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ON THE RENORMALIZED INTERACTING FIELD THEORIES - I

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On the renormalized interacting field theories - I

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Abstract

The usual renormalization scheme as proposed by Stevenson of the variational approximation with a trial Gaussian ansatz for the $\lambda\phi^4$ in 3+1 dimensions is re-analysed. The asymmetric phase (where $< vac|\phi|vac > \neq 0$) is considered focusing the attention to conditions that the values of the model parameters (mass and coupling constant) should respect. The minimizations of the energy density with relation to the renormalized coupling constant as well as for the renormalized mass are considered for a limited range of values of the renormalization mass scale parameter and the resulting expressions are faced as new GAP equations. Ranges of these values, mass and coupling constant, in which the approximation is expected to be more reliable as well as some instabilities are found. Conditions for the values of the coupling constant are found for a given energy density of the model at a given process scale. A different interpretation for the so called symmetry restoration is found.

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1 Introduction

The $\lambda\phi^4$ model has been extensively studied, for example, to shed light on non perturbative effects in quantum field theory (QFT). It's usually considered for the study of Inflationary models and it shares several properties with the linear sigma model which is an effective model of QCD such as a spontaneously symmetry breaking (generating a phase in which the condensate $\bar{\phi} =< vac|\phi|vac >$ is non zero) and asymptotic freedom [1]. These model nevertheless have the intriguing "triviality" in the symmetric phase.

It is only known exact solutions for the free case for which the ground state is described by a Gaussian wavefunctional. The most developed approach to solve the interacting case is perturbation theory which only works well for very small coupling constants as it occurs in Electrodynamics. Also in this approach one has a systematic and direct way of dealing with ultraviolet (UV) divergences, i.e., one knows precisely how to renormalize. However, non perturbative effects are important in many cases as for spontaneous symmetry breaking and bound states. The variational method with a Gaussian prescription for the wave functional has been extensively studied corresponding to a summation of "cactus" type diagrams for the energy [2, 3, 4, 5]. It takes into account more non linearities than perturbation theory although several difficulties arise [6]. Anyway, it offers an alternative powerful approach.

In the present article the usual renormalization scheme as proposed by Stevenson of the Gaussian approach for the $\lambda\phi^4$ model is re-analyzed. We argue that there are constraints to be obeyed by the parameters of the model to obtain a reliable description of the corresponding system in the energy scale under investigation. In the next section the Gaussian approximation is summarized: the GAP equations which are regularized with a cutoff are derived and subsequently compared to the renormalized ones. The usual renormalization procedure of the mass and coupling constant which was performed by Stevenson and others [3, 7] is considered. In sections 2 and 3 new renormalized GAP equations are analyzed: we seek for values of the renormalized mass and coupling constant which minimize the energy density. They could correspond to values for which the approximation is more appropriated but in some cases instabilities are found. The stability of the model for these values is analyzed. In section 4 we show that for a fixed value of the energy density and characteristic physical mass of the system the coupling constant exhibits only a range of physical values as precluded in [8]. In the last section the results are summarized.

2 Gaussian approximation for the $\lambda\phi^4$ model

The Lagrangian density for a scalar field $\phi(\mathbf{x})$ with bare mass m_0^2 and coupling constant λ is given by:

$$\mathcal{L}(\mathbf{x}) = \frac{1}{2}\left\{\partial_\mu\phi(\mathbf{x})\partial^\mu\phi(\mathbf{x}) - m_0^2\phi^2(\mathbf{x}) - \frac{\lambda}{12}\phi^4(\mathbf{x})\right\} \tag{1}$$

The corresponding Hamiltonian density is written as:

$$H = \frac{1}{2}\left(\pi^2(\mathbf{x}) + (\nabla\phi)^2 + m_0^2\phi^2(\mathbf{x}) + \frac{\lambda}{12}\phi^4(\mathbf{x})\right). \tag{2}$$

In the static Gaussian approximation at zero temperature, in the Schrödinger picture [9], the ground state wave functional Ψ is parametrized by:

$$\Psi\left[\phi(\mathbf{x})\right] = Nexp\left\{-\frac{1}{4}\int d\mathbf{x}d\mathbf{y}\delta\phi(\mathbf{x})G^{-1}(\mathbf{x},\mathbf{y})\delta\phi(\mathbf{y})\right\}, \tag{3}$$

Where $\delta\phi(\mathbf{x}) = \phi(\mathbf{x}) - \bar{\phi}(\mathbf{x})$; the normalization is N, the variational parameters are the condensate $\bar{\phi}(\mathbf{x}) =< \Psi|\phi|\Psi >$ and quantum fluctuations represented by the width of the Gaussian $G(\mathbf{x},\mathbf{y}) =< \Psi|\phi(\mathbf{x})\phi(\mathbf{y})|\Psi >$. In variational calculations the averaged energy calculated with $\Psi[\phi(\mathbf{x})]$ is to be minimized to obtain the GAP equations. It yields a maximum bound for the ground state averaged energy.

The average value of the Hamiltonian is given in terms of the variational parameters:

$$\begin{aligned}\mathcal{H} &= \tfrac{1}{2}\Big[\tfrac{1}{4}G^{-1}(\mathbf{x},\mathbf{x}) - \Delta G(\mathbf{x},\mathbf{x}) + m_0^2 G(\mathbf{x},\mathbf{x}) + \tfrac{\lambda}{4}G^2(\mathbf{x},\mathbf{x}) + \\ &+ m_0^2\bar{\phi}^2(\mathbf{x}) + (\nabla\bar{\phi}(\mathbf{x}))^2 + \tfrac{\lambda}{12}\bar{\phi}^4(\mathbf{x}) + \tfrac{\lambda}{2}\bar{\phi}^2(\mathbf{x})G(\mathbf{x},\mathbf{x})\Big].\end{aligned} \tag{4}$$

Variations of the averaged energy density with respect to the variational parameters and their conjugate yield the following GAP equations which define the ground state of the model:

$$\begin{aligned}\frac{\delta\mathcal{H}}{\delta G(\mathbf{x},\mathbf{y})} &\to 0 = \left(-\frac{1}{8}G^{-1}(\mathbf{x},\mathbf{z})G^{-1}(\mathbf{z},\mathbf{y})\right) + \left(\frac{\Gamma(\mathbf{x},\mathbf{y})}{2} + \frac{\lambda}{2}\bar{\phi}(\mathbf{x})^2\right) \\ \frac{\delta\mathcal{H}}{\delta\bar{\phi}(\mathbf{x})} &\to 0 = \Gamma(\mathbf{x},\mathbf{y})\bar{\phi}(\mathbf{y}) + \frac{\lambda}{6}\bar{\phi}^2(\mathbf{x}),\end{aligned} \tag{5}$$

Where $\Gamma(\mathbf{x},\mathbf{y}) = -\Delta + \left(m_0^2 + \frac{\lambda}{2}G(\mathbf{x},\mathbf{x})\right)\delta(\mathbf{x}-\mathbf{y})$ with implicit integrations in these equations. The quantum fluctuations in the vacuum are therefore described by the two point function G:

$$G_0(\mathbf{x},\mathbf{y}) =< \mathbf{x}|\frac{1}{\sqrt{-\Delta + \mu_0^2}}|\mathbf{y} > \tag{6}$$

where μ_0^2 is given by the self consistent GAP equation:

$$\mu_0^2 = m_0^2 + \frac{\lambda}{2}TraceG(x,x,\mu^2) + \frac{\lambda}{2}\bar{\phi}^2. \tag{7}$$

From the above expressions we can see that the non zero solution for the condensate is given by:

$$\bar{\phi}^2 = -6\frac{m_0^2}{\lambda} - 3G(\mu_0^2) = \frac{3\mu_0^2}{\lambda}. \tag{8}$$

Given a mass, μ_0^2, and the coupling constant, λ, the bare mass m_0^2 is defined by the vacuum of the chosen phase (symmetric - $\bar{\phi} = 0$ - or asymmetric- $\bar{\phi} \neq 0$).

The above expression for the Gaussian width (6) (and its inverse G_0^{-1}) can be calculated in the momentum space with a regulator Λ (cutoff) yielding:

$$\begin{aligned} G(\mu_0^2) &= \frac{1}{8\pi^2}\left(\Lambda^2 - \mu_0^2 Ln\left(\frac{d\Lambda}{\mu_0}\right)\right), \\ G^{-1}(\mu_0^2) &= \frac{1}{8\pi^2}\left(2\Lambda^4 + 2\mu_0^2\Lambda^2 - \frac{\mu_0^4}{4} - \mu_0^4 Ln\left(\frac{d\Lambda}{\mu_0}\right)\right) \end{aligned} \tag{9}$$

where $d = 2/\sqrt{e}$. In the (local) limit of infinite cutoff the average energy and other observables would diverge and the divergences must be eliminated. These solutions have been studied in three dimensions for example in [4, 3, 1, 5] as well as a renormalization procedure which will be considered in the present work.

2.1 Regularized equations

To understand consistently the role of the quantum effects as well as to perform consistent comparisons it is important to fix the parameters of the model. But instead of fixing physical mass and coupling, it is also possible to fix other variables as the energy density or particle number [8]. For the sake of the argument, let us consider the simpler case of homogeneous solutions in the vacuum. At the tree level ($G = 0$) we obtain, in the vacuum:

$$\mathcal{H}_{vac} = -\frac{3m_0^4}{2\lambda}. \tag{10}$$

Therefore we can fix, for example, the mass and the energy density of the vacuum and calculate the corresponding coupling constant. This can be useful for the study of the influence of the quantum effects because the inclusion of fluctuations (perturbatively or not) change the ground state which is defined by the mass and coupling constant [10, 11].

Let us consider the regularized energy density $\mathcal{H}_{G,vac}$ at the Gaussian level which can be particularly well suited for lattice calculation. We want to emphasize the the $\lambda\phi^4$ model can be considered as an effective model for which the cutoff can be fixed at some energy scale. Fixing the physical mass (it yields G) and the total energy density (from expressions (4) and (5)) we obtain a second degree polynomial expression for the coupling constant. In terms of the bare (regularized) quantities the corresponding solutions for the coupling constant are given by:

$$\lambda = \frac{-\delta \pm \sqrt{\delta^2 - 4\mu_0^4 G^2}}{2G^2}, \tag{11}$$

where $\delta = 8\mathcal{H}_{vac} - G^{-1} + 2\mu_0^2 G$. To obtain this expression we have used the GAP equations in the asymmetric phase. For the solutions of the above expression to be real we obtain the following condition:

$$\frac{\mu_0^2}{\lambda} \geq \frac{\frac{G^2}{2} + \frac{5\bar{\phi}^4}{6} + G\bar{\phi}^2}{2(G+\bar{\phi}^2).} \tag{12}$$

The renormalized version of $\mathcal{H}$ (as derived in [3]) can also be used for this exercise. This will be considered in section 3.

2.2 Renormalized equations - usual procedure

The usual renormalization procedure of the parameters of the model is done as follows. The divergent terms in the energy density of the symmetric phase as well as in the GAP equation (7) are subtracted from the corresponding terms in the expressions of the asymmetric phase. The resulting subtracted equations are written in terms of a renormalized mass ($m_R^2 = \mu_0^2$), coupling constant (λ_R) and a mass scale μ^2 which eliminates the cutoff dependent terms. The expressions are the following:

$$\begin{aligned} m_R^2 &= \frac{m_0^2 + \frac{\lambda\Lambda^2}{16\pi^2}}{1 + \frac{\lambda}{16\pi^2} log\left(\frac{d\Lambda}{\mu}\right)}, \\ g_R &= \frac{-\frac{\lambda}{2}}{1 + \frac{\lambda}{16\pi^2} log\left(\frac{d\Lambda}{\mu}\right)}, \end{aligned} \tag{13}$$

With which we can rewrite the (subtracted) energy density expression $\mathcal{H}_{sub} = \mathcal{H}(\bar{\phi}) - \mathcal{H}(\bar{\phi} = 0)$ as:

$$\mathcal{H}_{sub} = \frac{m_R^2}{2}\bar{\phi}^2 + \frac{1}{4g_R}\left(m_R^2 - \mu^2\right)^2 + \frac{1}{128\pi^2}\left(m_R^4 Ln\left(\frac{m_R^4}{\mu^4}\right) - m_R^4 + \mu^4\right), \tag{14}$$

where the mass scale can be written from the GAP equation as defined in expression (7):

$$\mu^2 = m_R^2 + g_R\left(\bar{\phi}^2 + \frac{m_R^2}{8\pi^2} Ln\left(\frac{m_R}{\mu}\right)\right). \tag{15}$$

It is seen from these expressions that in the limit of Λ to infinity the bare coupling constant would go to zero in order to keep g_R finite. Although this is known as the triviality it is possible to face this alternatively from the same equations by fixing the value Λ/μ instead of only varying the cutoff. Afterall, μ is just a mass scale to be fixed in the theory.

2.3 New Renormalized GAP equations

In this subsection we derive new GAP equations by minimizing the renormalized energy density $\mathcal{H}_{sub}$ with relation to the renormalized mass and to the condensate.

The derivative of the renormalized energy density with relation to m_R is required to be zero and the roots of the resulting expression can be calculated to find the minimum of the potential with relation to this parameter. The new renormalized GAP equation, is given by:

$$0 = m_R^3 \left[Ln\left(\frac{m_R}{\mu}\right)^2 a_1 + Ln\left(\frac{m_R}{\mu}\right) a_2 + a_3 \right], \tag{16}$$

where a_i can be given in terms of

$$J = 1 - \frac{g_R}{(8\pi)^2} = 1 - G_R,$$

by:

$$\begin{aligned} a_1 &= \frac{1}{g_R} J^2 + \frac{1}{32\pi^2}, \\ a_2 &= \frac{2}{g_R}\left(-1 + J + \frac{J^2}{(32\pi^2)}\right) + \frac{1}{128\pi^2}\left(1 + \frac{2J^2}{(8\pi)^2}\right), \\ a_0 &= -\frac{1}{g_R} - \frac{1}{32\pi^2} - \frac{1}{8.128\pi^4}\left(\frac{9}{8}\right). \end{aligned} \tag{17}$$

There are therefore five solutions for the renormalized mass m_R^2 which can be written in the following form:

$$\begin{aligned} m_R^3 &= 0, \\ m_R^{\pm} &= \mu exp(H^{\pm}), \end{aligned} \tag{18}$$

where:

$$H^{\pm} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1 a_0}}{2a_1}. \tag{19}$$

The (degenerated) zero mass solutions correspond to a saddle point, they are not minima of the energy density. The stability of the others solutions are checked via the positiveness of the second derivative:

$$\frac{d^2\mathcal{H}}{dm_R^2} = \frac{m_R^2}{(8\pi)^2}\left(2Ln\left(\frac{m_R}{\mu}\right) a_1 + a_2\right) > 0. \tag{20}$$

For the derivation of these expressions we have not used the completely self consistency of the Gaussian equations. There has been used a truncation on the dependence on μ: the dependence of $Ln(\mu)$ on μ was considered only to first order.

In figures 1a and 1b the solutions of the above equations $(m^{\pm}/\mu)$ are shown as a function of $G_R = g_R/(32\pi^2)$. Values between $-1 < G_R < 0$ do not correspond to physical stable values of the condensate as it will be shown below expression (24). All the solutions of figure 1a correspond to stable solutions ($d^2\mathcal{H}/dm_R^2 > 0$) but those of figure 1b are stable for G_R equal or smaller to nearly -1.45 or equal and greater than nearly 1.25. The point $g_R = 0$ was of no interest and is not plotted. In the limit of $g_R \to \pm\infty$ we obtain analytically that either $m_R = \mu$ or $m_R = 0$. While the m^- solution in the weak coupling regime can be identified to the renormalization point usually considered (for $\mu >> m_R$ or the cutoff to infinite) we found another solution m^+ for which $\mu < m^+$.

2.3.1 The condensate

The minimization of the $\mathcal{H}$ with respect to the condensate yields the following solutions of new GAP equations:

$$\begin{aligned} &\bar{\phi} = 0, \\ &\bar{\phi}^2 = -\frac{m_R^2}{g_R}\left(1 + \frac{1}{8\pi^2} Ln\left(\frac{m_R}{\mu}\right)\right). \end{aligned} \tag{21}$$

This expression may coincide with the expression of $\bar{\phi}_0$ obtained from the minimization of the regularized energy density depending on the values of the cutoff and mass scale μ.

The above expression for the condensate can also be equated to the previous (regularized) expression (8). Taking into account the flow of the renormalized coupling constant in terms of the bare one (expressions (13)) this equality will be satisfied whenever the following expression holds:

$$\lambda = \frac{16\pi^2}{Ln\left(\frac{\Lambda d}{\mu}\right)}\left(-1 + \frac{3}{2\left(1 + \frac{1}{8\pi^2} Ln\left(\frac{m_R}{\mu}\right)\right)}\right). \tag{22}$$

If the cutoff is sent to infinite the bare coupling constant may assume different values depending on the values of μ and m_R But the two expressions for the condensate are not necessarily equivalent. We have assumed, as it usually is, that the minimum of the effective potential with relation to the condensate coincides necessarily with its minimum in respect to the physical mass m_R^2. This is not neither the most general case and will be discussed further elsewhere [11].

Expression (21) can be written as:

$$g_R\bar{\phi}^2 = -m_R^2\left(1 + \frac{1}{8\pi^2} Ln\left(\frac{m_R}{\mu}\right)\right). \tag{23}$$

When $\mu = m_R exp(8\pi^2)$ we see that either $\bar{\phi} = 0$ or $g_R = 0$ in the asymmetric phase of the potential. This may correspond to the so called symmetry restoration when the condensate disappears at a particular high excitations energy.

Also, from the expression (21) we can deduce the following conditions from the sign of the coupling constant to obtain real values of $\bar{\phi}$:

$$\begin{aligned} if: g_R > 0 \to Ln\left(\frac{m_R}{\mu}\right) < -8\pi^2, \\ if: g_R < 0 \to Ln\left(\frac{m_R}{\mu}\right) > -8\pi^2. \end{aligned} \tag{24}$$

The energy density is expected be stable for the condensate values found in expression (21). This corresponds to calculating the second derivative of the energy density with relation to $\bar{\phi}$. Its positiveness corresponds to the condition:

$$g_R\left(1+\frac{g_R}{32\pi^2}\right) > 0. \tag{25}$$

From this we can state that for positive coupling constant g_R can assume any value (from this stability criterium) whereas if $g_R < 0$ one has to consider $g_R < -32\pi^2$.

3 Analysis of the renormalized coupling constant values

A relevant subject for any approximation method is to understand in which range of values of the parameters of the model (as mass and mainly coupling constants) the approximation is best appropriated. Let us check whether there are values for the renormalized coupling constant for which the renormalized energy density is minimum. Besides that this section can also be considered a preliminar calculation for the renormalization group of the model in the frame of the Gaussian approximation.

For this calculation we will consider that $d\mathcal{H}/dg_R = 0$ with a truncation of the self consistency. This is done by taking the scale parameter to be close to the renormalized mass $\mu^2 = m_R^2 + \delta$ where $\delta << m_R^2$ is determined from the GAP equation self consistently. We obtain the following equation:

$$(G'_R)^3 + (G'_R)^2\left(3 + (1+H)\frac{1}{16\pi^2}\right) + G'_R\left(3 + (1+H)\frac{3}{2} + (1+H)\frac{1}{32\pi^2}\right) + 1 + \frac{(1+H)^2}{2} = 0, \tag{26}$$

where $H = Ln\left(\frac{m_R}{\mu}\right)/(8\pi^2)$ and $G'_R = g_R/(16\pi^2)$. It is very interesting to notice that the solutions for the coupling constant depend only on the ratio m_R/μ and not on the absolute values of these parameters. This seems to correspond to a scale invariance for different physical processes at different energy scales and can be expected to be found in the standard renormalization group approach.

The stability of the solutions of the above equation can also be verified. This is done by analyzing the positiveness of the second derivative:

$$\frac{d^2\mathcal{H}_{sub}}{dg_R^2} > 0. \tag{27}$$

In figures 2a, 2b and 2c the solutions of expression (26) are showed as function of a limited range of H, i.e., $Ln(m_R/\mu)$. There are degenerated solutions in figures 2b and 2c. It is plotted only the region in which the above truncation scheme of the self consistency is expected to be reliable. All these solutions present a negative second derivative (expression (27)). These instabilities will be studied at length in the frame of the renormalization group elsewhere.

3.1 Fixing the energy density

An expression of constraint for the renormalized coupling constant involved in a physical process whose energy density is given by $\mathcal{H}$ can be found analogously of that obtained in expression (11). This corresponds to fix renormalized mass and energy scale ($\mathcal{H}_{sub}$) of a process and to calculate the allowed physical coupling constants. We obtain a third degree algebraic equation which can be writen as:

$$g_R^3\frac{H^2}{128\pi^2} + g_R^2\left(\frac{H^2}{4} - \frac{2H^2}{x}\right) + g_R\left(-\frac{H(H+1)}{2} + \frac{1}{x}\left(\frac{H}{4} - 1 + H^2\right) - \frac{\mathcal{H}_{sub}}{m_R^4}\right) - \frac{1}{4} + \frac{H^2}{4} = 0. \tag{28}$$

where $x = 128\pi^2$ and $H = Ln(m_R/\mu)/(8\pi^2)$. This expression presents the scale invariance for the parameters m_R/μ unless by the term which depends on the energy density if $\mathcal{H}_{sub}$ scales differently from m_R^4.

In figures 3a, 3b and 3c we show the solutions of this algebraic equation as function of H for a fixed energy density $\mathcal{H} = (100MeV)^{-4}$ and $m_R = 100MeV$. There is almost no deviation in the numerical results for a very large range of the ratio $\mathcal{H}/m_R^4$. g_R can be strong in the region of $\mu \simeq m_R$. And in the limit of $\mu = m_R$ we found an unique value -this is not evident in figure 3a- which is given by:

$$g_R = -\frac{1}{4\left(\frac{\mathcal{H}}{m_R^4} + \frac{1}{128\pi^2}\right)}. \tag{29}$$

Besides that for $H \to -\infty$, which may be equivalent to $\mu/m_R \to \infty$, whereas in figure 3a we see $g_R \to 0$ solutions of figure 3b and 3c does not correspond to $g_R \to 0$, but to finite values close to 10.

4 Summary

Consequences of the usual renormalization scheme for the variational Gaussian approximation were analyzed. The regularized and (new) renormalized derivations of the GAP equations were performed by minimizing the renormalized energy density. New values for the condensate in the vacuum were found by minimizing the effective potential with relation to $\bar{\phi}$. From this expression we notice that either the condensate or g_R disappears when the mass scale (introduced in the renormalization procedure) assumes the value $\mu = m_R exp(8\pi^2)$. This can be seen as a restoration of the spontaneously symmetry breaking. Constraints for the possible values of the renormalized coupling constant were found. We found, in particular, that g_R can be positive or negative and eventually very strong being the mass scale μ a relevant parameter to fix its value. The coupling constant fixes the values of the renormalized mass which yield a minimum of the effective potential. A minimization of the effective potential with respect to the coupling constant was also performed in the limiting case that the mass scale μ is close to the physical mass. This is a way of truncating the self-consistency of the approximation. No stable solution for g_R (considered without the whole renormalization group equations) was found within the truncation scheme done for the self consistency. Furthermore we have assumed along this work, as usually it is, that the minimum of the effective potential with relation to the condensate and with relation to m_R coincide. This is not necessarily true. This is, however, the first step towards a more complete analysis [11].

Acknowledgements

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Figure captions

Figure 1a - First solution of expression (16) - a new GAP equation - for the ratio of the renormalized mass to the mass scale μ as a function of $g_R/(8\pi^2)$.

Figure 1b - Second solution of expression (16) - a new GAP equation - for the ratio of the renormalized mass to the mass scale μ as a function of $g_R/(8\pi^2)$.

Figure 2a - First solution of expression (26) for the renormalized coupling constant - from the minimization of the energy density with relation to the renormalized coupling constant - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 2b - Second solution of expression (26) for the renormalized coupling constant - from the minimization of the energy density with relation to the renormalized coupling constant - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 2c - Third solution of expression (26) for the renormalized coupling constant - from the minimization of the energy density with relation to the renormalized coupling constant - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 3a - First solution for the renormalized coupling constant of expression (28) - fixing $\mathcal{H} = (100MeV)^4$ and $m_R = 100MeV$ - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 3b - Second solution for the renormalized coupling constant of expression (28) - fixing $\mathcal{H} = (100MeV)^4$ and $m_R = 100MeV$ - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 3c - Third solution for the renormalized coupling constant of expression (28) - fixing $\mathcal{H} =$

$(100 MeV)^4$ and $m_R = 100 MeV$ - as a function of $H = Ln(m_R/\mu)/(8\pi^2)$.

Figure 1a

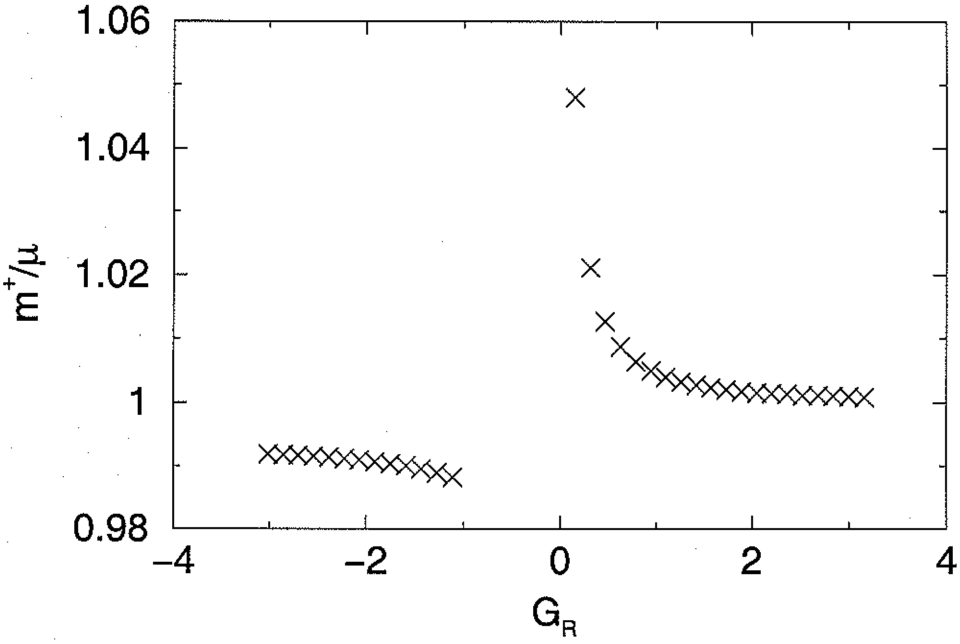

Figure 1b

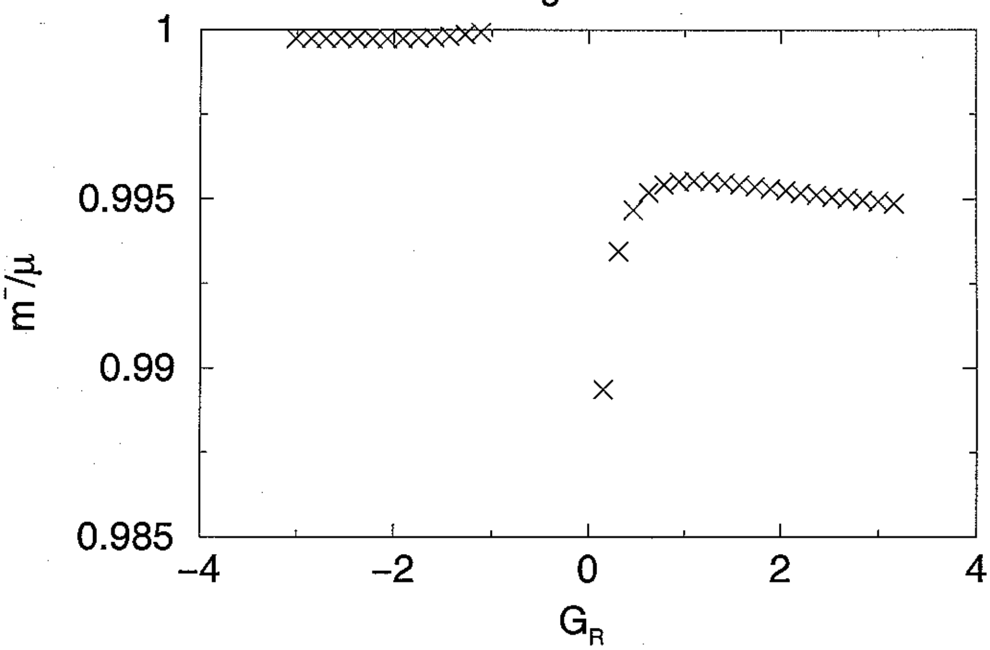

Figure 2a

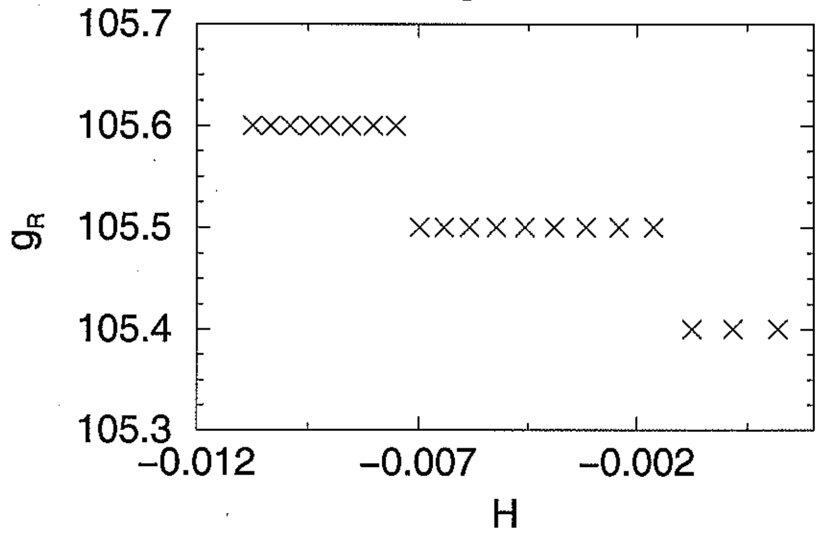

Figure 2b

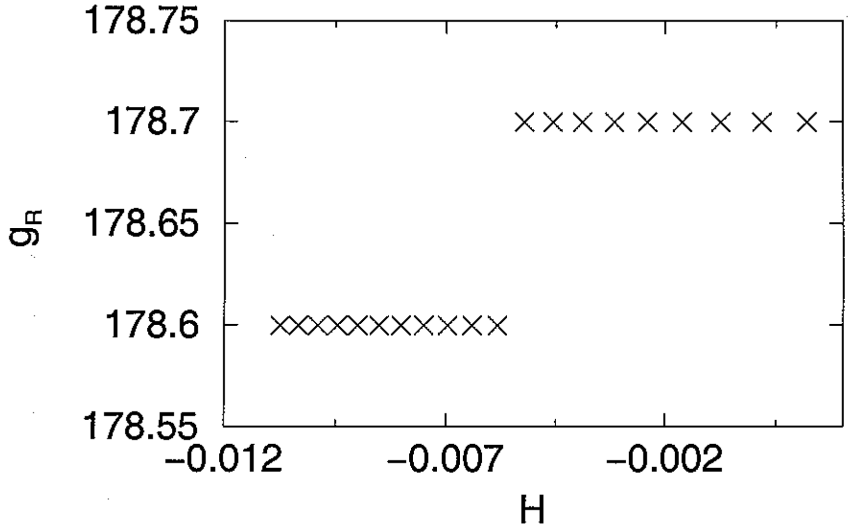

Figure 2c

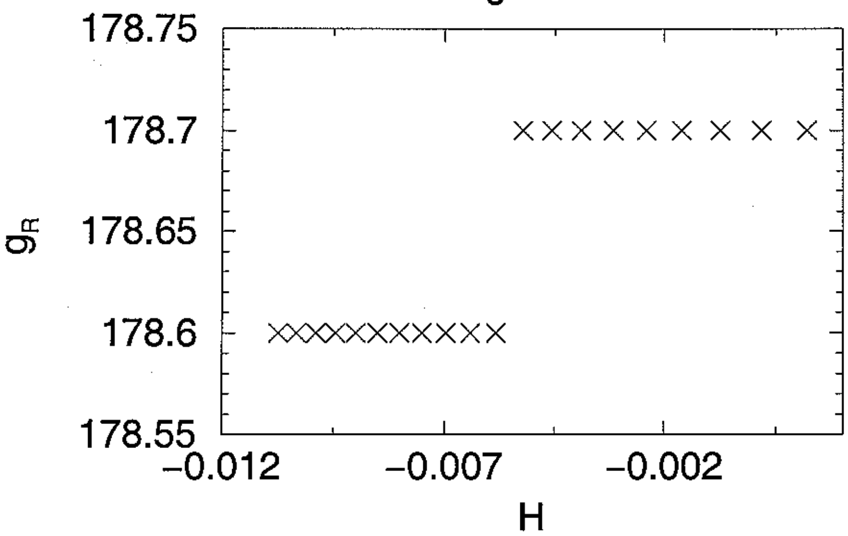

Figure 3a

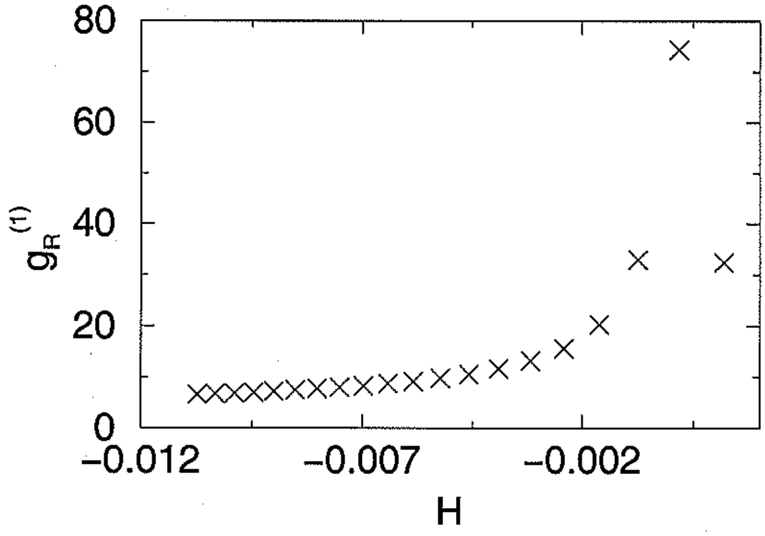

Figure 3b

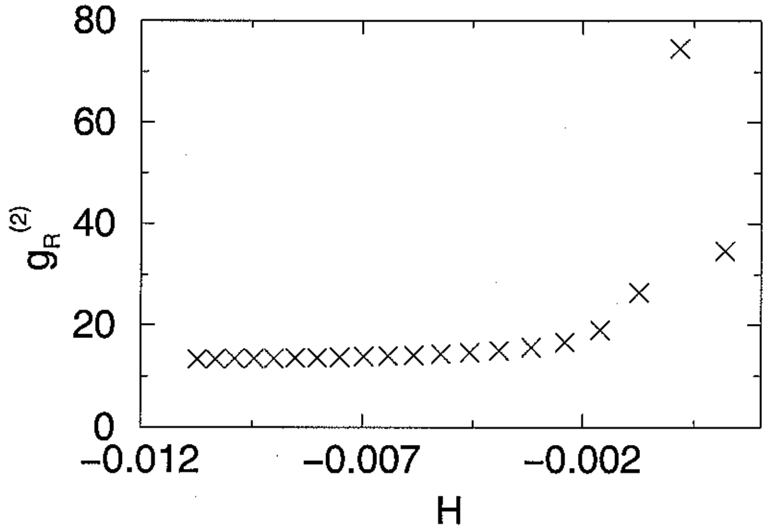

Figure 3c