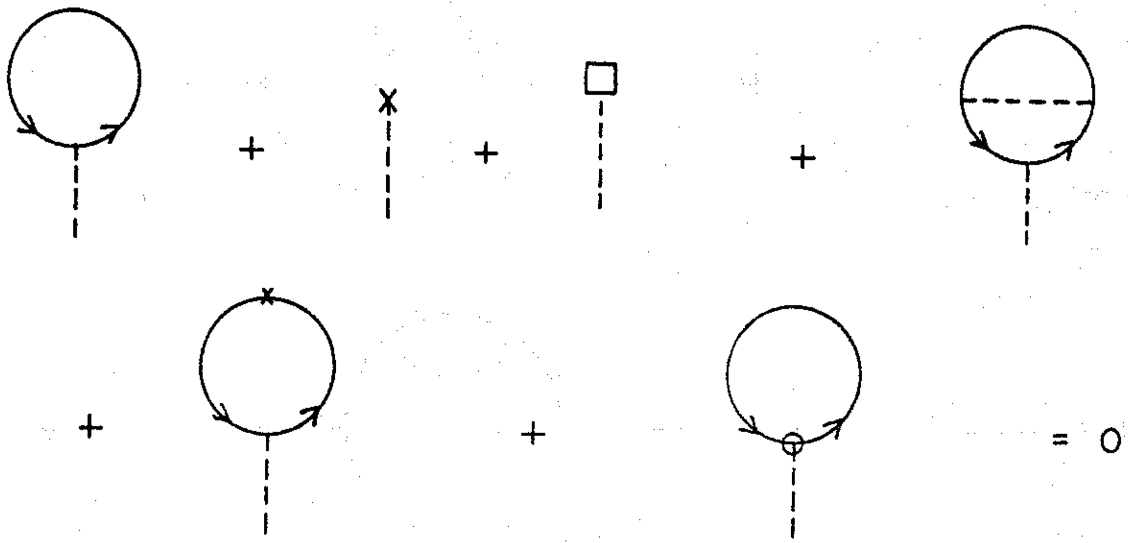


Fig. 5

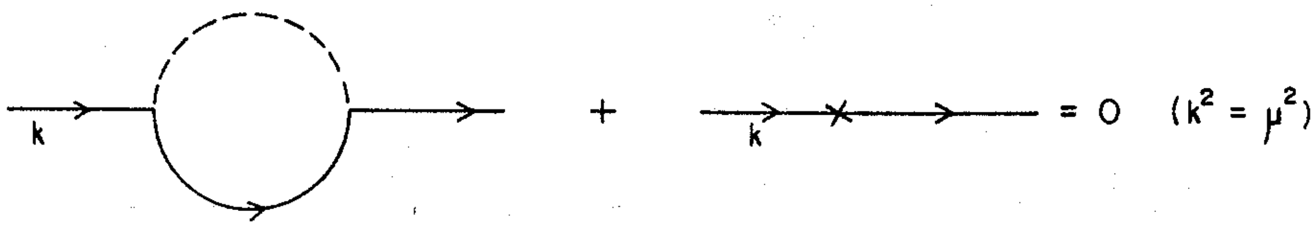
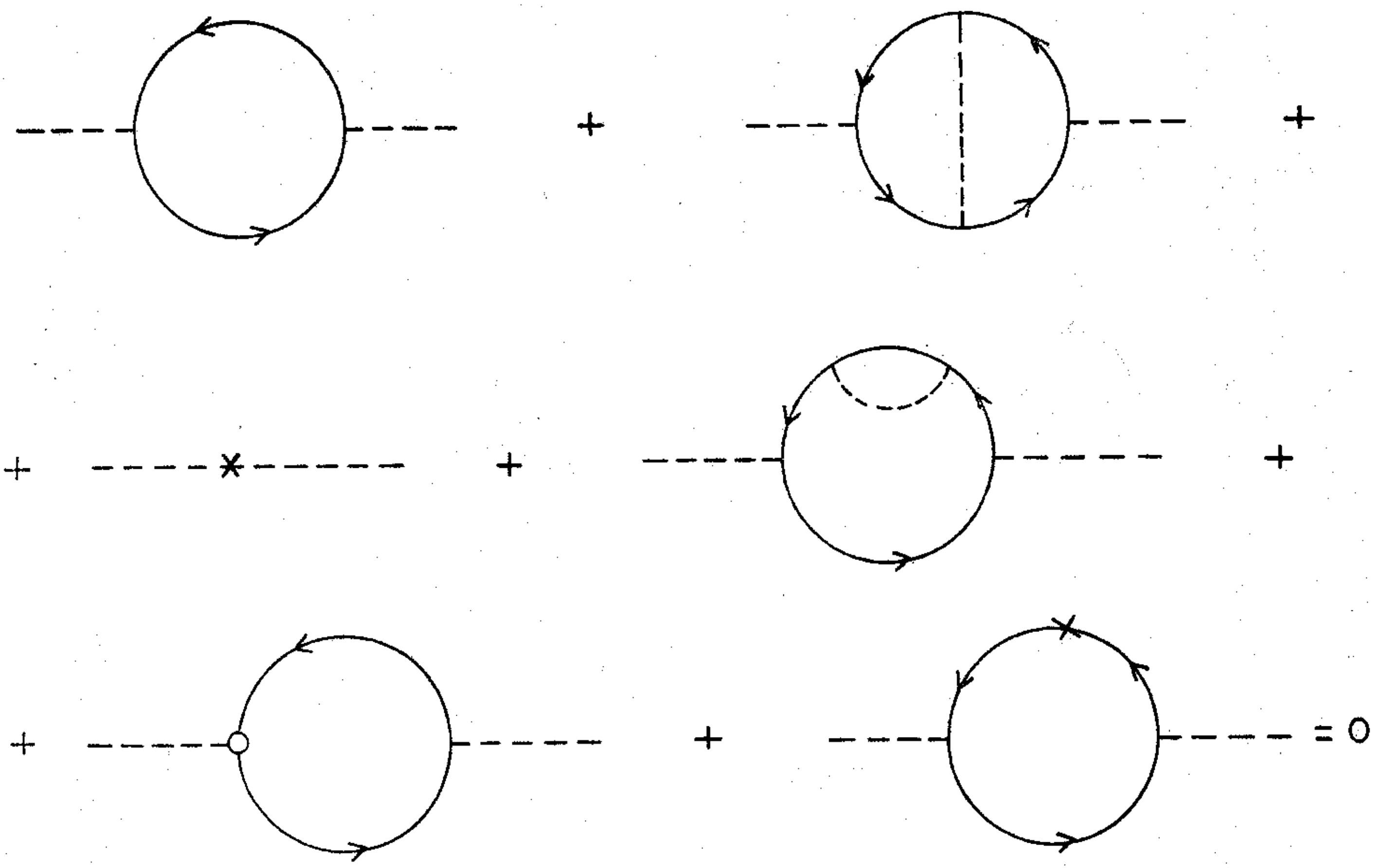


Fig. 6



$(k^2 = \mu^2)$

Fig. 7