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preprint

IFUSP/P-158

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ELECTROMAGNETIC MASS DIFFERENCE OF PIONS AND THE RADIAL EXCITATIONS OF THE ρ-A_1-π SYSTEM.

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A B S T R A C T

We revise the calculation of the electromagnetic mass difference of pions taking into account radial excitations. A very good result for $\delta m_\pi = m_{\pi^+} - m_{\pi^0}$ is obtained.

* On leave of absence from Pisa University-Italy.

** Sponsored by the Italian Goverment, CNPq and FINEP under the Technical Cooperation Agreement Brasil-Italia of November 30,1972.

JAN/79

Some time ago Das et al.[1], calculated the electromagnetic mass difference of pions using chiral symmetry which is a very useful scheme for translating algebraic relations of currents into properties of spin zero and one particles.

Most successful applications of chiral symmetry were obtained at a time when only one particle of each kind was thought to exist. That is not the case, as we know now, and radial excitations could in principle exist for every type of particle. A natural question to be asked would then be: how do the radial excitations modify the old results ?

In a recent paper[2] we have searched for a simple way of taking radial excitations into account. This search lead us, among other things, to finite-energy Weinberg-like sum-rules where one integrates up to a maximum finite value (intead of infinite) of the argument of the spectral functions[3]. A similar thing was also done for the three point functions.

The main purpose of ref. 2 was to obtain the mass and width of the A_1 .Reasonable results were obtained on the assumption that the $\rho'(1250)$ exists and that the radial excitations of ρ and A_1 are almost equally spaced. The A_1 mass and width obtained in ref. 2 are $m_{A_1}=1193$ MeV and $\Gamma(A_1)=193$ MeV .

The experimental situation is still unclear as to the actual value of the A_1 mass with recent reviews favoring lower masses than our 1193 MeV [4]. Let us point out, however, that with this last mass, the A_1 forms, together with D(1286), E(1416) and $K_A(1340)$, an almost ideal nonet (mixing angle $\theta=29º - 30º$).

In the present note we would like to show that the assumptions we have made in ref. 2 yield also a good result for the electromagnetic mass difference of pions δm_π . It is interesting to note that alternative hypothesis [like for instance: no $\rho'(1250)$, first ρ

excitation at ~ 1600 MeV] lead to unacceptable values for δm_π .

Let us refer once again to the classic paper of Das. et al[1]., where it is shown that the mass squared difference is given by

$$m_{\pi^+}^2 - m_{\pi^0}^2 = \frac{3i\alpha}{4\pi^3 C_\pi^2} \int \frac{d^4k}{k^2} \int_0^\infty ds\, \frac{\rho_A(s) - \rho_V(s)}{k^2 - s + i\varepsilon} \quad , \tag{1}$$

where α is the fine structure constant, C_π the π decay constant and the ρ's are the spectral functions for the isovector currents: ρ_A for the axial vector spin one part and ρ_V for the vector current .

Years ago, it was a standard procedure to neglect everything but the ρ pole contribution to ρ_V and the A_1 to ρ_A. This is no longer justified and we should, in principle, add the contribution of many (possible infinite) states when the integration goes up to infinity as in eq.(1).

Nevertheless, we can still take into consideration only a small number of resonances whenever suitable cancelations take place. Such a situation was described in ref. 2. There, we gave arguments in favour of finite-energy Weinberg-type sum-rules of the form

$$\int^{s_{max}} ds\, \frac{\rho_V(s)}{s} = \int^{s_{max}} ds \left[\frac{\rho_A(s)}{s} + \rho_0(s)\right] \tag{2}$$

and

$$\int^{s_{max}} ds\, \rho_V(s) = \int^{s_{max}} ds\, \rho_A(s) \quad . \tag{3}$$

This type of sum-rules make sense whenever we can find a s_{max} located approximately midway between two resonances both in the vector and axial vector spectra.

Given the existence of $\rho'(1250) = \rho^{(1)}$, $\rho''(1600) = \rho^{(2)}$, ... $\rho^{(n)}$ and excitations $A_1^{(n)}$ of the axial family, the finite- energy sum-rules (2) and (3) are compatible with the original Weinberg sum-rules[3] if the following relations hold

$$m^2_{\rho(n)} = m^2_\rho + a\,n \ , \quad m^2_{A_1(n)} = m^2_{A_1} + a\,n \ , \quad m^2_{A_1(n)} \approx m^2_{\rho(n+1)} \ , \tag{4}$$

$$\frac{m_\rho^2}{f_\rho^2} = \frac{m^2_{\rho(n)}}{f^2_{\rho(n)}} = \frac{m^2_{A_1(n)}}{f^2_{A_1(n)}} \neq \frac{m^2_{A_1}}{f^2_{A_1}} \qquad (n = 1, 2 \dots) \tag{5}$$

and

$$\frac{m^4_{A_1}}{f^2_{A_1}} = \frac{m^4_\rho}{f^2_\rho} + \frac{m^4_{\rho(1)}}{f^2_{\rho(1)}} \quad , \tag{6}$$

and furthermore the integrals in (2) and (3) can be cut at $s_{max} \sim 2\mathrm{GeV}^2$ (between $\rho^{(1)} - \rho^{(2)}$ and between $A_1 - A_1^{(1)}$).

In the narrow widht approximation, what appears in eq.(1) is

$$\rho_A(s) - \rho_V(s) = \frac{m^4_{A_1}}{f^2_{A_1}} \delta(s - m^2_{A_1}) - \frac{m^4_\rho}{f^2_\rho} \delta(s - m^2_\rho) + \sum_{n=1} \left[\frac{m^4_{A_1(n)}}{f^2_{A_1(n)}} \delta(s - m^2_{A_1(n)}) - \frac{m^4_{\rho(n)}}{f^2_{\rho(n)}} \delta(s - m^2_{\rho(n)}) \right] , \tag{7}$$

which on account of (4) and (5) reduces to

$$\rho_A(s) - \rho_V(s) \approx \frac{m^4_{A_1}}{f^2_{A_1}} \delta(s - m^2_{A_1}) - \frac{m^4_\rho}{f^2_\rho} \delta(s - m^2_\rho) - \frac{m^2_{\rho(1)}}{f^2_{\rho(1)}} \delta(s - m^2_{\rho(1)}) \ . \tag{8}$$

This allows eq.(1) to be written as

$$m^2_{\pi^+} - m^2_{\pi^0} = \frac{3 i \alpha}{4 \pi^3 C^2_\pi} \int \frac{d^4 k}{k^2} \left[\frac{m^4_{A_1}}{f^2_{A_1}(k^2 - m^2_{A_1})} - \frac{m^4_\rho}{f^2_\rho (k^2 - m^2_\rho)} - \frac{m^4_{\rho(1)}}{f^2_{\rho(1)}(k^2 - m^2_{\rho(1)})} \right] . \tag{9}$$

After using eq.(6) and performing the integration in eq.(1) we get

$$m^2_{\pi^+} - m^2_{\pi^0} = \frac{3\alpha}{4\pi C^2_\pi} \frac{m^2_\rho}{f^2_\rho} \left[m^2_\rho \, \ln\left(m^2_{A_1} / m^2_\rho \right) - m^2_{\rho(1)} \, \ln\left(m^2_{\rho(1)} / m^2_{A_1} \right) \right] , \tag{10}$$

which can be further reduced to

$$m_{\pi^+}^2 - m_{\pi^0}^2 = \frac{3\alpha}{2\pi}\left[m_\rho^2 \ln\left(m_{A_1}^2 / m_\rho^2 \right) - m_{\rho(1)}^2 \ln\left(m_{\rho(1)}^2 / m_{A_1}^2 \right)\right] , \tag{11}$$

by the use of the KSFR[5] relation

$$2\, c_\pi^2 = m_\rho^2 / f_\rho^2 \quad ,$$

very well satified experimentally.

Using $m_{\rho(1)}$=1256 MeV [4], m_{A_1}=1193 MeV obtained in ref. 2 and the charged (neutral) pion mass we get, from eq.(11),

$$\delta m_\pi = m_{\pi^+} - m_{\pi^0} = 4.4\,(4.6)\ \mathrm{MeV}. \tag{12}$$

It is hard to say how much of the agreement with the experimental $(\delta m_\pi)_{exp}$= 4.60 MeV[4] is meaningful and how much of it is accidental. One thing that can be said is that the original expression derived by Das et al., (which is eq.(14) without the $\rho^{(1)}$ contribution) does not yield such nice values: δm_π=5.3(5.4)MeV for m_{A_1}=1100 and δm_π=6.5(6.7)MeV for m_{A_1}=1193 MeV.

Now we have to comment on the validity of eq.(1). In order to arrive at this equation Das et al. had to neglect the term

$$\Delta_0 = \frac{3 i \alpha}{2\pi^3 c_\pi^2} \int \frac{d^4k}{k^2} \int_0^\infty \frac{s\,\rho_0(s)\, ds}{k^2 - s} \quad , \tag{13}$$

where $\rho_0(s)$ is the spectral function for the spin zero part of the axial current.

When the only π-like particle was the pion itself one had, in the narrow width approximation,

$$s\,\rho_0(s) = c_\pi^2\, m_\pi^2\, \delta(s - m_\pi^2) \quad . \tag{14}$$

With this, eq.(13) can be written as

$$\Delta_0 = \frac{3 i \alpha}{2\pi^3}\, m_\pi^2 \int \frac{d^4 q}{q^2 (q^2 - 1)} \quad , \tag{15}$$

which could be neglected in the spirit of the massless pion limit. Assuming there are other π-like particles $\pi^{(n)}$ $(n=1,2\ldots)$ we have now

$$s\,\rho_0(s) = c_\pi^2 m_\pi^2\,\delta(s-m_\pi^2) + \sum_{n=1} c_\pi^2(n)\, m_\pi^2(n)\,\delta[s-m_\pi(n)] \quad . \tag{16}$$

None of these new particles nave yet been discovered. If they exist, the success of PCAC indicates that all the $C\pi^{(n)}$ should be very small. For the sake of argument only, let us consider the bounds $|c_\pi(n)| \leq [C_\pi m_\pi / n m_\pi(n)]$ which together with eqs. (15) and (16) give

$$\Delta_0 \leq \frac{3i\alpha}{2\pi^3}\,\frac{\pi^2 m_\pi^2}{6} \int \frac{d^4q}{q^2(q^2-1)} \quad , \tag{17}$$

to be get rid of in the massless limit.

The point of this last exercise was to show that even when we still are in the unpleasant situation of having to calculate mass difference for massless pions, the existence of more π-like particles does not necessarily make things worse.

If, as it seems to be the case, each type of particle has its radial recurrences many of the old chiral symmetry calculations will have to be revised. With ref.2 and this note we have started such a program, first calculating the A_1 mass and width and now the eletromagnetic mass difference of pions. Reasonable results are obtained in both cases if the first radial excitation of the ρ is the $\rho'(1250)$. That strengthens our expectation that this resonance will be experimentally confirmed[7].

FOOTNOTES AND REFERENCES

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