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EXTERNAL CHROMO MAGNETIC FIELD IN COLOR SUPERCONDUCTIVITY WITH A TEST MODEL

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External chromo magnetic field in color superconductivity with a test model

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- Understanding strong interactions: phase diagram
- Matter at high densities: other states (color/flavor degrees of freedom)
- Neutron stars and dense astrophysical objects

- Superconductivity (idea)
- in Quantum Chromodynamics (idea)
- External chromo magnetic field
- Test model: interaction due to scalar boson exchange
- Preliminar results

APRESENTADO NA / PRESENTED IN
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This work has been initiated with the investigation of the effect of classical vector fields (for example as those considered in relativistic model of nuclear systems) which leads naturally to a redefinition of chemical potentials for finite density systems. The external chromofield was done, in part but not only, due to a sort of mathematical generalization. Modifications of the phase diagram does appear due to this fields like exemplified in this seminar and will be discussed with more details in a forthcoming paper.

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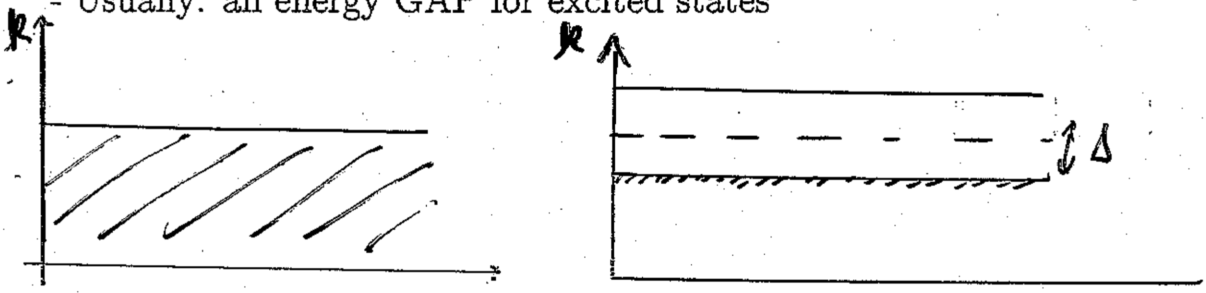
1 Superconductivity: idea

- Fermi system at finite density (usual Bardeen-Cooper-Schrieffer)
  - Very low temperatures: zero voltage difference with finite current
  - There appears a condensate (pairing between charged particles) $<\psi_i\psi_j>\equiv \Delta_{i,j}\neq 0$: it "breaks (spontaneously) a gauge symmetry" ATTRACTIVE INTERACTION DESTABILIZE FERMI SURFACE
  - Usually: an energy GAP for excited states



- In principle: $<\Phi|\bar\psi\bar\psi\psi\psi|\Phi>\to \underline{<\Phi|\bar\psi\bar\psi|\Phi>}<\Phi|\psi\psi|\Phi>$ $\qquad |\Phi\rangle = \sum_i a_i |N_i\rangle$

particle-particle cond. = anti-particle-anti-particle cond. unless for a phase

- However $\bar\psi$ are far from Fermi surface: $\underline{<\bar\psi\bar\psi>}\to\ 0$
- Δ constant (exclusion of magnetic field)
- $\Delta=\Delta(k)$, eg. Larkin-Orchinnikov-Fulde-Ferrell (crystaline)

when used with care, the notion of spontaneous gauge symmetry breaking can be extremely convenient fiction

Rajagopal and Wilczek (about color superconductivity),

2 Color superconductivity in QCD

At finite densities: $\Delta \sim < q_i^T q_j > \neq 0$ (color antitriplet channel)

- Barrois (1977) / Bailin-Love (1984) $\Delta \sim 1MeV$
- Alford *et al*, Rapp *et al* 1998, $\Delta \sim 100MeV$

  Attractive interaction in color anti-triplet channel: $< q_i^T \Gamma q_j >$

- (Effective) Interactions between quarks usually considered:

1) Instanton induced interactions ('t Hooft 1978) (or similar Nambu-Jona-Lasinio like int.) generates $< qq >$

$$\mathcal{L} = \frac{G}{4(N_c^2-1)} \left\{ \frac{1}{4N_c} (\bar{\psi}\sigma_{\mu\nu}\tau_\alpha^- \psi)^2 + \frac{(2N_c-1)}{2N_c} \left((\bar{\psi}\tau_\alpha^- \psi)^2 + (\bar{\psi}\gamma_5 \tau_\alpha^- \psi)^2 \right) \right\} \quad (1)$$

eg. Rapp *et al* 1998

2) One gluon exchange: very high densities, asymptotic freedom, leading processes with g weak

- extensively studied: eg. Schafer and Wilczek / Pisarski and Rischke (1999/2000)
- $T_C \simeq .567\Delta_0$ $\quad \Delta(T)/\Delta(T=0)$

  the same as in BCS-type superconductivity

- but $\Delta \sim exp(-\pi/2\bar{g})$ in contrast to $\Delta \sim exp(-a/g^2)$ in usual BCS

- At the end: Δ_{QCD} seems to be gauge invariant (Pisarski,Rischke)

- QCD vacuum: condensates

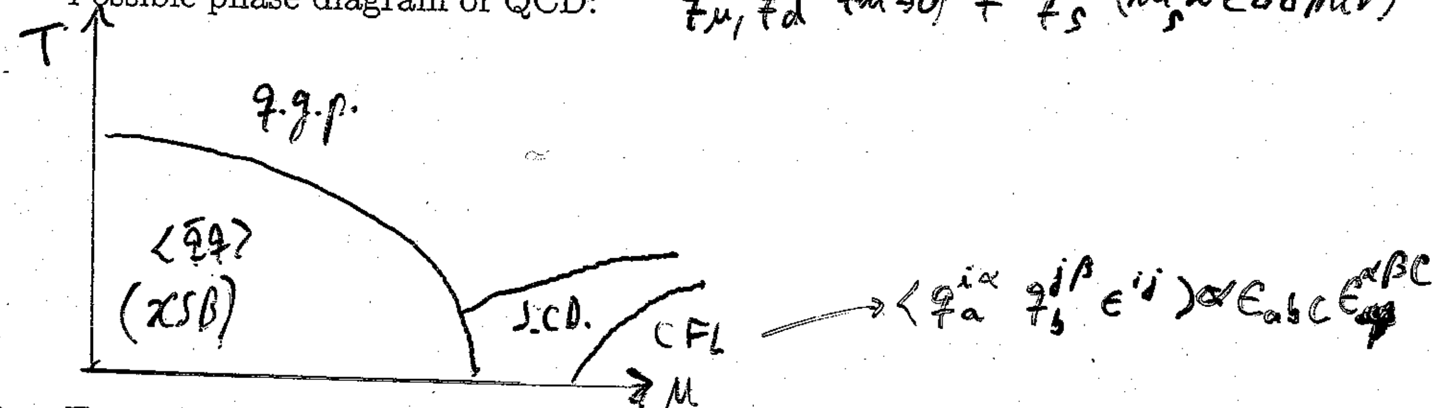
1) scalar $< 0|\bar{q}^i_r q^i_l|0 >$ (chiral SSB)

2) $< 0|F^a_{\mu\nu}F^{\mu\nu}_a|0 >$ (trace anomaly)

- Chromo electric and magnetic classical fields: $E^a_i \equiv F^a_{i,0} \quad \epsilon_{i,j,k}B^a_k \equiv F^a_{i,j}$ where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu$

- As (fermionic/baryonic) density increases $< \bar{q}q >$, $< F_{\mu\nu}F^{\mu\nu} >$ change and there appears $< qq >$: non trivial phase diagram

- Possible phase diagram of QCD: q_u, q_d $(m \to 0)$ + q_s $(m_s \sim 200\,MeV)$



3 External chromo magnetic field

- Ebert and Volkov (PLB 272, (1991)): in a Nambu-Jona-Lasinio type model $< F_{\mu\nu}F^{\mu\nu} >$ can be mimicked by means of external classical A^a_μ

- In a dense medium (relat. heavy ions collisions) A^a_μ may differ considerably from vacuum values (however high T destroys $< qq >$ as well as other condensates)

- Inequivalent vector potential yield same (chromo) magnetic field

- D. Ebert *al* with NJL type interaction - external $\vec{H}^a$ may enhance Δ (even at $\mu = 0$)

- several works, eg. Alford, Berges, Rajagopal, Gorbar, Ebert *et al*

4 Superconductivity due to a scalar boson exchange with an external chromo magnetic field

Scalar boson (ϕ) exchange generates difermions $< q_a q_a >$

Extension of the Approach of Pisarski-Rischke Phys. Rev. **D 60** 094013 (1999)

$$\mathcal{L} = \overline{\Psi}\left(i\gamma_\mu D^\mu + \gamma_0\mu - g\phi\right)\Psi + \frac{1}{2}D_\mu\phi D^\mu\phi - \frac{1}{2}M^2\phi^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2}$$

Where $D_\mu = \partial_\mu - g' A^a_\mu \lambda_a/2$.

Scalar field in chromo electromagnetic field:

$$\begin{aligned} &Tr\mathcal{D}^\mu\phi(\mathrm{x})\mathcal{D}_\mu\phi(\mathrm{x}) \to \partial^\mu\phi(\mathrm{x})\partial_\mu\phi(\mathrm{x}) + Tr\frac{g'^2}{4}A^a_\mu A^\mu_a\phi^2(\mathrm{x}) \\ &M^2 \to \tilde{M}^2 \end{aligned} \tag{3}$$

Integrating out the scalar field:

$$I\left[\bar{\psi},\psi\right] = \int d^dx d^dy \left(\bar{\psi}(\mathrm{x})(G_0^+)^{-1}(\mathrm{x},\mathrm{y})\psi(\mathrm{y}) + \frac{g}{2}\bar{\psi}(\mathrm{x})\psi(\mathrm{x})D(\mathrm{x},\mathrm{y})\bar{\psi}(\mathrm{y})\psi(\mathrm{y})\right) \tag{4}$$

where $D(\mathrm{x},\mathrm{y})$ is the scalar field propagator

In the mean field approach:

$$\begin{aligned} &\psi\bar{\psi} \to \psi\bar{\psi} - <\psi\bar{\psi}>; \quad (\psi\bar{\psi})^\dagger \to (\psi\bar{\psi})^\dagger - <\psi\bar{\psi}>^\dagger, \\ &\psi\psi \to \psi\psi - <\psi\psi>; \quad (\psi\psi)^\dagger \to (\psi\psi)^\dagger - <\psi\psi>^\dagger. \end{aligned} \tag{5}$$

For which the quartic interaction may be

$$g_a\frac{g^2}{2}\bar{\psi}(\mathbf{x})\psi(\mathbf{y})D(\mathbf{x},\mathbf{y})\bar{\psi}(\mathbf{x})\psi(\mathbf{y})+g_b\frac{g^2}{2}\bar{\psi}(\mathbf{x})\psi(\mathbf{x})D(\mathbf{x},\mathbf{y})\bar{\psi}(\mathbf{y})\psi(\mathbf{y}) \tag{6}$$

g_a for the formation of $\boxed{<\psi\bar{\psi}>}$ cond. // g_b for $\boxed{<\psi\psi>}$ cond.

The action in the frame of the mean field approach:

$$I=\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\left(\bar{\Psi}(\mathbf{x})A(\mathbf{x},\mathbf{y})\Psi+\bar{\Psi}(\mathbf{x})B(\mathbf{x},\mathbf{y})\Psi_C+\bar{\Psi}(\mathbf{x})_C C(\mathbf{x},\mathbf{y})\Psi+\bar{\Psi}(\mathbf{x})_C D(\mathbf{x},\mathbf{y})\Psi_C\right),$$

$$\Psi=\begin{pmatrix}\psi\\ \psi_c\end{pmatrix} \qquad \psi_c = C\,\bar{\psi}^T(-k) \tag{7}$$

Where:

$$\begin{aligned}
A&=(G^{+}_{0,ff})^{-1}\equiv -i\left(i\gamma_\mu(\partial^\mu-ig'A^\mu_a\lambda_a/2)+\gamma_0\mu+g_a g\int D(\mathbf{x},\mathbf{y})<\psi\bar{\psi}(\mathbf{y})>\right)\delta(\mathbf{x}-\mathbf{y})\\
D&=(G^{-}_{0,ff})^{-1}\equiv -i\left(i\gamma_\mu(\partial^\mu-ig'A^\mu_a\lambda_a/2)-\gamma_0\mu+g_a g\int D(\mathbf{x},\mathbf{y})<\psi\bar{\psi}(\mathbf{y})>\right)\delta(\mathbf{x}-\mathbf{y})\\
B&=\Delta^+=g_b g^2<\Psi_C\bar{\Psi}>D(\mathbf{x},\mathbf{y})\\
C&=\Delta^-=g_b g^2\gamma_0(\Delta^+(\mathbf{x},\mathbf{y}))^\dagger\gamma_0.
\end{aligned} \tag{8}$$

DOES [redacted]

The condensate $<\bar{\psi}\psi>$ (break[redacted] chiral symmetry): quark mass ?

By performing a Fourier transformation (restricting possible configurations)

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi exp\int_{|\mathbf{k}|>0}d^3\mathbf{k}(\bar{\psi}\bar{\psi}_C)S^{-1}(\mathbf{k})\,(\psi\psi_C)^T=\mathcal{N}det\left(S^{-1}\right)^{\frac{1}{2}} \tag{9}$$

The full propagator:

$$S=\begin{pmatrix} G^+ & -G_0^+\Delta^-G^- \\ -G_0^-\Delta^+G^+ & G^- \end{pmatrix}, \tag{10}$$

where

$$G^{\pm}=((G_0^{\pm})^{-1}-\Delta^{\mp}G_0^{\mp}\Delta^{\pm})^{-1} \tag{11}$$

Using identity:

$$< \Psi(k)\bar{\Psi}(k) >= -S(k) \tag{12}$$

The GAP equations are:

$$\begin{aligned}
&\Delta^{\pm}(k) = g^2 T \int_p D(k-p) G_0^{\mp}(p)\Delta^{\pm}(p) \left((G_0^{\pm})^{-1}(p) - \Delta^{\mp}(p)G_0^{\mp}(p)\Delta^{\pm}(p)\right) \\
&< \psi\bar{\psi}(q) >= \int_p D(k-p) \left((G_0^{+})^{-1}(p) - \Delta^{-}(p)G_0^{-}(p)\Delta^{+}(p)\right)^{-1}, \\
&< \bar{\psi}\psi(q) >= \int_p D(k-p) \left((G_0^{-})^{-1}(p) - \Delta^{+}(p)G_0^{+}(p)\Delta^{-}(p)\right)^{-1}.
\end{aligned} \tag{13}$$

where

$$(G_{\bar{f}f}^{\pm})_0^{-1}(k) = -\left(\gamma_\mu(k^\mu - igA_a^\mu\lambda_a/2) \mp \gamma_0\mu + g_a g \int_k D(k) < \psi\bar{\psi}(k) >^{\pm}\right). \tag{14}$$

To compare with one gluon exchange:

$$\Delta(k) = g^2 \int \frac{d^4p}{(2\pi)^4} \Gamma_a^\mu D_{\mu,\nu}^{a,b}(k-p) G_0(p)\Delta(p) \left((G_0^{\pm})^{-1} - \Delta^{\mp}G_0^{\mp}\Delta^{\pm}\right)^{-1} \Gamma_b^\nu \tag{15}$$

The external field equation is:

$$\partial_\mu F_a^{\mu\nu} = g' < \bar{\psi}(\mathrm{x})\gamma^\nu\lambda_a\psi(\mathrm{y}) > \tag{16}$$

dynamical/non homogeneous external field: vector condensates

Spinors

- $$\Delta = \underline{\Delta_1\gamma_5} + \underline{\Delta_2\vec{\gamma}.\vec{k}}\gamma_0\gamma_5 + \Delta_3\gamma_0\gamma_5 + \underline{\Delta_4} + \underline{\Delta_5\vec{\gamma}.\vec{k}\gamma_0} + \Delta_6\vec{\gamma}.\vec{k} + \Delta_7\vec{\gamma}.\vec{k}\gamma_5 + \Delta_8\gamma_0 \tag{17}$$

However:

1) $<\bar{q}q>$ plays the role of mass for the quarks in the equations

2) quark masses mix states of chiralities and duplicate the number of possible different di-quark condensates

In a first analysis $<\bar{q}q>$ neglected, resulting in four possible states of helicity and chirality:

$$\Delta(k) = \phi^+_{r,+}(k)P^+_{r,+}(k) + \phi^+_{l,-}(k)P^+_{l,-}(k) + \phi^-_{r,-}(k)P^-_{r,-}(k) + \phi^-_{l,+}(k)P^-_{l,+}(k) \tag{18}$$

Using projectors for quarks with helicities $\pm$ and chiralities r, l:

$$P_\pm(k) = \frac{1 \pm \gamma_5\gamma_0\vec{\gamma}.\hat{k}}{2} \qquad P_{r,l} = \frac{1 \pm \gamma_5}{2} \tag{19}$$

For each of the condensates $\phi_{r,l;\pm}$ there is an equation such as:

$$\overset{(+)}{\phi^a_{r,+}}(k) = \frac{g^2}{2}\int \frac{d^4p}{(2\pi)^4} D(k-p, M)\left(\frac{A - \hat{k}.\hat{p}}{p_0{}^2 - \epsilon^+_a\left[\phi_{r,+}(p)\right]^2}\overset{(+)}{\phi^a_{l,-}} + \frac{B + \hat{k}.\hat{p}}{p_0{}^2 - \epsilon^-_a\left[\phi_{l,+}(p)\right]^2}\overset{(-)}{\phi^a_{l,+}}\right) \tag{20}$$

a is for color (always diagonal: $<q_a q_a>$)

For zero external field $\epsilon^\pm_a[\phi_{r,+}] = \sqrt{(|\vec{p}| \mp \mu)^2 + |\phi|^2}$

A, B are also dependent on the choice of the external field

GAP equations in the form:

$$\phi^+_{r,+} = \phi^+_{l,-}(F^+_0(\phi_{l,-}) - F^+_1(\phi_{l,-})) + \phi^-_{l,+}(F_0(\phi_{l,+} + F_1(\phi_{l,+})), \tag{21}$$

For non zero: $\phi^+_{r,+}$, $\phi^-_{r,-}$, $\phi^+_{l,-}$, $\phi^-_{l,+}$

Integrals of the type:

$$(F^\pm_0, \quad F^\pm_1) = \frac{g^2}{2}\int \frac{dp_0}{2\pi}\int \frac{d\vec{p}}{(2\pi)^3}\frac{1}{(k-p)^2 - \tilde{M}^2}(1\,, \quad \hat{k}\hat{p})\frac{1}{p_0^2 - (\epsilon^\pm[\phi])^2} \tag{22}$$

(a) Poles from scalar boson propagator $\rightarrow$ if $\tilde{M}^2$ is small the approximation of punctual interaction (for which $\phi^- = \phi^+$) is not necessarily reliable

(b) Poles from fermions

For the cases of small k or zero momentum exchange: the integral $F_1 \rightarrow 0$

The F_0^+ integral:

$$\begin{aligned}
F^+_0 = \frac{g^2}{16\pi^2}\int_0^{\Lambda,\infty} dp\frac{p}{k\epsilon^+}\Bigg(&-\frac{1}{2}\left\{ln\left(\frac{\epsilon^+ - k_0 - \sqrt{(k-p)^2 + M^2}}{\epsilon^+ + k_0 - \sqrt{(k-p)^2+M^2}}\right) + \right.\\
&\left. - ln\left(\frac{\epsilon^+ - k_0 - \sqrt{(k+p)^2 + M^2}}{\epsilon^+ + k_0 - \sqrt{(k+p)^2+M^2}}\right)\right\}\\
&+\frac{1}{2}\left\{ln\left(\frac{\epsilon^+ + k_0 - \sqrt{(k-p)^2 + M^2}}{\epsilon^+ - k_0 - \sqrt{(k-p)^2+M^2}}\right) - ln\left(\frac{\epsilon^+ + k_0 - \sqrt{(k+p)^2 + M^2}}{\epsilon^+ - k_0 - \sqrt{(k+p)^2+M^2}}\right)\right\}\\
&+\frac{1}{4}ln\left(\frac{(\epsilon^+ + k_0)^2 - (k+p)^2 + M^2}{(\epsilon^+ + k_0)^2 - (k-p)^2 + M^2}\right) + \frac{1}{4}ln\left(\frac{(\epsilon^+ - k_0)^2 - (k+p)^2 + M^2}{(\epsilon^+ - k_0)^2 - (k-p)^2 + M^2}\right)\Bigg)
\end{aligned} \tag{23}$$

There are ultraviolet divergences, eliminated by:

$$F^\pm_0 \rightarrow F^\pm_0 + \frac{g^2}{4}\int^{\mu_R} \frac{d^3\mathbf{p}}{(2\pi)^2}\frac{1}{(k-p)^2}\frac{1}{|\mathbf{p}|} - F_{c.t.} \tag{24}$$

Example:

constant chromo magnetic field $A_i^a = \sqrt{H/g'}\delta_{i,a}$ (i=1,2,3), else $A_i^a = 0$, leading to $H_i^a = \delta_i^a H =$ const.

- poles (for equal chemical potential):

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{\vec{p}^2} \mp \mu)^2 + \phi_i^2$$

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{\vec{p}^2} \underline{\pm A_1^1 g'/2 \mp \mu})^2 + \phi_i^2$$

$$(\epsilon^{\pm}[\phi_i])^2 = (\sqrt{(|\vec{p}| + g' A_3^3)^2} \underline{\pm A_2^2 g'/2 \mp \mu})^2 + \phi_i^2$$

Different "effective" chemical potential for each colored quark

(Other external magnetic field may induce terms with role of quark mass) (?)

- We consider (approximatively): $|\phi|$ as constant reliable for small k, but $\phi = \phi(k)$
- Strong A_μ^a (or H) may enhance $|\phi|$ (not simultaneously for all the colors)

- For punctual interaction $M^2 \to \infty$ (NJL): $\phi^+ = \phi^-$

- Limit of **weak coupling**: in principle $\phi << \mu$ ($\phi \sim \mu exp(-c/g^2)$)

-

$$F_0^-/F^+ \sim \frac{\epsilon^+}{\epsilon^-} << 1 \qquad \rightarrow \phi^- > \phi^+$$

- pairing dominated by fermions close to Fermi surface $\langle \Psi\Psi \rangle$
- H^a_{ext} induces $\tilde{\mu} < \mu$ enhancing ϕ

-

- Limit of **strong coupling**:

contributions far from Fermi surface

- increasing $|\phi|$
- if $\tilde{M}^2 \propto g'^2$, then $D \sim 1/M$ and $\phi^+ \sim \phi^-$ (BUT $\phi^- > \phi^+$)
- H^a_{ext}: non degeneracy in $|\phi^a|$

-

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- Using approximations: massless fermions, momentum independent GAP

- Scalar boson exchange (effective degree of freedom of the system) as a test model for color superconductivity
along the lines proposed by Pisarski-Rischke

- Non zero energy exchange (non punctual interactions) modify GAP equations

- Detailed study of complete phase diagram $< qq >$, $< \bar{q}q >$ depend on considering mixed chirality/helicity states

- External chromo (electro) magnetic fields may lead to several different effects:
- changing fermion masses
- effective chemical potential (usually smaller) $\rightarrow \tilde{\mu} \sim \Delta \rightarrow < \bar{q}\bar{q} > \sim < qq >$
- may cause non degeneracy of ϕ_a

- Towards derivation of effective interactions for q-q which takes into account $< qq >$