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Skyrmion and chiral symmetry breaking

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Abstract. A chiral coupling is considered for the Skyrme model to a light scalar meson which develops a classical component, eventually representing the scalar quark-anti-quark condensate of the spontaneous breakdown of chiral symmetry. This scalar field leads to a modification in the chiral radius which becomes a dynamical variable and tends to acquire values close to zero inside the topological soliton. A chiral rotation of the scalar and pseudoscalar fields can lead to the linear sigma model favoring the identification of the scalar field to the scalar sigma. The role of the scalar field mass is discussed.

The Skyrmion model [1] is an effective model for the low energy QCD since it exhibits some of the fundamental properties of the Quantum Chromodynamics in which the baryon emerges as a topological soliton [2, 3] and it predicts observables close to experimental data for the nucleon static properties [2]. Several attempts have been made in order to improve its phenomenology including the coupling to the relevant vector meson fields and also scalar fields representing gluons and quarks degrees of freedom in a not very complete physical basis [4, 2]. There are some experimental evidences of light scalar mesons in several processes including that with large width in the scalar isoscalar channel of the pion-pion scattering at energies of the order of 600 MeV [5, 6]. In spite of the strong criticism on the idea of a $q-\bar{q}$ bound scalar state, new developments are indicating it is not unreasonable [7]. Several works however claim other structures [8]. In [9] it is argued that above a critical large color number, $N_c = 6$, it is reasonable to expect such (broad) state with crossing and unitary symmetries. In the present work we revisit the problem of coupling the Skyrmion to a scalar field $\eta(\mathbf{r})$ which may correspond to chiral partner of the pion. This contribution is based on Reference [10] with a discussion about the role of the scalar field mass.

The Lagrangian of the model coupled to a scalar meson and to a vector- isovector massive field is given by:

$$\mathcal{L} = \frac{\eta^2(r)}{4} Tr(\mathcal{D}_\mu U^\dagger \mathcal{D}^\mu U) + \frac{1}{2}\mu^2 f_\pi^2 \left(\frac{\eta(r)}{f_\pi}\right)^m \left(TrU + TrU^\dagger - 2\right) + \mathcal{D}_\mu \eta \mathcal{D}^\mu \eta - V(\eta) + \mathcal{L}_\rho, \tag{1}$$

where all the parameters and couplings are those considered and discussed in [10] being common in the literature. m in the pion mass term is model depedent [1] The couplings shown above respect scale invariance and the covariant derivative $\mathcal{D}_\mu$ is to include coupling to isovector fields (eventually with a chiral invariant fashion, with axial fields).

[1] In particular: $m = 1$ the usual model is found, $m = 4$ this massive term becomes scale invariant [10].

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For the quadratic term the coupling shown above is implemented by the following chiral transformation from the non linear sigma model: $U(r) \to \frac{\eta(r)}{f_\pi} U(r)$. This model alone is not stable however [10]. Different ways of obtaining the coupling to the scalar field in the quartic Skyrme term were envisaged [10]. Firstly an *ad hoc* substitution of the constant $1/e$ by $\eta^4/(e f_\pi^4)$ was considered. Secondly the modification of the chiral function in the Skyrmion Lagragian was done like in the quadratic term. However, these replacements yield scale non-invariant Lagrangian terms. The lack of stability was also checked by the Derrick's argument [10]. Concerning the non linear sigma model with the scalar field alone, it is worth to remember that a chiral rotation of the non linear sigma model yields the modified version of the linear sigma model as presented in the Lagrangian above [11]. A seemingly more fundamental way of implementing this is the following [10]. The quartic stabilizing term can be obtained in the limit of very large isovector meson mass by means of the KSFR relation in the usual Skyrmion model [12], when $\eta(r) = f_\pi$ and the mass $m_\rho^2 \propto f_\pi^2$. With the coupling to the scalar field, from the equation of the vector meson we would have: $m_\rho^2 \simeq a g^2 \eta^2$ where a is a coefficient for the "gauged" kinetic term. Assuming everything is OK with this, there appear (scale non invariant) new terms in the resulting "effective" Lagrangian and differential equations. These terms make the numerical solutions less stable [2]. Secondly, and more difficult to get rid of, inside the topological soliton $\eta < f_\pi$ or $\eta << f_\pi$. Therefore the KSFR should not to be expected to work in the same way and the elimination of the vector field for the quartic stabilizing term would not be allowed anymore, although there may occur a scaling among the variables. Numerical calculation were done with the usual Skyrme term, by assuming that the changes in the parameters due to the coupling to the η field cancel each other allowing the use of the quartic Skyrme term with coupling e. Adopting the hedgehog ansatz, for which $\hat{\pi} = \hat{\mathbf{r}}$, we re-write the static lagrangian in terms of F and then calculate the Euler- Lagrange equations. Boundary conditions are discussed in [10].

While there is a minor change in the function $F(r)$ the condensate $\eta(r)$ becomes non homogeneous reaching values close to zero inside the topological soliton (like a "hole"). It can mean that the chiral symmetry is restored inside the nucleon. The nucleon masses and observables obtained in this modified Skyrmion model were shown more extensively discussed in [10] and will be discussed further elsewhere. For the lowest values of sigma mass, $\eta(r \to 0) \simeq 7$ MeV. A zero value of $\eta(r \to 0)$ does not seem to be reachable in this model. We can partially understand the dependence of the solution close to the origin $(r = 0)$ on the sigma mass with the following argument. Given that the scalar condensate $\eta(r \to \infty) \to f_\pi$ smoothly, we can consider that, for large r, its differential equation [10] is written with variable $\delta(r) = f_\pi - \eta(r) > 0$, with $\delta(r \to \infty) \to 0$. In a linear approximation in $\delta(r)$ (for $r \to \infty$), assuming a "decoupling" of the differential equations, we find:

$$\delta'' \simeq \delta(-2m_\eta^2 + 3\mu_\pi^3), \qquad \delta(r) \simeq Cexp\left(\pm r(3\mu_\pi^2 - 2m_\eta^2)^{\frac{1}{2}}\right), \tag{2}$$

[2] There appears from both stability analysis which we have done (in numerical calculations and with Derrick's "dimensional" arguments) that scale invariance and the (consequent) stability of the solutions may be related deeperly. The solutions become more unstable when scale invariance is broken beyond a certain level which would depend on the free parameters [10].

where C is a constant. Since the solution is real we choose the minus signal in the exponential. For this we have the following condition: $m_\eta^2 < \frac{3}{2}\mu_\pi^2$, which corresponds to a bound on the value of the scalar meson (sigma) mass for these stable solutions. According to this, the parameter m_η^2 could be positive although the spontaneously broken symmetry potential would, in principle, require negative values. A further bound on the scalar meson mass is given in [10] requiring the negativity of the energy density of the (stable) spontaneously broken potential of the η field, $m_\eta \simeq \sqrt{3}\mu_\pi > 0$. These two conditions shown above correspond to the bounds, respectively, of $m_\sigma \geq 0$ and $m_\sigma^2 \geq -3\mu_\pi^2$, a "negative"/imaginary mass bound (maybe an unstable state). With this value and $e = 5$ the scalar condensate reaches $\eta(r \to 0) \simeq 7$ MeV. By considering a Skyrmion in finite density of skyrmions we are lead to the following usual picture. Provided that inside the baryons the quarks are deconfined and chiral symmetry is not spontaneously broken the ("overall") order parameter (scalar condensate) tends to be closer to zero as the baryonic density increases, and asymptotic freedom, already valid inside the "holes", starts to become predominant everywhere. The $SU(3) \times SU(3)$ extended skyrmion with scalar mesons will be developped elsewhere.

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