Instituto de Física
Universidade de São Paulo

SPONTANEOUS SYMMETRY BREAKINGS IN THE LINEAR SIGMA MODEL AT FINITE BARYONIC DENSITY

BRAGHIN, FL

Instituto de Física, Universidade de São Paulo, CP 66.318
05315-970, São Paulo, SP, Brasil

Publicação IF – 1573/2001

UNIVERSIDADE DE SÃO PAULO
Instituto de Física
Cidade Universitária
Caixa Postal 66.318
05315-970 - São Paulo - Brasil

Spontaneous Symmetry Breakings in the Linear Sigma Model at finite baryonic density

Fábio L. Braghin *

Nuclear Theory and Elementary Particle Phenomenology Group,

Instituto de Física, Universidade de São Paulo

C.P. 66.318; CEP 05315-970; São Paulo - SP, Brazil.

Abstract

The O(4) invariant linear sigma model coupled to baryons and a massive vector gauge meson is considered for the description of the finite baryonic density system. The stability equation is analyzed in the homogeneous limit considering quantum fluctuations for the scalar and pseudo scalar fields in a variational way. Truncated solutions are sought semi-analytically. All the bosonic components are found to have non zero expected classical values at finite density corresponding to dynamical symmetry breakings. There also exists a non trivial solution for the classical vector meson field, as another condensate, indicating the occurrence of a gauge dynamical symmetry breaking at finite density.

PACS numbers: 11.15.Ex; 11.30.Rd; 12.38.Aw; 12.39.Fe.

Key-words: Spontaneously symmetry breaking, finite density, gauge symmetry, chiral symmetry, condensates.

IF- USP - 2001

*e-mail: braghin@if.usp.br

1 Introduction

Because the fundamental theory for the strong interacting systems, the Quantum Cromodynamics (QCD), has a very complex non abelian structure and strong coupling constants at low energies it is very difficult to obtain exact solutions. One is therefore lead to construct effective models which respect the main properties and symmetries of the QCD for those energy ranges. In the vacuum, the lightest strong interacting particles are known to respect, approximately at least, chiral symmetry $SU_L(2) \times SU_R(2)$ which is spontaneously broken down to $SU(2)$. Pions, whose masses are small in the hadronic scale, are viewed as the Goldstone bosons of such SSB. The vacuum acquires a non trivial structure due to the formation quark-anti quark condensate $< \bar{q}q >$, the order parameter of the Chiral SSB. These features can be taken into account via sigma models which, in the linear realization with mesons, implement chiral symmetry with two fields: the (pseudo-scalars) pions and the (scalar) sigma. In the vacuum the scalar field acquires a non zero expected value in the vacuum, $< \sigma > \propto < \bar{q}q >$, which lowers the energy density of the vacuum, but $< \vec{\pi} >_{vac} = 0$. At finite density, QCD is known, and expected, to have a very complex phase diagram with the appearance of other condensates [1]. With different approaches finite density QCD and effective models have been intensively studied in the last years.

In this work we study the O(N) Linear Sigma Model (LSM) coupled to baryons and a massive vector meson. The exact field equations are truncated to allow for analytical solutions by considering particular prescriptions for the stability condition of the system. We found that there is a (isospin) spontaneously broken symmetry generating non zero expected value for the pion field at not necessarily very high densities. In the next section the linear sigma model with an Abelian massive gauge vector boson is presented and their dynamical equations are shown considering quantum fluctuations of the sigma and pion fields in a truncated version of a variational approach. Their solutions are discussed and some other possible consequences are analyzed.

2 Linear sigma model at finite density

The Lagrangian density of the Linear Sigma Model for nucleons $N(\mathbf{x})$, sigma and pions $(\sigma, \vec{\pi})$ coupled to a massive gauge vector meson V_μ is given by:

$$\begin{aligned} \mathcal{L} = & \bar{N}(\mathbf{x}) \left(i\gamma_\mu \mathcal{D}^\mu - g_S(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi})\right) N(\mathbf{x}) + \frac{1}{2}\left(\partial_\mu \sigma.\partial^\mu \sigma + \partial_\mu \vec{\pi}.\partial^\mu \vec{\pi}\right) + \\ & -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{4}\left((\sigma)^2 + (\vec{\pi})^2 - v^2\right)^2 + \frac{1}{2} m_V^2 V_\mu V^\mu, \end{aligned} \quad (1)$$

where the covariant gauge derivative is: $\mathcal{D}^\mu = \partial^\mu - ig_V V^\mu$, the gauge invariant tensor is: $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. g_V, g_S and λ are the coupling constants and $v = f_\pi$ in the vacuum. The gauge invariant coupling of the vector meson to the baryons is equivalent to the definition of a chemical potential. If the pion mass were considered to break the chiral symmetry explicitly we could add another term $\mathcal{L}_{sb} = c\sigma$. This would have no further relevance for our original results. We consider the possible existence of condensates for all the mesonic fields and look for solutions. In this work the whole baryon masses come from the the coupling to the scalar mesonic field by the Higgs mechanism ($M^* = g_S\bar{\sigma}$). The assumption of the existence of an explicit mass term for the baryons in the Lagrangian due to the quark content does not change the original results of this work. This would lead to an *in medium* mass: $M^* = M + g_S\bar{\sigma}$ [3]. From expression (1) the Hamiltonian can be calculated.

The baryon (nucleon) field is quantized in terms of creation and annihilation operators. Its wave function can be written as superposition of spinor (χ), isospinor (η) and coordinate components $u(\mathbf{p})$, $v(\mathbf{p})$. It generates non zero scalar, baryonic and pseudo-scalar densities (ρ_s, ρ_B and ρ_{PS}). We will not explicitly evaluate here all these quantities but only make use of ρ_B. The resulting expressions for the quantized fermionic energy density ρ_f and for the baryonic density ρ_B are given respectively by:

$$\rho_f = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + (M^*)^2}, \qquad \rho_B = 4\int^{k_F} \frac{d^3k}{(2\pi)^3}. \tag{2}$$

In these expressions 4 is the degeneracy of fermions and k_F is the nucleon momentum at the Fermi surface.

To take into account the quantum fluctuations of the sigma and pion fields we consider a truncated version of the variational approach using a Gaussian trial wave-functional in the Schroedinger picture [2]. This variational principle states that a maximum bound for the energy density of the vacuum can be obtained by calculating the averaged energy with trial wave-functions ($\mathcal{H}_\phi =< \Phi|\hat{H}|\Phi >$) whose (trial) parameters are fixed when the energy density is minimum with relation to its parameters. The averaged value of the Hamiltonian is calculated with trial Gaussian wave-functionals for the scalar and pseudo-scalar fields: $< \tilde{\Psi}[\sigma, \bar{\vec{\pi}}]|H(\sigma, \vec{\pi})|\tilde{\Psi}[\sigma, \bar{\vec{\pi}}] >$. For the sigma we can write:

$$\Psi\left[\sigma(\mathbf{x})\right] = N exp\left\{-\frac{1}{4}\int d\mathbf{x}d\mathbf{y}\delta\sigma(\mathbf{x})G_S^{-1}(\mathbf{x},\mathbf{y})\delta\sigma(\mathbf{y})\right\}, \tag{3}$$

Where $\delta\sigma(\mathbf{x}) = \sigma(\mathbf{x}) - \bar{\sigma}$; the normalization is N, the variational parameters are the condensate $\bar{\sigma} =< \Psi|\sigma|\Psi >$, the quantum fluctuations represented by the width of the Gaussian $G_S(\mathbf{x},\mathbf{y}) =< \Psi|\sigma(\mathbf{x})\sigma(\mathbf{y})|\Psi >$. An analogous expression for the pion sector is considered with variational parameters given by: $\bar{\vec{\pi}} =< \vec{\pi} >$ and $G_P^{a;b}$, which is a matrix that can be considered to be diagonal as a particular case along this work ($G_P^{a,a} = G_P$). This reduces the corresponding functional space but it guarantees the explicit chiral invariance. We will assume that these quantum fluctuations only intervene for the meson masses, as shown below, as well as for a shift of the respective fields:

$$\tilde{\vec{\pi}}^2 = \bar{\vec{\pi}}^2 + G_P, \qquad \tilde{\sigma}^2 = \bar{\sigma}^2 + G_S. \tag{4}$$

This corresponds to a truncation on the self consistency with a particular renormalization energy point. More exact calculations are being done for this model in [3].

The minimizations of the averaged energy with respect to the Gaussian variational parameters yield the GAP and condensate equations which define the minimum of the potential for these fields. The following set of equations is obtained for the sigma sector:

$$\begin{aligned} &\frac{\delta \mathcal{H}}{\delta \bar{\sigma}} = 0 \to \bar{\sigma}\lambda\left(\bar{\sigma}^2 + 3G_S + \bar{\vec{\pi}}^2 + G_P - v^2\right) + \frac{d\rho_f}{d\bar{\sigma}} = 0; \\ &\frac{\delta \mathcal{H}}{\delta G_S} = 0 \to \frac{d\rho_f}{dG_S} - \frac{G_S^{-2}}{8} - \frac{\Delta}{2} + \frac{\lambda}{4}\left(6\bar{\sigma}^2 + 2\bar{\vec{\pi}}^2 + 6G_S + 2G_P - 2v^2\right) = 0. \end{aligned} \tag{5}$$

From this last expression we can write the following expression for the sigma mass:

$$\mu_S^2 = \lambda\left(3\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2 + ...\right) \tag{6}$$

For the sake of clearness we truncate the exact solution considering that:

$$\frac{d\rho_f}{dG_i} = 0, \qquad G_P = G_S = G \simeq constant. \tag{7}$$

The corresponding GAP equations for the pion field are:

$$\begin{aligned} &\lambda\bar{\pi}_a\left(\tilde{\vec{\pi}}^2 + 2G_P + \tilde{\sigma}^2 - v^2\right) + \frac{d\rho_f}{d\bar{\pi}_a} = 0; \\ &\mu_P^2 = \lambda\left(3\tilde{\vec{\pi}}^2 + \tilde{\sigma}^2 - v^2 + ...\right) \end{aligned} \tag{8}$$

The first of these expressions contains one of the most relevant results of the present work. As far as the fermionic density depends on $\bar{\sigma}$, via the *in medium* mass M^*, and the equation for $\bar{\sigma}$ (expression (5) does depend on $\bar{\pi}_a$ we find that the pion condensate $\bar{\pi}_a$ must be nonzero whenever the fermionic density (and therefore baryonic density) is not zero. We have found therefore that:

$$\frac{d\rho_f}{d\bar{\pi}_a} = \frac{d\rho_f}{d\bar{\sigma}}\frac{d\bar{\sigma}}{d\bar{\pi}_a} \neq 0. \tag{9}$$

As discussed above, the expressions (6,8) for the meson masses, with the shift of the fields (4), are the only effects of the quantum fluctuations in the present work. Since we will not explicitly calculate the two point function G_i, there will be no concern with Ultra Violet divergences here. Assuming the usual hypothesis of a explicit symmetry breaking ($\mathcal{L}_{sb} = c\sigma$) to generate the pion mass the results of this work are almost unchanged.

The Euler Lagrange equation for the vector meson was calculated for a gauge in which only the component V_0 is non zero and homogeneous. If we consider the other components $V_i \neq 0$ the equations are slightly changed, but the conclusions remain valid with another coupled equation. The equation is given by:

$$g_V \left(\rho_B + V_0 \frac{d\rho_B}{dV_0} \right) - m_V^2 V_0 = 0. \tag{10}$$

The total averaged energy density can be written as:

$$\mathcal{H} = \rho_f + g_V V_0 \rho_B - \frac{1}{2} m_V^2 V_0^2 + \frac{\lambda}{4} (\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)^2. \tag{11}$$

3 Truncated Solutions

In this section solutions for the above equations are given such that the main properties of a (bound) finite density system are consistently described. The stability condition of the system can be written as:

$$\left. \frac{d\mathcal{H}}{d\rho_B} \right|_{\rho_B = \rho_0} = \left. \frac{\mathcal{H}}{\rho_B} \right|_{\rho_B = \rho_0} < 0, \tag{12}$$

where ρ_0 is the stable density. To guarantee that this expression is satisfied we consider some prescriptions for the dependence of the involved variables in on the baryonic density. Namely:

$$\begin{array}{l} \dfrac{d\rho_f}{d\rho_B} = \dfrac{\rho_f}{\rho_B}, \\ \dfrac{d(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{d\rho_B} = \dfrac{(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2)}{\rho_B}, \\ \dfrac{d\mathcal{H}_V}{d\rho_B} = \dfrac{\mathcal{H}_V}{\rho_B}. \end{array} \tag{13}$$

In this last expression $\mathcal{H}_V$ is the energy density with contributions of the vector meson.

From the first of the differential equations (13) we find a solution for the dependence of ρ_f on the baryonic density ($\rho_f = \rho_f(\rho_B)$) which is in excellent agreement with that resulting from the integration of expression (2) [3].

From the second expression in (13) we find a constraint which can be defining a symmetry radius in the medium:

$$(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - v^2) = \tilde{C} \sqrt{\rho_B}. \tag{14}$$

In this expression $\tilde{C}$ is a constant. Therefore in the vacuum: $\tilde{\sigma}^2 = v^2 = f_\pi^2$ as discussed above.

The GAP and condensate equations (5, 8), for $\tilde{\sigma}$ and for the $\tilde{\vec{\pi}}$ can be faced as differential equations for ρ_f and they can be written as:

$$
\begin{aligned}
(\tilde{\sigma}^2 + \tilde{\vec{\pi}}^2 - \tilde{v}^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\sigma}^2} \simeq 0, \\
(\tilde{\vec{\pi}}^2 + \tilde{\sigma}^2 - \tilde{v}^2) + \frac{2}{\lambda}\frac{d\rho_f}{d\tilde{\vec{\pi}}^2} \simeq 0.
\end{aligned}
\tag{15}
$$

Where $\tilde{v}^2 = v^2 - G$. These equations are isomorfic and show an equal dependence of ρ_f with each of these (chiral) fields. An approximated solution for the two condensate equations can be found for as if ρ_f were a function of these fields independently. This yields $\rho_f = \rho_f^{(1)}(\bar{\sigma})$ and $\rho_f = \rho_f^{(2)}(\bar{\vec{\pi}})$. Eliminating the ρ_f from the solution of the first expression for the second we find the following approximated value for the pion condensate (if $|\tilde{\vec{\pi}}^2| << v^2$):

$$
\tilde{\vec{\pi}}^2 \simeq \frac{\tilde{\sigma}^2(\tilde{\sigma}^2 - \tilde{v}^2)}{4(-\frac{\tilde{\sigma}^2}{2} \pm \tilde{v}^2)},
\tag{16}
$$

In these solutions, as well as in others more exact, $\tilde{\vec{\pi}}^2$ may be either positive or negative. More exact solution for the condensate equations will be given elsewhere [3].

We can also add the two differential equations (15) which can be seen as partial differential equations. An approximated solution is:

$$
\begin{aligned}
\rho_f^{(1)} &\simeq \frac{\lambda}{2}\tilde{\sigma}^2\left(\tilde{v}^2 - \tilde{\sigma}^2\right) + C_f(\tilde{\sigma}^2 - \tilde{v}^2), \\
\rho_f^{(2)} &\simeq \frac{\lambda}{2}\tilde{\vec{\pi}}^2(\tilde{v}^2 - \tilde{\vec{\pi}}^2) - C_f\tilde{\vec{\pi}}^2,
\end{aligned}
\tag{17}
$$

Where C_f is a constant. With the above expressions for ρ_f and the symmetry radius $\tilde{C}\sqrt{\rho_B}$ (14) we obtained a consistent basis for the study of the behavior of the condensates (as order parameters) from a Quantum Field Theory which has a SSB with density. Expecting that these solutions are valuable in some range of ρ_f as well as ρ_B we can equate expressions (17) to obtain the following constraint expression for the condensates:

$$
C_f\tilde{C}\sqrt{\rho_0} = \frac{\lambda}{2}\left((\tilde{\sigma}^2 - \frac{v^2}{2})^2 - (\tilde{\vec{\pi}}^2 - \frac{v^2}{2})^2\right).
\tag{18}
$$

This also expresses the dynamical symmetry breaking which occurs in the medium mainly because $C_f\tilde{C}\sqrt{\rho_B} \neq 0$.

Finally, considering the equation of V_0 - expression (10) - as a differential equation of the baryonic density ρ_B as a function of V_0 we obtain the following solution:

$$
V_0(\rho_B) = \frac{-g_V\rho_B \pm \sqrt{g_V^2\rho_B^2 - 2C_V\rho_B m_V^2}}{m_V^2},
\tag{19}
$$

where C_V is a constant. This constant will be the only contribution of the vector meson sector to the energy density $\mathcal{H}_V = C_V \rho_B$. In the limit of zero density we get $V_0 \to 0$. It is seen that the baryonic density generates a non zero value of V_0 - which can be viewd as a condensate. This may be another dynamical symmetry breaking. Requiring the energy to be stable with relation to the solution V_0 we find that: $C_V = 2g_V\bar{V}_0$. Where $\bar{V}_0$ is the value of V_0 at a stable density ρ_0. In this point we have $m_V^2 = -2g_V\rho_0/\bar{V}_0$. From this we see that the mass of the vector meson is proportional to the density, i.e., it is an *in medium* effect. This also seems to suggest the existence of still other QCD condensate(s) at finite density associated with vector meson(s).

3.1 Discussion

With the present results it is not possible yet to decide, unambiguously, the symmetry behavior with density as precluded in other works [4]. Besides the appearance of another condensate, $\bar{\vec{\pi}}$ due to another SSB, we find the possibility, although not very appealing, of further chiral symmetry breaking with $\bar{\sigma} > v$ at finite baryonic density [3]. This depends on the values and signs of the coupling constants g_S and λ [3].

From the coupling of pions to fermions in the Lagrangian, we can see that the non zero pion condensate will induce a difference between the *in medium* baryonic isospin states (which can be the neutron and proton). This non trivial solution corresponds to a non invariant ground state under an isospin transformation, although the Lagrangian is symmetric. Zero energy (Goldstone) collective modes are therefore expected to occur [3]. To what extend this SSB and chiral SSB as well as the underlying gauge symmetry for the vector meson are related is not yet clear. The present model will be considered for the description of hadronic (nuclear) matter properties and eventually of atomic nuclei elsewhere [3].

Acknowledgement

This work was supported by FAPESP, Brazil.

References

[1] R. Rapp, T. Schäfer, E. Shuryak, M. Velkovsky, Phys. Rev. Lett., 53 (1998); M. Alford, K. Rajagopal, F. Wilczek, Phys. Lett. **B 422**, 247 (1998).

[2] P.M. Stevenson, Phys. Rev. **D 32**, 1389 (1985); W.A. Bardeen, M. Moshe, Phys. Rev. **D 28**, 1372 (1983); A.K. Kerman, C. Martin, D. Vautherin, Phys. Rev. **D 47**, 632 (1993) and references therein.

[3] F.L. Braghin, *in preparation*; F.L. Braghin, Phys. Rev. **D 64**, 1250001 (2001) - hep-ph/0106338.

[4] G.E. Brown and M. Rho, Phys. Rep. **269**, 333 (1996).