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PROJECTILE IN ASTROPHYSICAL  
FUSION REACTIONS**

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# Role of virtual break-up of projectile in astrophysical fusion reactions

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## Abstract

We study the effect of virtual Coulomb break-up, commonly known as the dipole polarizability, of the deuteron projectile on the astrophysical fusion reaction  ${}^3\text{He}(d,p){}^4\text{He}$ . We use the adiabatic approximation to estimate the potential shift due to the  $E1$  transition to the continuum states in the deuteron, and compute the barrier penetrability in the WKB approximation. We find that the enhancement of the penetrability due to the deuteron break-up is too small to resolve the longstanding puzzle observed in laboratory measurements that the electron screening effect is surprisingly larger than theoretical prediction based on an atomic physics model. The effect of the  ${}^3\text{He}$  break-up in the  ${}^3\text{He}(d,p){}^4\text{He}$  reaction, as well as the  ${}^7\text{Li}$  break-up in the  ${}^7\text{Li}(p,\alpha){}^4\text{He}$  reaction is also discussed.

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The problem of electron screening effect on nuclear fusion reactions measured at a laboratory at very low incident energies has not yet been fully understood. The rise of the astrophysical  $S$ -factor for reactions such as  ${}^3\text{He}(d, p){}^4\text{He}$  and  $D({}^3\text{He}, p){}^4\text{He}$  as the incident energy goes down below about 50 keV has been attributed to the screening effect of the bound electrons in the target atom (or molecule), which shields the Coulomb potential between the colliding nuclei [1,2]. It has been found, however, that the amount of the enhancement of the  $S$ -factor can be accounted for only when an unrealistically large electron screening energy  $U_e$  is used in a calculation [3-7]. The value of  $U_e$  required to fit the data ranges between 0.88 and 14.5 times the adiabatic value (see Table 1 in Ref. [8] for a summary), where the screening energy is given by a difference of electron binding energies between the unified and isolated systems [1,9]. Since the adiabatic approximation should provide the upper limit of the screening energy [10-13], the mechanism of the low energy enhancement in the  $S$ -factor remains an open problem. A noteworthy recent paper is by Barker, who refitted the experimental data by including the screening correction as a free parameter and obtained smaller screening energies which are consistent with the adiabatic value in many systems [14]. However, for some cases, the optimum screening energy still exceeds the adiabatic value, and the fitted  $S$  factors are somewhat in disagreement with the experimental result of the Trojan-horse method [15-17], which is believed to provide the bare cross sections without the influence of bound electrons. Thus, the problem has not been resolved completely yet.

Besides the electron screening effects, several small effects on astrophysical fusion reactions have also been examined. These include vacuum polarization [18], relativity [18], bremsstrahlung outside the barrier [18], atomic polarization [18], radiation correction during the tunneling [19], zero point fluctuation of nuclei in the atom and the molecule [8], and the effect of finite beam width [8]. All of these effects have been found much smaller than the screening effect.

In this paper, we consider more corrections to astrophysical fusion reaction. An important fact is that the classical turning point of interest is much larger than the nuclear size (for instance, it is 288 fm for the  $d+{}^3\text{He}$  reaction at  $E_{c.m.}=10$  keV), and effects which are relevant to the reaction have to be associated with the Coulomb interaction or otherwise very long ranged. The effects associated with the nuclear interaction will be washed out by a careful choice of effective nuclear potential between the projectile and the target, unless the energy dependence is very strong [19]. In this sense, the nuclear absorption under the barrier [20-22], which has been discussed in connection with the sharp rise of nuclear  $S$ -factor for the  ${}^{12}\text{C} + {}^{12}\text{C}$  fusion reaction at low energies, for instance, is not helpful for the astrophysical reactions. One may also think about the non-local effects of the internuclear potential on the tunneling phenomena [23-25]. However, this effect does not seem significant either, as one can cast from the result of microscopic cluster model calculation for the  $d+{}^3\text{He}$  reaction at low energies [26], where the exchange effect has been included both in the nuclear and in the Coulomb interactions.

As another example of small effect on fusion, we here consider the effect of Coulomb break-up of colliding nuclei. At energies which we are interested in, the break-up channel is most likely kinematically forbidden. However, the tunneling probability is still influenced through the virtual process [27], and it is important to estimate the size of its effect for a complete understanding of the reaction mechanism. This effect is also known as dipole

polarizability. To our knowledge, this effect has not yet been computed in the literature, although a few calculations have existed based on the continuum-discretized-coupled-channels (CDCC) method for transfer reactions at much higher energies, which were performed in aiming at extracting the cross section of the astrophysical radiative capture reactions at zero incident energy [28,29]. We also notice that the enhancement of tunneling probability due to the break-up coupling has been extensively discussed in the context of subbarrier fusion reaction of a halo nucleus [30–34].

We use a three-body model in order to estimate the effect of the virtual Coulomb excitation of projectile on the tunneling probability. Denoting the coordinate between the target and the center of mass of the projectile by  $\mathbf{R}$  and the coordinate between the projectile fragments by  $\mathbf{r}$ , the Coulomb interaction in this system reads

$$V_C(\mathbf{R}, \mathbf{r}) = \frac{Z_1 Z_T e^2}{|\mathbf{R} + m_2 \mathbf{r} / (m_1 + m_2)|} + \frac{Z_2 Z_T e^2}{|\mathbf{R} - m_1 \mathbf{r} / (m_1 + m_2)|}, \quad (1)$$

$$\sim \frac{Z_P Z_T e^2}{R} + \frac{4\pi}{3} \frac{Z_T e}{R^2} \sum_{\mu} Y_{1\mu}^*(\hat{\mathbf{R}}) \hat{T}_{\mu}^{E1}, \quad (2)$$

where  $\hat{T}_{\mu}^{E1}$  is the  $E1$  operator given by

$$\hat{T}_{\mu}^{E1} = e_{E1} r Y_{1\mu}(\hat{\mathbf{r}}). \quad (3)$$

Here,  $e_{E1}$  is the  $E1$  effective charge given by  $(m_2 Z_1 - m_1 Z_2)e / (m_1 + m_2)$ , where  $m_1$  and  $m_2$  are the mass of the projectile fragments while  $Z_1$  and  $Z_2$  are their charges ( $Z_P = Z_1 + Z_2$  is the total charge of the projectile). For a head-on collision, the incident channel is a  $s$ -wave bound state  $\phi_0(\mathbf{r})$  of the projectile coupled to the relative angular momentum  $L = 0$  for the  $\mathbf{R}$  coordinate. This channel couples to a  $p$ -wave state  $\phi_1(\mathbf{r})$  of the projectile via the coupling interaction (2). The relative angular momentum for the  $\mathbf{R}$  coordinate has to be  $L = 1$  in the excited state channel so that the total angular momentum is conserved. For simplicity, we have neglected the spin of the projectile fragments. The matrix element of the coupling potential between these channels is given by

$$F(R) = \frac{\sqrt{4\pi}}{3} \frac{Z_T e^2}{R^2} \sqrt{\frac{B(E1) \uparrow}{e^2}}, \quad (4)$$

where  $B(E1) \uparrow = |\langle \phi_0 | \hat{T}^{E1} | \phi_1 \rangle|^2$  is the strength of the electric dipole transition of the projectile.

For an exponential wave function for the bound state  $\phi_0$  together with the plane wave function for the scattering state  $\phi_1$ , a simple and compact expression for  $B(E1) \uparrow$  has been derived by Bertulani, Baur, and Hussein [35], which is given by

$$\frac{dB(E1) \uparrow}{dE_{\gamma}} = \frac{3\hbar^2 e_{E1}^2}{\pi \mu_{12}^2} \frac{\sqrt{\epsilon} (E_{\gamma} - \epsilon)^{3/2}}{E_{\gamma}^4}, \quad (5)$$

where  $\mu_{12} = m_1 m_2 / (m_1 + m_2)$  is the reduced mass of the projectile system and  $\epsilon$  is the binding energy. This function has a peak at  $E_{\gamma} = 8\epsilon/5$ , and the total dipole strength is given by [35]

$$B(E1) \uparrow = \frac{3\hbar^2 e_{E1}^2}{16\pi\mu_{12}\epsilon}. \quad (6)$$

In this work, for simplicity, we assume that the  $E1$  strength is exhausted by a single state at  $E_\gamma = 8\epsilon/5$  with the strength given by Eq. (6). With this prescription, the problem is reduced to the two dimensional coupled-channels calculation with the coupling matrix given by

$$V_{\text{coup}}(R) = \begin{pmatrix} 0 & F(R) \\ F(R) & E_\gamma + \frac{2\hbar^2}{2\mu R^2} \end{pmatrix}, \quad (7)$$

where  $\mu$  is the reduced mass for the  $R$  motion.

In order to estimate the coupling effect, we use the adiabatic approximation and derive the adiabatic potential shift by diagonalizing the coupling matrix (7) at each  $R$ . Taking the smaller eigenvalue, the potential shift is given by

$$\Delta V_{ad}(R) = \frac{\left(E_\gamma + \frac{2\hbar^2}{2\mu R^2}\right) - \sqrt{\left(E_\gamma + \frac{2\hbar^2}{2\mu R^2}\right)^2 + 4F(R)^2}}{2}. \quad (8)$$

Note that this potential shift coincides with the adiabatic polarization potential which Alder *et al.* derived using the second order perturbation theory [36] (see also Ref. [37] for a derivation using the Feshbach formalism), in the limit of  $E_\gamma \gg F(R)$  and when one ignores the angular momentum transfer. We then compute the tunneling probability using the WKB formula for low energy,

$$P(E) = \exp \left[ -2 \int_{R_0}^{R_1} dR \sqrt{\frac{2\mu}{\hbar^2} (V_0(R) + \Delta V_{ad}(R) - E)} \right], \quad (9)$$

where  $V_0(R) = Z_P Z_T e^2/R$  is the bare Coulomb interaction, and  $R_0$  and  $R_1$  are the inner and the outer turning points, respectively. Notice that the adiabatic approximation provides the upper limit of the potential penetrability [10–12]. Therefore, our results should be regarded as the upper limit of the virtual break-up effects in the astrophysical reactions, although the adiabatic approximation should work well at astrophysical energies.

The effect of the target break-up can also be taken into account in a similar manner. In this case, one considers five channel states: i) the incident channel, ii) the projectile break-up channel, iii) the target break-up channel, iv) the mutual break-up channel with the relative angular momentum  $L = 0$ , and v) the mutual break-up channel with  $L = 2$ . Both of the channels iv) and v) are coupled to the channels ii) and iii) by the  $E1$  operator of the target and of the projectile, respectively. The adiabatic potential  $\Delta V_{ad}(R)$  is given as the lowest eigenvalue of the  $5 \times 5$  coupling matrix at each  $R$ . Here we neglect the dipole-dipole term in the interaction, which we assume to be much smaller than the monopole-dipole term.

Let us now numerically estimate the effect of the virtual break-up coupling on astrophysical fusion reactions. We first consider the effect of deuteron break-up on the  $d+{}^3\text{He}$  reaction. The break-up  $Q$ -value is  $\epsilon=2.22$  MeV, and Eq.(6) leads to the total  $B(E1) \uparrow$  strength of  $0.558$  ( $e^2 \text{ fm}^2$ ). The dashed line in Fig. 1 shows the enhancement factor  $f$  of the penetrability,  $P(E)/P_0(E)$ , as a function of the center of mass energy  $E_{c.m.}$ , where  $P_0(E)$

is the penetrability of the bare Coulomb interaction  $V_0(R)$ . We take  $R_0 = 4.3$  fm for the inner turning point [18]. We see that the enhancement factor slowly decreases as the energy decreases. The value of the enhancement factor is about 0.21 % at  $E_{c.m.} = 5.8$  keV, which is smaller than the effect of vacuum polarization [18] by one order. Therefore, the effect of the dipole polarizability of deuteron seems to be negligible as compared to the vacuum polarization effect. The dotted line shows the effect of  ${}^3\text{He}$  break-up, where the binding energy is 5.49 MeV from the threshold of the  $d + p$  system. Although this effect is much smaller than the deuteron break-up effect, the combined effect of the mutual excitations increases the penetrability in a non-negligible way (see the solid line). Figure 2 shows the effect of dipole break-up of  ${}^7\text{Li}$  nucleus (into  $\alpha + t$ ) on the  $p + {}^7\text{Li}$  reaction. For this system, the  $E1$  effective charge is small, and the effect of break-up is even much smaller than the  $d + {}^3\text{He}$  system. Notice that the  $E1$  effective charge vanishes for a similar projectile,  ${}^6\text{Li}$ , which predominantly breaks into  $\alpha + d$ .

In summary, we have studied the effect of virtual Coulomb break-up process of colliding nuclei (i.e., the dipole polarizability) on astrophysical fusion reactions. For the deuteron break-up, we have found that the enhancement of the tunneling probability is about 0.2 % for the  $d + {}^3\text{He}$  system. The effect is much smaller for the  ${}^7\text{Li}$  break-up in the  $p + {}^7\text{Li}$  system, where the enhancement factor was found to be about  $2.7 \times 10^{-3}$  %. Therefore, the break-up effect alone does not resolve the large screening puzzle. We have a feeling that we have almost exhausted the list of small effects in astrophysical reactions. Of course, there are still some exotic effects such as the deformation of proton [38] or the color van der Waals force [39], but these effects should be extremely small in the astrophysical reaction. We may now be at a stage where the atomic physics based model has to be re-examined with a more careful and consistent treatment of few-body dynamics of charged particles including electrons.

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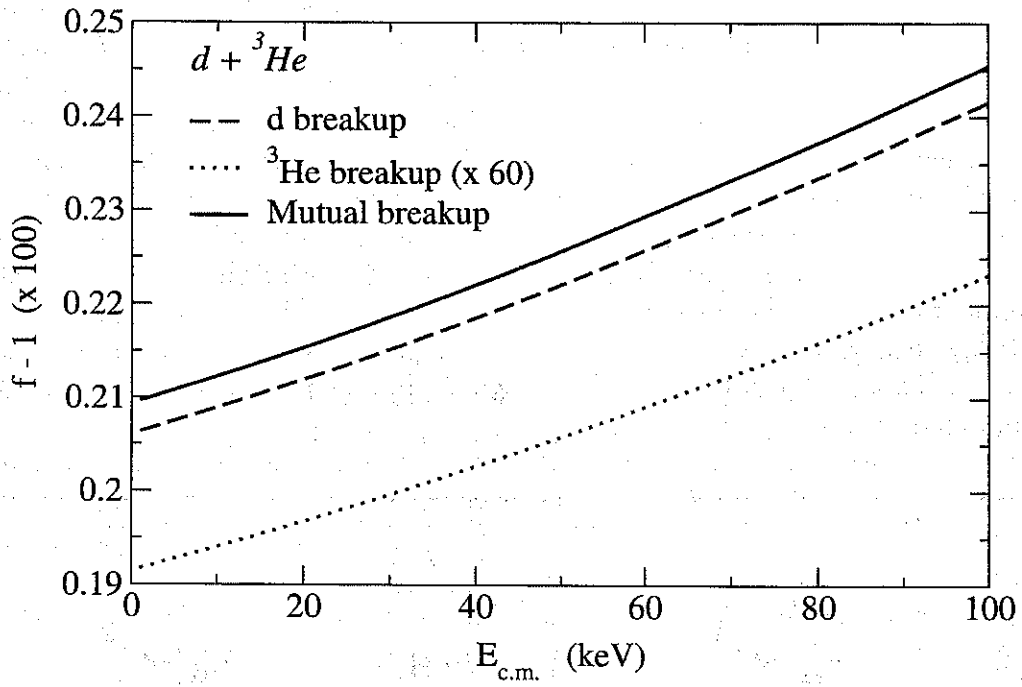


FIG. 1. The effect of virtual Coulomb break-up of colliding nuclei on the  $d+^3\text{He}$  reaction.  $f$  is the enhancement factor of penetrability due to the break-up, measured from unity. The dashed and the dotted lines show the effect of break-up of the  $d$  and the  $^3\text{He}$  nuclei, respectively. The solid line is the combined effect of mutual break-up of both the projectile and the target nuclei.

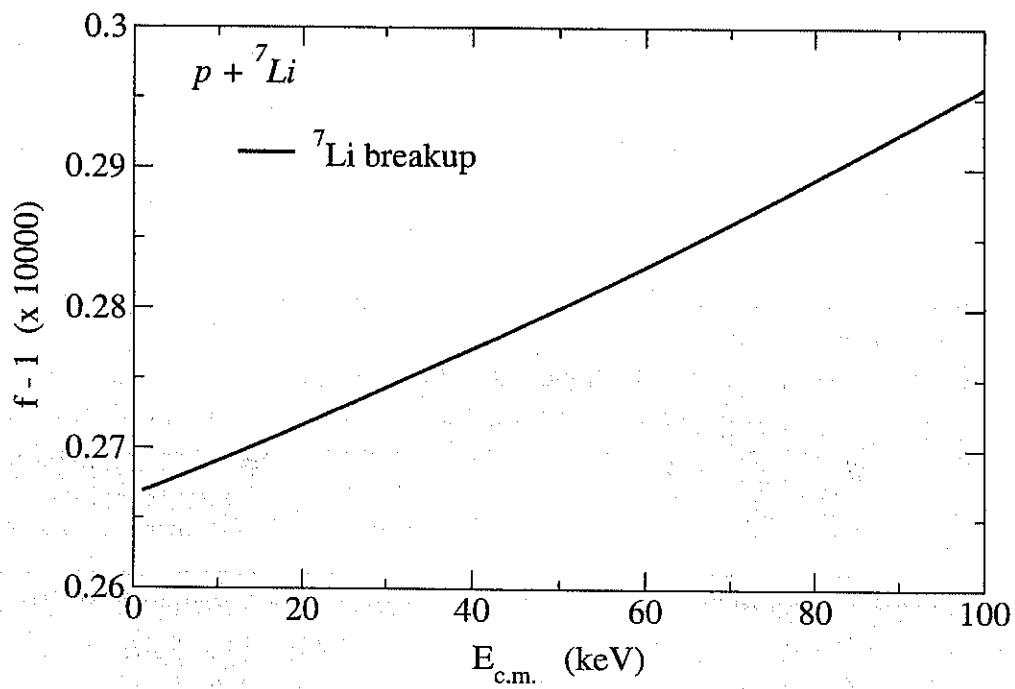


FIG. 2. Same as fig. 1, but for the effect of  ${}^7\text{Li}$  break-up on the  $p+{}^7\text{Li}$  reaction.



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