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Abstract

In this work we discuss aspects of interest for a general phase diagram of matter in specific conditions (neutron-proton asymmetry and with anti-matter) which may be relevant for the description of dense stars. The inclusion of light isovector spin zero mesons in relativistic models of nuclear systems is discussed and deviations from the usual results for the symmetry energy term are found seemingly in qualitative agreement with developments recently done for non relativistic description of nuclear matter. Remarks for the description baryonic and anti-baryonic bound states are done.

Keywords: Asymmetric nuclear matter, scalar fields, relativistic model, anti-matter, symmetry energy, condensate, spontaneous symmetry breaking.

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1 Introduction

In this work basically two aspects of nuclear matter (n.m.) properties at specific conditions are investigated: with neutron-proton density asymmetry and n.m. with anti-matter component(s). For the former aspect, the symmetry energy of nuclear matter is the relevant parameter being basically the difference in the binding energy of a neutron and a proton in the nuclear medium. It is useful to parametrize the asymmetry with parameter such as $d = N - Z$ (for finite systems) or $\alpha = (\rho_n - \rho_p)/\rho$. Usually, at the saturation density where nuclear matter matchs finite nuclei in several respects, it is given/represented basically by two terms: $c_z\alpha + a_\tau\alpha^2$, where c_z and a_τ (the symmetry energy coefficient - s.e.c.) symmetry are constants coefficients. The first one breaks isospin symmetry explicitely and it is found to be smaller than the second. The value of a_τ at the saturation density is expected to be around $28-34$ MeV and its behavior with the nuclear density has been quite studied [1, 2]. The s.e.c. can be obtained as a function of the nuclear matter isovector polarizability [3] and it can be expected to depend itself on the n-p asymmetry leading to different more involved dependence of the symmetry energy on α, mainly for large n-p asymmetries [4]. Concerning the presence of anti-matter bound states in the known Universe there is very scarse information [5].

It is kown that nuclear dynamics plays a relevant role in the structure of dense stars and Supernovae, which can therefore be considered as sources of information for the phase diagram of matter. A large symmetry energy (coefficient), a_τ, leads to smaller electron capture (quasi-static phase) which may eventually lead to greater energy realese of Supernovae, to larger final proton fraction and faster cooling (via neutrino emission) [6, 4]. Nuclear interactions are dependent on the temperature and baryonic density, depending as well as on the neutron-proton (or flavor) asymmetries which may be result of the breakdwon of isospin symmetry, knowledge that should be extended to encompass the eventual presence of antimatter fields.

In this work the (isovector) symmetry energy term is studied in the framework of a relativistic model of nuclear matter which includes massive iso-scalar and iso-vector fields, the chiral partners $(\sigma, \vec{\pi})$ and $(\vec{\delta}, f_0)$, which are considered here in an "effective quadriplet". All the bosonic components are considered to develop classical counterparts in the lines of usual relativistic models and extensions. The n-p asymmetry is introduced with the inclusion of the isovector rho mesons, what is not considered fully here. The effective neutron and proton masses, and therefore the respective densities, differ in isospin and n-p asymmetric medium. In the next section the model is exhibited for an expected value (classical part) of the bosonic fields. Then the nuclear densities are briefly commented as well as the dynamical equations and the solutions for the bosons. Finally the possible formation of anti-matter bound states is discussed.

2 Finite density linear sigma model with other light mesons

The linear realizations of chiral symmetry, eventually flavor symmetry for strange systems, are implemented in Lagrangian densities containing (lighter) scalars and pseudo-scalars mesons from a QCD octet/nonet [7]. We simplify the usual $SU(3) \times SU(3)$ version of the Linear sigma model, shown for example in [8] which includes some strange light mesons [9]:

$$\mathcal{L} = \bar{N}_i(\mathbf{x}) \left(i\gamma_\mu \mathcal{D}_t^\mu - g_s(\sigma + i\gamma_5 .\vec{\tau}.\vec{\pi}) + g_\delta(\vec{\tau}.\vec{\delta} + a_0) - \gamma_0 \hat{\mu}_c \right) N_i(\mathbf{x}) + \frac{1}{2} \left(\partial_\mu \Sigma . \partial^\mu \Sigma + \partial_\mu \Pi . \partial^\mu \Pi \right) + \\ + \frac{\mu^2}{2} Tr|\Sigma + i\Pi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda_1 Tr(|(\Sigma + i\Pi)|^2)^2 - \lambda_2 (Tr(|\Sigma + i\Pi|^2))^2 + \frac{1}{2} m_V^2 V_\mu V^\mu + \mathcal{L}_\rho + \mathcal{L} \tag{1}$$

where the three last terms stand for the isovector mesons, which for $SU(2)$ chiral picture are the rho and A_1. These degrees of freedom are necessarily more relevant below the deconfinement phase transition. The scalar (pseudoscalar) matrix was chosen to be written as: $\Sigma = \sum_a \lambda_a \sigma_a$ ($\Pi = \sum_a \lambda_a "\pi_a"$) where the λ_a are the Gell-Mann matrices in the adjoint representation and σ_a ("π_a") nine scalar (nine pseudoscalar) fields, reducing to eight each multiplet. As stated above only some of these fields will be considered in this work. The covariant derivative (with the coupling to the vector and isovector fields) is: $\mathcal{D}_t^\mu$. The rho and A1 mesons contributions will not be considered for quantitave analysis in the present work. The complete (self consistent) calculation it is left for a forthcoming paper. The minimization of the spin zero potential for the $SU(3)$ symmetric limit yields the strange-chiral radius v_3 that can be defined with a coupling: $\lambda_3 = \lambda_2 + \lambda_1/2$, reading: $v_3^2 = \mu^2/(4\lambda_3)$, where μ^2 is the mass from the Lagrangian. This is the analogous to the chiral radius in $SU(2) \times SU(2)$ with the Lagrangian term written as: $-\lambda_3(Tr|\Sigma^2 + \Pi^2| - v_3^2)^2$. The vector fields are introduced with the usual "gauge invariant" kinetic terms $F_{\mu\nu}$.

We will choose to deal with four spin zero mesons: the $SU(2)$ quadriplet $(\sigma, \boldsymbol{\pi})$ and another one containing strangeness degrees of freedom: $(a_0, \boldsymbol{\delta})$.

All these four fields are considered to develop classical components (denoted with overlines) which shift them: $\sigma \to \bar{\sigma} + \sigma$, $\pi \to \bar{\pi} + \pi$, $a_0 \to \bar{a_0} + a_0$, etc. The masses of these mesons are found from the Lagrangian density after these shiftings as the coefficient of the shifted fields.

2.1 Nuclear densities and pions

The nucleon effective masses are to depend on the classical scalar and pseudo-scalar fields. They can be written as:

$$M^*_{a,b;s} = g_S < N_{a,s}|(\bar{\sigma}_a + i\gamma_5 \vec{\tau}.\vec{\pi})|N_{b,s}> + o(\delta) \simeq g_S(\bar{\sigma} \pm \bar{\pi}_0 \tilde{M}_s) \pm g_\delta \delta_3, \tag{2}$$

where a, b are isospin indexes, $\tilde{M}_s$ is a matrix dependent on the nucleons spins in the medium which (probably) have small components because g_S is large. We can forsee that different effective masses for protons and neutrons should lead to different densities in addition to the usual different momenta at the Fermi surfaces. Furthermore, for the other components of the classical pion field π_1 and π_2 different from zero, the neutron may oscillate into proton, and vice versa, in a dense medium [10].

The expressions for the baryonic density and quantized fermionic density are considered to be those from the usual Fermi liquid picture in terms of the Fermi momenta. They are written in terms of the components of the solution of the Dirac equation for the nucleon. The pseudoscalar baryonic density emerges from the solution of *in medium* Dirac equation as: $\rho_{ps} \propto (\bar{u}v - \bar{v}u)(E + m)\vec{\sigma}.\vec{p}$ where u, v are the spinor components of the baryon field solution coupled to vector fields and $\bar{u}, \bar{v}$ the transposed hermitian components. This quantity, although much smaller than the baryonic and scalar densities, should not be (necessarily) expected to be zero. This quantity in the equation of motion of the (classical part of the) pseudoscalar mesons may yield non trivial non zero results.

2.2 Stability Equations

The equations of motion for the fermionic and spin zero bosonic fields are the Euler-Lagrange equations [11]. The vector fields are considered to satisfy modified (variational) equations which take into account the interaction with finite density baryons [10]. The stability of the bound system is garanteed by the following conditions:

$$\left.\frac{dE/A}{d\rho_B}\right|_{\rho_0} = 0, \qquad \frac{d^2E/A}{d\rho_B^2} \propto K > 0, \tag{3}$$

where K is the nuclear incompressibility and ρ_0 is the stability density. The usual procedure is to solve movement equation numerically and with these solutions one verify whether these conditions are satisfied. Now, instead of doing this, we search analytic solutions by considering the second of these equations to be separated into three nearly independent equations for each component of the system: 1) Hamiltonian terms with the fermionic density, 2) Hamiltonian terms mostly with spin-zero bosons, 3) Hamiltonian terms which contain explicit vector fields. The solutions satisfy the equations of motion [9, 10]. The equations are given by:

$$\begin{aligned} &\frac{d\rho_f}{d\rho_B} = \frac{\rho_f}{\rho_B}, \\ &\frac{d(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2)}{d\rho_B} = \frac{(\tilde{\sigma}^2 + \tilde{\pi}^2 - v^2)}{2\rho_B}, \\ &\frac{d\mathcal{H}_V}{d\rho_B} = \frac{\mathcal{H}_V}{\rho_B}. \end{aligned} \tag{4}$$

This last equation is considered for the vector field, being $\mathcal{H}_V$ represents the terms with vector fields. From the resulting stability equations [10, 9] we have found an extended symmetry radius ($\tilde{C}_3$) which, together with the sums of the pion and sigma masses (μ_{tot}) as well as of the four spin zero fields ($\mu_{s.z.}$) at finite density, are given respectively by:

$$\begin{aligned} &\tilde{C}_3 = \frac{\lambda}{\sqrt{\rho_B}}(\bar{\sigma}^2 + \bar{\pi}^2 + \bar{a_0}^2 + \bar{\delta}^2 - v_3^2), \\ &\mu_{tot}^2 = \lambda(2(\bar{\sigma}^2 + \bar{\pi}^2) + \bar{a_0}^2 + \bar{\delta}^2 - v_3^2), \\ &\mu_{s.z.}^2 = \lambda(4(\bar{\sigma}^2 + \bar{\pi}^2 + \bar{a_0}^2 + \bar{\delta}^2) - 2v_3^2). \end{aligned} \tag{5}$$

We impose that the non strange limit of this picture (without Λ,a_0 and $\boldsymbol{\delta}$) yields the usual relations of the SU(2) linear sigma model. This is implemented by fixing the values of v_3 such that: $\bar{a_0}^2 + \bar{\boldsymbol{\delta}}^2 - v_3^2 = -v^2$, with $M_\Lambda = g_{S,3}\bar{a_0} = 1115$ MeV. Besides that, the masses of a_0 and $\boldsymbol{\delta}$ are also fixed by expressions together with the other meson masses. The mass of the delta meson derived from the Lagrangian terms shown above is smaller than the sigma mass because for symmetric nuclear matter one should have $\bar{\boldsymbol{\delta}} = 0$ from its equation of movement. An extra *ad hoc* term was consider to "mend" this. The "symmetry radius", $\tilde{C}_3$, is thus proportional to the sum of the in medium masses of the spin zero particles.

Usually the neutron-proton asymmetry is parametrized by: $\alpha = (\rho_n - \rho_p)/\rho_B$, which determines the fermionic and scalar densities. Furthermore the nucleon effective masses (neutron and proton) are to be different due to the coupling to (classical) fields of pions and delta as shown above. To deal with these two different ways we develop the following approach. Effects of the classical scalar - isovector mesons on the effective masses are discussed leading to different baryonic (neutron,proton) densities. Secondly, the effect of these different effective masses as well as of the different Fermi momenta of neutrons and protons on the fermionic (and total energy) densities are estimated. With this we give more reasons, from the point of view of a relativistic model, for the non relativistic analysis with Skyrme forces of References [4] (although they are not fully equivalent since we are not calculating the isovector polarizability here).

Numerical results are shown in the following. The values of the parameters were considered to be: $\lambda = 40$, $M = 940$ MeV, $\mu_\pi = 140$ MeV, $\mu_\sigma = 450$ MeV, $g_S = 10$, $g_\delta = 10$, $\mu_\delta = 980$ MeV, $\mu_{a_0} = 980$ MeV. In Figure 1 the effective masses of neutron and proton are shown as functions of the asymmetry parameter α for two cases, namely for the limit of their effective masses in symmetric nuclear matter equal to $M^* = 0.7M$ and to $M^* = 0.8M$ (dashed and solid lines respectively) taking into account the contributions of the delta and of $\bar{\pi}$ - by their expected values, neutron (proton) effective mass with thick (normal) lines. For the contribution of $\bar{\pi}$ it was normalized such that in symmetric nuclear matter this expected value is zero, $\bar{\pi} = 0$, by subtracting the value found in this limit of every solution, at any neutron-pronton asymmetry. This is done for the sake of the main argument. The total fermionic densities are shown in Figure 2 as functions of α with the different effective masses for protons and neutrons of Figure 1 (normal lines) and with equal effective masses (usual calculations) - $M^* = cte$ (thick lines). The effective masses in symmetric nuclear matter are $M^* = 0.7M$ and $M^* = 0.8M$ (dashed and solid lines respectively in both figures). It is seen that the inclusion of different effective masses M_n^*, M_p^* amplifies the symmetry energy for higher n-p asymmetries, as it could be expected.

The dynamics of the dense core of the Supernovae involves URCA processes for which the symmetry energy is determinant in several aspects as discussed in several references [2, 6]. For a large value of the symmetry energy (a_τ or $A_{0,1}$ [4]) the deleptonization (electron capture) in the quasi-static phase of the supernovae mechanism should be smaller, what yields a larger final proton fraction. The cooling via neutrino emission is to be modified as well [6]. This conclusion goes compatibly with the analysis done for the SN 1987a. Further discussion is found in [2, 6, 4, 12].

2.3 Remarks on other channels of nuclear forces and Supernovae

Besides the isovector channel the spin dependent part of the effective nucleon interactions may also be relevant for astrophysics. For example, the spin channel of effective nuclear forces is relevant for the neutrino interaction with matter (coupling to axial vector current with the scalar channel) [12] and it also can lead to contributions to Electro-Magnetic fields in dense stars, although they seem to provide large values [13]. As such, another symmetry energy coefficients (terms) can be defined as for example:

$$\frac{E}{A} \propto A_{s=1,t=0}(\rho_{up} - \rho_{down})^2 + ..., \tag{6}$$

where ρ_{spin} is the density of nucleons with spin up/down. An increase of the spin susceptibility (roughly the inverse of the spin s.e.c) lead to the suppression of Gamow Teller transitions (supernovae) and it may lead to instabilities of ferromagnetic polarized states. The spin-isospin channel has been associated with instabilities which would lead to "pion condensation", although this is not completely understood [14]. However all the symmetry energy coefficients should be expected to depend on the neutron proton asymmetry [4].

In the isoscalar channel it is possible to define other incompressibilities than the usual one, K, measuring the stiffness (stability) of the phase diagram matter with respect to deformations compressions. This is known to depend on the neutron proton asymmetry [4]. These four parts of the nuclear interactions have been discussed in the frame of a non relativistic model in these last references.

3 Anti-matter component(s)

The solution of the Dirac equation for the baryons with couplings to vector fields (scalar/pseudoscalar classical fields contribute to change -or endow the baryon with - masses) may be a mixture of nucleons and anti-nucleons states which yield finite density state(s). Solutions for Dirac Equation with classical vector fields do not have the symmetry of the matter-anti-matter states characteristic from the free system [15, 5]. However it may be possible to have both coexisting in dense systems such as post-Supernovae or even anti-stars (of any kind) constituted (mainly if not competely) by anti-matter in the Universe. These conclusions hold at finite temperature. We define the following densities respectively for baryons and anti-baryons $N, \bar{N}$: $\bar{N}\gamma_\mu N = j_\mu \to \rho_B \to \rho_f$ and $N^T\gamma_\mu \bar{N}^T = \bar{j}_\mu \to \rho_{\bar{B}} \to \rho_{\bar{f}}$. A coefficient which measures the ratio of these components, if they do not annihilate themselves, may be defined as: $\omega = (\rho_{\bar{B}} - \rho_B)/(\rho_{\bar{B}} + \rho_B)$. Furthermore the classical vector fields, with different -variable- values, lead to "effective" chemical potentials for each nucleon (positive and negative energies) component. For particular values of V_0 the eingenvalues associated to anti-matter may be favored leading preferentially to anti-matter (bound) states. Moreover there are solutions for which the energy eigenvalue becomes complex which should, in principle, corresponding to an unstable state which should decay if it exists. Anti-matter bound states (mixed with matter or not) could be considered in the phase diagram of strong interacting systems as a third axis (besides density and temperature) or eventually associated to the matter density - linear independent or not. In Reference [16] the effect of classical tensor and vector fields were considered to the formation of superconductive states at very high densities yielding condensates of di-antifermions besides the usual di-fermions (di-quarks) condensates precluding the ideas of this article and of Reference [9].

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Captions

Figure 1 Effective masses of neutron (bold lines) and proton (normal lines) in asymmetric nuclear matter for two cases when in symmetric nuclear matter $M^* = 0.7M$ (dashed lines) and $M^* = 0.8M$ (solid lines).

Figure 2 Total fermionic energy density (ρ_f (fm^{-4})) with effective nucleon masses of symmetric nuclear matter $M^* = 0.7M$ (dashed lines) and $M^* = 0.8M$ (solid lines) with and without different proton and neutron masses, respectively normal and bold lines, for $\lambda = 40$. Without different masses the asymmetry is only due to $k_F^n \neq k_F^p$.

Figure 1

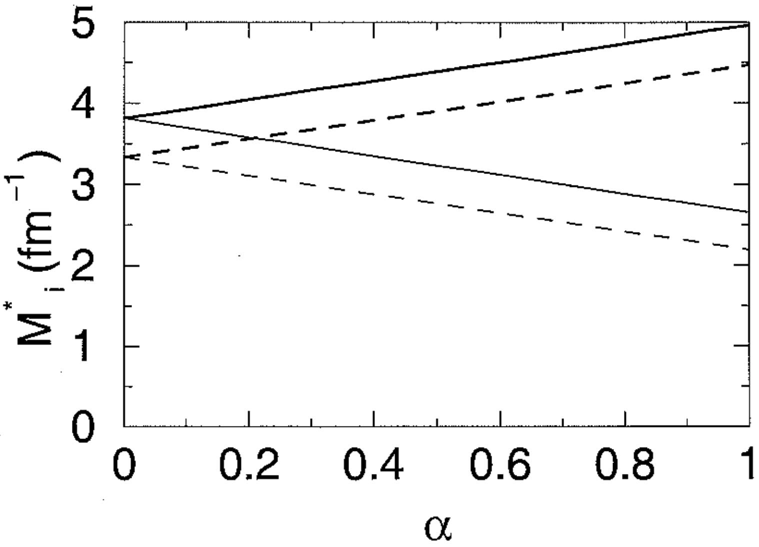

Figure 2