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Nuclear matter symmetry energy from generalized polarizabilities: dependences on momentum, isospin, density and temperature

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Abstract

Symmetry energy terms from macroscopic mass formulae are investigated as generalized polarizabilities of nuclear matter. Besides the neutron-proton (n-p) symmetry energy the spin dependent symmetry energies and a scalar one are also defined. They depend on the nuclear densities (ρ), neutron-proton asymmetry (b), temperature (T) and exchanged energy and momentum (q). Based on a standard expression for the generalized polarizabilities, a differential equation is proposed to constrain the dependence of the symmetry energy on the n-p asymmetry and on the density. Some solutions are discussed. The q-dependence (zero frequence) of the symmetry energy coefficients with Skyrme-type forces is investigated in the four channels of the particle-hole interaction. Spin dependent symmetry energies are also investigated indicating much stronger differences in behavior with q for each Skyrme force than the results for the neutron-proton one

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1 Introduction

The symmetry energy terms and their dependences on the density are of relevance for the nuclear structure and in many nuclear processes including the structure and dynamics of proto-neutron and neutron stars. The neutron-proton symmetry energy is the best known in spite of the different values at the saturation density in the literature (in the range of 25MeV up to 36MeV). It is basically

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represented by a squared power of the neutron-proton (number or density) asymmetry in usual macroscopic/microscopic mass formula [1], the *parabolic approximation.* With a symmetry energy coefficient (s.e.c), a_τ, the binding energy is, in the simplified versions, usually written as:

$$E/A = H_0(A,Z)/A + a_\tau (N-Z)^2/A^2, \tag{1}$$

where the energy density H_0 does not depend on the asymmetry, Z, N and A are the proton, neutron and mass numbers respectively. Neutrons and protons occupying the same total volume yield a term proportional to the squared asymmetry density, $a_\tau(\rho_N - \rho_Z)^2/\rho^2$ which appears in the nucleonic matter equation of state. The neutron and proton densities may not be exactly equal to each other in nuclei [2]. Different polynomial terms of the asymmetry in this expression (proportional to $(N-Z)^n$ for $n \neq 2$) are usually expected to be less relevant [1, 3, 4, 5, 6, 7, 8]. However it is not well known whether and how this parabolic approximation is to be modified for very asymmetric systems, such as nuclei far from the stability line or for (asymmetric) nuclear matter above and below the saturation density [9]. In large stable nuclei such as ^{208}Pb the n-p asymmetry ($(N-Z)^2/A^2 \simeq 1/9$) is not so large as it would be in neutron matter. The n-p symmetry energy coefficient (s.e.c.) a_τ is given by the static polarizability of the system [10] (the inverse of the "isovector screening function") which can also depend on the asymmetry of the medium [11]. This may lead to slightly different forms for the symmetry energy for very asymmetric n-p systems.

Other symmetry energy coefficients may also be defined in nuclear matter, for instance, the spin A_σ and spin-isovector $A_{\sigma\tau}$ ones. Extending the n-p symmetry energy, the other symmetry energy coefficients can be defined in macroscopic mass formulae as:

$$\frac{E}{A} = \frac{H_0(A,Z)}{\rho} + A_\tau \frac{(\rho_N - \rho_Z)^2}{\rho^2} + A_\sigma \frac{(S_{up} - S_{down})^2}{(S_{up} + S_{down})^2} + A_{\sigma\tau} \frac{(\rho^N_{up} - \rho^N_{down} + \rho^Z_{down} - \rho^Z_{up})^2}{\rho^2}, \tag{2}$$

where the density (and eventually number) of neutrons and protons is denoted by ρ_N, ρ_Z, of nucleons with spin up (down) by $S_{up}(S_{down})$ and $\rho^i_{up,down}$ the neutron/proton densities with spin (up, down). The spin channel may lead to the appearance of polarized nucleonic matter which has been investigated within different approaches with controversial results [12, 13, 14, 15, 16, 11, 17, 18, 19, 20, 21]. The spin channel is also relevant for the study of the neutrino interaction with matter because it couples to the axial vector current together with the scalar channel in dense stars [22, 23, 24]. The spin-isospin channel has been associated to pion condensation [25, 26] and also to anti-ferromagnetic states [20]. A nuclear dipolar incompressibility was also defined in [11], being related to the nuclear matter incompressibility as discussed below, and which varies accordingly with the n-p asymmetry being eventually relevant for the isoscalar dipole resonances [27]. These coefficients and their corresponding dependences on the asymmetry of neutron-proton densities have been investigated in several other works, as for example in [28, 29, 21]. A different way of obtaining the symmetry energy has been proposed by means of the linear response method for the dynamical polarizabilities. The static limit of these generalized polarizabilities are proportional to the inverse of the symmetry energy coefficients in symmetric matter [10]. Therefore it becomes reasonable to consider the polarizabilities as a suitable and sound framework to determine the behavior of the symmetry energy with the parameters of the nuclear equation of state. Developments with relativistic models also yield strong effects with the isovector mesons, see for example references [30, 31, 32] among others. The density dependence of the neutron-proton symmetry energy and the isospin dependence of the nuclear equation of state are being extensively investigated for several reasons and several experimental tests are being done and prepared nowadays mainly in intermediary and high energy heavy ion collisions [33, 34, 35, 36, 37, 38, 39, 40, 41, 42]. The investigation of the possible effects with their particularities and the consequences for the observables is extense involving new experimental facilities such as RIA and GSI [36, 37, 43, 44] besides many other works [45]. Any definitive realistic investigation at really high densities (several times the saturation density) should take into account baryonic structure with the internal (quark and gluon)

In the present work some aspects of the symmetry energy terms are investigated as provenient from the generalized polarizabilities of nuclear matter for different ranges of the density, n-p asymmetry and momentum exchange, in the zero energy limit of the dynamical polarizabilities, within general arguments and with Skyrme forces at finite temperature. The case in which there is also non zero energy exchange corresponds to the analysis of the dynamical response function. The density dependence of the equation of state is not well known and it is reasonable to ask whether and how the n-p symmetry energy (and more generally other symmetry energies in the other channels of the nuclear interaction) depends on isospin at different densities / very high n-p asymmetries. The parabolic approximation, usually appropriated for a restricted range of densities (very) close to the saturation ρ_0 and small asymmetries, may be modified for lower and/or higher densities.

The parameters of the forces which are used (SLyb and SKM) were fitted (i) from results of asymmetric nuclear matter and neutron matter properties obtained from microscopic calculations [46] and (ii) from properties of giant collective modes in ^{208}Pb [47]. Other forces will be investigated elsewhere. Skyrme forces can be obtained from a reduction of the nuclear density matrix [48] and their basic structure is also present in non relativistic reductions of relativistic models for nuclear systems by passing to relativistic point coupling models or not [49, 50] such that the necessary density dependence of each of the terms are expected to be stronger than considered in the earlier parametrizations [51, 52].

This work is, in part, an extension of previous works and it is organized as follows. In the next section general aspects for the investigation of symmetry energy within the approach of the general polarizabities are discussed including the stability of nucleonic matter with respect to external perturbations. In section 3 an analysis of simultaneous dependence of the polarizabilities on the neutron-proton asymmetry and on the density is proposed with a differential equation that constrains these two behaviors of the symmetry energies. In sections 4 and 5 the q-dependence (exchanged momentum between the components of nuclear matter, eg. neutrons and protons) of static generalized polarizabilities at finite temperatures with Skyrme forces is investigated in the limit of symmetric nuclear matter. In the last section results are summarized.

2 Symmetry energy and nuclear matter polarizabities

Basically, in this section, arguments from previous works are reproduced. Consider that with the inclusion of an external source of amplitude ϵ, which separates nucleon densities with quantum numbers (s,t) (where $(0,1)$ is for spin up-spin down and $(0,1)$ for neutron-proton), the energy density of nuclear matter can be written as:

$$H = H_0 + \mathcal{A}_{s,t} \frac{(\rho_{(s,t)_1} - \rho_{(s,t)_2})^2}{\rho} + \epsilon' \beta, \tag{3}$$

where H_0 does not depend on the density asymmetry $(\rho_{(s,t)_1} - \rho_{(s,t)_2})^2$, $\mathcal{A}_{s,t}$ is the corresponding symmetry coefficient ($\mathcal{A}_{1,0}$ the spin one, $\mathcal{A}_{0,1}$ the neutron-proton one) and the total density fluctuation is $\beta = \delta\rho_{(s,t)_1} - \delta\rho_{(s,t)_2}$, for these two cases. For the spin-isospin external perturbation $(s,t=1,1)$ the simultaneous fluctuations of the spin (up/down) and neutron/proton densities are to be considered just like it is shown in expression (2). In the case of $(s=0, t=0)$, the scalar channel, there is a change in the total nuclear density and $\mathcal{A}_{0,0}$ is associated to a dipolar incompressibility [11]. For equal volumes the densities become the nucleon-numbers.

In the ground state, the variation of the energy with respect to the density fluctuation of a channel

(s,t), $\delta\rho \equiv \beta$, yields the condition of minimum:

$$\epsilon' + 2\frac{\mathcal{A}_{s,t}}{\rho}(\rho_m + \delta\rho) \;=\; \epsilon + 2\frac{\mathcal{A}_{s,t}}{\rho}\,\delta\rho \;=\; 0, \tag{4}$$

where $\rho_m = \rho_0^n - \rho_0^p \neq 0$ is for an n-p asymmetric matter (or correspondingly $\rho_m^s = \rho_0^{up} - \rho_0^{down} \neq 0$ for spin polarized matter) and the total "inducing perturbation" for n-p asymmetric systems is denoted by

$$\epsilon = \epsilon' + 2\mathcal{A}_{s,t}\,\rho_m. \tag{5}$$

The ground state can be considered to have a (polarized) spin up-down asymmetric density given by $\rho_m^s \neq 0$ simultaneously to (or instead of) the n-p asymmetry (also denoted by ρ_m above). ρ_m will be considered in most part of this paper . The nuclear matter polarizabity in the channel (s,t) can be written for ϵ' or for the total (inducing) perturbation, ϵ, respectively as:

$$\Pi_a^{s,t} \equiv \frac{\beta}{\epsilon'} = -\frac{\rho}{2\mathcal{A}_{s,t}^a}, \quad (i) \qquad \Pi^{s,t} \equiv \frac{\beta}{\epsilon} = -\frac{\rho}{2\mathcal{A}_{s,t}} \quad (ii). \tag{6}$$

The stability condition for these expressions are different. This will be discussed below.

The main development will be focused for the neutron-proton symmetry energy $(s,t = 0,1)$ although it is analogous for the other channels. The neutron proton asymmetry used in the present work is defined by the neutron and proton densities ρ_n, ρ_p as:

$$b = \frac{\rho_n}{\rho_p} - 1. \tag{7}$$

An asymmetry coefficient which is probably more familiar to the reader is given by:

$$\alpha = \frac{(2\rho_n - \rho)}{\rho}. \tag{8}$$

They are related by: $b = 2\alpha/(1-\alpha)$. The coefficient b varies from $b = 0$, in symmetric nuclear matter, up to $b \to \infty$, in neutron matter. For the sake of generality the coefficient $\mathcal{A}_{s,t}$ is considered to be a function of the density fluctuation β. The fluctuation β is considered to depend on the n-p asymmetry b. These parameters may be related to each other and therefore it will be written that $\mathcal{A}_{s,t} = \mathcal{A}_{s,t}(\beta)$. Relations between b and β have been investigated by means of prescriptions. Among those, one which leads to reasonable results is:

$$\beta = \delta\rho_n \left(\frac{2+b}{1+b}\right), \tag{9}$$

Where $\delta\rho_n$ is the neutron density fluctuation. In the n-p symmetric limit $\beta = 2\delta\rho_n$ and in another limit, in neutron matter, $\beta = \delta\rho_n$. This ansatz (expression (9)) is based on the assumption that the density fluctuations are proportional to the respective density of neutrons and protons, i.e., $\delta\rho_n/\beta = \rho_n/\rho$, being ρ the total density.

The resulting expression for the symmetry energy coefficient $\mathcal{A}_{s,t}$ for the prescription above is given by [11]:

$$\mathcal{A}^{s,t} = \mathcal{A}^{s,t}_{sym}\frac{2+2b}{2+b}, \tag{10}$$

for a general s,t (spin,isospin) channel of the effective interaction. In this expression $\mathcal{A}_{sym} = a_\tau \simeq 30 MeV$ is the s.e.c. of symmetric nuclear matter ($b = 0$). The (generalized) n-p symmetry energy term can be rewriten as:

$$\mathcal{A}^{0,1} = \mathcal{A}^{0,1}_{sym}(1+\alpha),$$

which corresponds to a third order term in the binding energy, being smaller than the quadratic term because $\alpha < 1$. In not very n-p asymmetric systems, those with n-p asymmetry close to the stability line, $\alpha^3 << 1$. For $b = 2$ ($\alpha = 0.5$, neutron density three times larger than the proton density) it follows $\mathcal{A} = 1.5\mathcal{A}_{sym}$. In the limit of neutron matter $\mathcal{A}(b \to \infty) = 2\mathcal{A}_{sym}$. For proton excess $b < 0$. Prescription (9) is therefore model-dependent and different choices yield other forms for the the (asymmetric) static generalized "screening functions". The dynamical response functions are less sensitive to this prescription.

So far it has been assumed that the stable density ρ is independent of b (or α). Below it is envisaged a development to guide the simultaneous variations of these two variables. Several investigations of the role of the symmetry energy on observables in Radioactive Ions are being prepared for RIA and GSI. For this the density dependence of the symmetry energy is extremely relevant. However for mass formulae of very asymmetric nuclei and for the equation of state at densities different from ρ_0 the isospin dependence of the symmetry energy may be different from the usual one given by the *parabolic approximation*, expression (1). Furthermore, more elaborated pictures in relativistic mean field calculations, which considers isovector mesons, $\vec{\delta}$, yield a qualitative increase of the relevance of the neutron-proton asymmetry, with a larger difference of neutron and proton effective masses [54, 55, 30, 32]. Experimental bounds on the neutron and proton effective masses [56] may shed light on this. The spin and spin-isospin symmetry energies can be investigated analogously. For example, the behavior of $\mathcal{A}_{1,0}$ of the spin channel (which has already been written as a_σ in the static limit in the framework of the Landau's Fermi liquid theory), at variable densities was investigated in different works [21, 17, 13].

2.1 Stability conditions

In the usual case in which the density ρ is not dependent neither on b nor on β there are two ways of writing a solution for the polarizability $\Pi_{s,t}$ from expression (3). They correspond to the different definitions of the external source shown before, respectively ϵ' (i) and $\epsilon = \epsilon' + \rho_m$ (ii). They allow for defining polarizabilities given respectively by:

$$\frac{\mathcal{A}^a_{s,t}}{\rho_0} = \frac{C^{s,t}}{(\Pi^a_{s,t})^2} - \frac{1}{\Pi^a_{s,t}} \quad (i), \qquad \Pi_{s,t} = -\frac{\rho_0}{2\mathcal{A}_{s,t}} \quad (ii), \tag{11}$$

where $C^{(s,t)} = -\frac{\rho_0}{4(\mathcal{A}^{sym}_{s,t})^a}$ is a constant, with the usual value of the symmetry energy coefficient. In the n-p channel: $(\mathcal{A}^{sym}_{0,1})^a \simeq 30$ MeV (symmetric limit). These two polarizabilities (11) are equal in the limit of symmetric nucleonic matter $\rho_m = 0$. This derivation applies for any of the channels (s,t).

Consider that the binding energy is to be minimized with respect to the density fluctuation β. From this an equilibrium condition for nuclear matter is obtained with $\delta^2(E/A)/\delta\beta^2 > 0$, being different from other ones and complementary to them [57]. To be a stable minimum of the binding energy the coefficients, of both definitions of the polarizabilities, satisfy respectively:

$$\frac{\delta^2 E/A}{\delta\beta^2_{s,t}} = -\frac{2C_{s,t}}{(\Pi^a_{s,t})^2} > 0, \quad (i), \qquad \frac{\delta^2 E/A}{\delta\beta^2_{s,t}} = 2\mathcal{A}_{s,t} > 0, \quad (ii), \tag{12}$$

where $\Pi^a_{s,t}$ and $\mathcal{A}_{s,t}$ given in expressions (11 (i)) and (11 (ii)) respectively. The constant $C^{(s,t)}$ may be negative (stable symmetric nuclear matter [21]) or positive, $\mathcal{A}_{s,t}$ and $\Pi_{s,t}$ also may be negative or positive. While the second expression yields the more expected result, i.e. the stability is directly shown by the signal of $\mathcal{A}_{s,t}$ in each channel (s,t), the first expression has a more involved behavior due to the complicated form of expression (11 (i)). From the polarizability (11-(i)) two conditions for real $\mathcal{A}$ and stable system follow:

$$(i)\ (1)\ \ \mathcal{A}^a_{s,t} < (\mathcal{A}^a_{s,t})_{sym}, \qquad (i)\ (2)\ \ (\mathcal{A}^a_{s,t})_{sym}\left(2 - \frac{\mathcal{A}^a_{s,t}}{(\mathcal{A}^a_{s,t})_{sym}} \pm 2\sqrt{1 - \frac{\mathcal{A}^a_{s,t}}{(\mathcal{A}^a_{s,t})_{sym}}}\right) < 0. \tag{13}$$

From condition (13 (i)-(1)) the neutron-proton asymmetry can only lower the value of the generalized coefficient $\mathcal{A}_{s,t}$ to keep the system stable with the use of polarizability (11 (i)). It is worth emphasizing that the two conditions (12) with the respective definitions for $\Pi_{s,t}$ should not be mixed. If the polarizability from expression (11-(ii)) is considered the condition (12-(ii)) is to be applied, otherwise inconsistent results arise. Expression (11- (ii)) is the usual form. However if one considers solution (11-(i)) the condition (12-(i)) is to be applied, otherwise inconsistent results arise. In particular in the case of the polarizabity given by expression (11-(i)) there are several possibilities for the stability of the symmetric and the corresponding asymmetric matter depending on $\mathcal{A}_{s,t} > 0$ (or $\mathcal{A}_{s,t} < 0$) and $\mathcal{A}^{s,t}_{sym} > 0$ (or $\mathcal{A}^{s,t}_{sym} < 0$) in each of the channel (s,t). The microscopic *in medium* nucleon interactions, in an exact calculation, would give the correct one. Expression (10) for $\mathcal{A}_{0,1}(b)$ was found with the solution (11 (ii)) for $\Pi_{s,t}$.

The stability conditions of a Fermi liquid in the leading order, in each channel of the interaction, correspond to a particular case of the above expression (12 - (ii)). They are given by the denominator of a particular limit of the response function $\Pi_{s,t}$ which can be written as:

$$a_{s,t} = N_0(1 + J_0^{s,t}) > 0, \tag{14}$$

where $J_0^{s,t}$ stands for any of F_0, F_0', G_0, G_0' respectively for the scalar $(s = 0, t = 0)$, isovector $(s = 0, t = 1$ with $a_\tau)$, spin $(s = 1, t = 0$ with $a_\sigma)$ and spin-isovector $(s = 1, t = 1$ with $a_{\sigma\tau})$ channels [58, 59]. These expressions contain the leading terms of the more general calculation. Within a non relativistic formalism with Skyrme type interactions they can be written in terms of Landau parameters [10, 60]. Other considerations can be associated in different formalisms [61, 17, 13, 62]. A complementary and more general discussion for particular models will be done in a forthcoming work.

3 Simultaneous dependence on isospin and density

Next it will be assumed that there is an implicit and *a priori* unknwon dependence of the saturation density on the n-p asymmetry without any supposition about the microscopic origin for this, $\rho_0 = \rho_0(b)$. From the general and usual expression for the polarizability (6) (or (11 (ii))) a differential equation for the simultaneous isospin and density dependence of the symmetry energy (coefficients) $\mathcal{A}_{s,t}$ will be derived. Although expression (6) was also derived without considering a dependence of ρ on b it will be considered that this simple form is more general. The derivative of the polarizability $\Pi_{s,t}$, expressions (6), with respect to b is given by:

$$\frac{\partial \beta}{\partial b} = \beta \left\{ \left(\frac{1}{\rho} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial \rho} \right) \frac{\partial \rho}{\partial b} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial b} \right\}. \tag{15}$$

The variation $\delta\beta/\delta b$ is given by expression (9), the prescription for the relation between the fluctuations. This equation has other three derivatives a priori unknown which have to be consistent with the equation of state: the derivatives $\partial\mathcal{A}/\partial\rho$, $\partial\mathcal{A}/\partial b$ and $\partial\rho/\partial b$. This expression is therefore to be equated to that of prescription (9) or, more generally,:

$$\frac{\delta\beta}{\delta b} \equiv -\beta f(b). \tag{16}$$

The resulting equation is:

$$\left\{ \left(\frac{1}{\rho} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial \rho} \right) \frac{\partial \rho}{\partial b} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial b} \right\} = -f(b). \tag{17}$$

This is one of the most relevant results of this paper. This equation constrains the simultaneous dependence of the symmetry energy on the density and on the nucleon density asymmetry (through the generalized coefficient $\mathcal{A}_{s,t}$, in the channel s,t, which is not a constant anymore) [1].

The following cases correspond to the derivation of section 2 (expressions (11)):

$$\frac{\partial \rho}{\partial b} = 0, \quad \text{and / or} \quad \frac{\partial \mathcal{A}}{\partial \rho} = \frac{\mathcal{A}}{\rho}.$$

These correspond to the use of prescription given by expression (9) which yields the function:

$$f(b) = \frac{1}{(1+b)(2+b)}. \tag{18}$$

For this prescription, which yields expression (10) for $\mathcal{A}(b)$, it has been assumed that ρ was independent of b, therefore, in that case:

$$\frac{1}{\beta}\frac{\partial \beta}{\partial b} = -\frac{1}{\mathcal{A}}\frac{\partial \mathcal{A}}{\partial b}. \tag{19}$$

In this case the behavior of $\mathcal{A}(\rho)$ can be the one typical of relativistic models with the increase of (any of the) symmetry energy coefficient with the increase of the nuclear density, i.e., $\mathcal{A}_{s,t} \propto \rho$ and in part of microscopic approaches, for which its value usually tends to a constant, [13, 29, 63, 64, 65, 66, 67]. However this is not the most general and interesting case because condition (19) holds when ρ is independent on b.

A slightly more general parametrization can be investigated. For example, Heiselberg and Hjorth-Jensen [68, 69] have used the following expression for the density dependence of the symmetry energy which nearly summarizes results obtained from relativist models:

$$E_{sym} = E_{sym}(\rho_0) \left(\frac{\rho}{\rho_0} \right)^{\gamma}, \tag{20}$$

[1]The parameter b however may be replaced by the equivalent one for the spin up-spin down asymmetry in polarized nuclear matter. The same form is obtained for spin-polarized nuclear matter, by interchanging the neutron-proton variables (from $s,t = 0,1$) to spin up- spin down ones (with $s,t = 1,0$). An equivalent prescription has to be provided for the spin density fluctuations.

where γ is a constant and ρ_0 the saturation density. A variational calculation favor values of the order of $\gamma \simeq 0.6$ whereas an analysis of heavy ion collisions experiments at low energies $\gamma \simeq 2$ [68, 69]. From the differential equation (17) it will be considered parametrizations given by:

$$\begin{aligned} \frac{\partial \mathcal{A}}{\partial \rho} &= \gamma \, \frac{\mathcal{A}}{\rho}, \\ \mathcal{A} &= \mathcal{A}_{sym} \frac{\alpha_1 + \alpha_2 b}{\alpha_3 + \alpha_4 b}, \end{aligned} \tag{21}$$

where α_i (i=1,2,3,4) are constants. When the asymmetry coefficient in expression (22) reaches the value

$$b = -\frac{\alpha_1}{\alpha_2}$$

the symmetry energy coefficient $\mathcal{A}_{0,1}$ changes the sign making the system unstable according to the stability condition (12 (ii)). The resulting equation, from the general equation (17), for the density as a function of b with the above parametrizations is:

$$\frac{\partial \rho}{\partial b} = \frac{\rho}{\gamma} \left(-f(b) + \frac{\alpha_1 + \alpha_2 b}{\alpha_3 + \alpha_4 b} \right). \tag{22}$$

For positive α_3 and α_4 the general solution is given by:

$$\rho(b) = (b+2)^{(\frac{1}{\gamma})} \, (1+b)^{(-\frac{1}{\gamma})} \, (\alpha_3 + \alpha_4 \, b)^{(\frac{\alpha_1 \alpha_4 - \alpha_2 \alpha_3}{\gamma \alpha_4{}^2})} \, e^{(\frac{\alpha_2 \, b}{\gamma \alpha_4})} \, B. \tag{23}$$

In the limit of symmetric nucleonic matter, for b the neutron-proton density asymmetry, $b = 0$, the constant B can be fixed in terms of ρ_0. In neutron matter $\rho \to 0$ or $\rho \to \infty$. For this to be finite, $\rho(b \to \infty) \to 0$, one must have $\alpha_2/\alpha_4 \leq 0$ and $\alpha_1\alpha_4 - \alpha_2\alpha_3 \leq 0$. A particular solution appears for $\alpha_3/\alpha_4 = -b$ which yields $\rho = 0$. When this occurs α_1 and α_2 from expression (23) have different signs.

For the usual form for the symmetry energy term in which $\mathcal{A} = a_\tau$ is independent of b, the resulting density as a function of the asymmetry b from the differential equation (17) is given by:

$$\rho(b) = C_0 \left(\frac{2+b}{1+b} \right)^{\frac{1}{\gamma}}, \tag{24}$$

where C_0 is fixed by a boundary condition, for example $\rho(b=0)$, with a fixed value of γ. From this limit: $C_0 = \rho_0 2^{1/\gamma}$ whereas in neutron matter $\rho(b \to \infty) = C_0$. The ratio of the density in these two limits is given :

$$\rho(b \to \infty) = \frac{\rho(b=0)}{2^{\frac{1}{\gamma}}}. \tag{25}$$

However, the parameter γ from the parametrization (20) may be (assumed to) depend on the neutron-proton asymmetry coefficient b (or equivalently α). In this sense, a modification in the usual symmetry energy dependence on the n-p asymmetry can be expected to be equivalent to different values for the parameter γ, at different densities, in different experimental situations.

A different form for the above equation (17) can be written by considering that: $\frac{\partial \rho}{\partial b} \equiv g(b,\rho)$, and $\frac{\partial \mathcal{A}}{\partial \rho} \equiv h(\rho, b) \neq \frac{\mathcal{A}}{\rho}$. The following differential equation appear for $\mathcal{A}(b,\rho)$ with these functions:

$$\left\{ \left(\frac{1}{\rho} - \frac{1}{\mathcal{A}_{s,t}} h(\rho, b) \right) g(b,\rho) - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial b} \right\} = -f(b). \tag{26}$$

Considering the particular prescription (19) it is obtained the following expression:

$$\left(\frac{1}{\rho} - \frac{1}{\mathcal{A}_{s,t}} h(\rho, b) \right) g(b,\rho) = -2f(b). \tag{27}$$

These expressions can be considered for any channel (s,t). They generate one differential equation for each channel of the nuclear effective interaction with $\mathcal{A}_{s,t}$, and therefore the final ρ dependence on b is to be the same for each of these equations, for b representing the same asymmetry (neutron-proton, spin-up-spin-down). For this, the choices for $\mathcal{A}_{s,t}$ and $\beta(b)$ should be associated, otherwise there will appear different $\rho(b)$.

4 Generalized "Screening functions" with Skyrme forces

In this section the analysis done previously [10, 53, 11, 21] is extended with the static limit of the expression for the dynamical polarizability of a non relativistic hot asymmetric nuclear matter with Skyrme effective interactions, $lim_{\omega \to 0} \Pi_{s,t}(\omega, q)$. These polarizabilities were obtained by the calculation of the response function of hot asymmetric nuclear matter in terms of three densities: neutron and proton densities (ρ_i), momentum density (τ_i) and kinetic energy ($\mathbf{j}_i$) densities from the time dependent Hartree Fock approximation with Skyrme forces [10, 11]. These densities appear in reductions from relativistic models in which the scalar density is written in terms of them [49, 50]. The time dependent approach introduces CP violating terms proportional to j which are larger in asymmetric nuclear matter. Four asymmetry coefficients are defined, a, b, c and d for the effective masses and densities, and they are given by:

$$a = \frac{m_p^*}{m_n^*} - 1, \quad b = \frac{\rho_{0n}}{\rho_{0p}} - 1, \quad c = \frac{1+b}{2+b}, \quad d = \frac{1}{1 + (1+b)^{\frac{2}{3}}}, \tag{28}$$

where m_i^* are the neutron and proton effective masses. Small approximations were done: (i) to equate the asymmetry coefficient defined for the momentum density to the density asymmetry coefficient (ii) to choose a particular prescription for the fluctuations of the asymmetry density - expression (9). The second approximation is in fact a choice with dynamical content and it deserves more attention.

At the Hartree Fock level, the symmetry energy coefficient, $a_\tau = \partial^2(E_0/A)\partial\alpha^2$, can be written from the expansion:

$$\frac{E_0}{A} = H_0 + \alpha \left.\frac{\partial(E_0/A)}{\partial\alpha}\right|_{\alpha=0} + \frac{\alpha^2}{2}\left.\frac{\partial^2(E_0/A)}{\partial\alpha^2}\right|_{\alpha=0} + ..., \tag{29}$$

where higher order terms are not written. There may appear (small) higher order terms. By calculating the general polarizability, within the linear response approach, a whole class of ring diagrams contribute beyond the Hartree-Fock [70]. Therefore more complete symmetry energy terms can be obtained.

For the calculation of the response function an external source is introduced in the Hartree Fock time-dependent equation which induces small amplitude density fluctuations. The general form of the source is the plane wave one, it is given by:

$$V_{ext} = -\epsilon\, \hat{O}_{s,t}\, \mathcal{D}\, e^{-i(\omega t - \boldsymbol{q}.\boldsymbol{r})}, \tag{30}$$

with an amplitude ϵ (usually a small parameter), an associated dipole moment $\mathcal{D}$ (equal to the unit from here on) and the operator $\hat{O}$ acts on the nucleon states. In particular, for the isovector interaction the third component of the isospin (Pauli) matrices is considered yielding neutron-proton density fluctuations. With the above external source, the Hartree Fock equation for nuclear matter is written as:

$$\partial_t \rho_i = -i\left[W_i + V_i^{ext}, \rho_i\right], \tag{31}$$

where W_i is the Hartree-Fock energy of protons or neutrons. The induced density fluctuations $\delta\rho$ are to have the same spatial and temporal plane waves behavior of the external source.

The resulting expression is more appropriatedly written in terms of generalized Lindhard functions whose real parts, at zero temperature F_{2i}, were defined as [10, 53]:

$$\Re e\Pi^i_{2N}(\omega, \mathbf{q}) \equiv \frac{gM^*}{2\pi^2}\Re e \int \mathrm{d}^3k \frac{f_q(\mathbf{k}+\mathbf{q}) - f_q(\mathbf{k})}{\omega + i\eta - \epsilon_p'(\mathbf{k}) + \epsilon_p'(\mathbf{k}+\mathbf{q})} (\mathbf{k}.(\mathbf{k}+\mathbf{q}))^N = \frac{gM^*}{2\pi^2}\int df_i(k)\Re e\, F_{2i}. \tag{32}$$

In these expressions $f_i(k)$ are the fermion occupation numbers for neutrons ($i = n$) and protons ($i = p$) which will be considered only for the zero temperature limit (when $df_i(k) \to -\delta(k - k_F)$), g is the degeneracy factor for spin and isospin, M^* is the effective mass in symmetric nuclear matter. In the limit of zero energy exchange ($\omega \to 0$) the Lindhard functions yield the (q-dependent) proton and neutron densities, momentum and kinetic energy densities are given by:

$$
\begin{aligned}
N_q &= \frac{\gamma M^*}{2\pi^2} \int df_i(k)\ \Re e F_0(\omega \to 0), \\
\rho_q &= \frac{\gamma M^*}{2\pi^2} \int df_i(k)\ \Re e F_2(\omega \to 0), \\
M_q &= \frac{\gamma M^*}{2\pi^2} \int df_i(k)\ \Re e F_4(\omega \to 0).
\end{aligned}
\tag{33}
$$

In the symmetric nuclear matter the momentum-dependent polarizability (6) in the channel s, t for the Skyrme effective force parametrization is written as:

$$
A_{s,t}(q) = \frac{\rho^q}{N^q} \left\{ 1 + 2\overline{V_0^{s,t}} N^q + 6 V_1^{s,t} M^* \rho^q + (V_1^{s,t})^2 (M^*)^2 \left(9{\rho_q}^2 - 4 M_q N_q \right) \right\}, \tag{34}
$$

Where $\overline{V_0}(q^2)$ and V_1 are functions of the Skyrme forces parameters for each of the (s, t) channel shown in the Appendix. The nuclear matter incompressibility modulus is related to $A_{0,0}(q^2 = 0)$ in the Appendix. The q-dependent densities N^q, ρ^q, M^q are the total densities from expressions (33). The term proportional to V_1^2 can be re-written in a homogeneous nuclear matter at zero temperature, as $\rho\tau - \vec{j}^2$ which is to be zero in the Galilean invariant (homogeneous and static) limit [50]. This invariance is broken in these cases and it is amplified in asymmetric nuclear matter. q in the neutron-proton channel is the exchanged momentum between the neutron and proton components, and similarly, in the spin channel, the corresponding exchanged momentum for spin up and down nucleons. The stability condition for this expression is given by (12 - (ii)). The finite temperature calculation of the densities lead to finite temperature symmetry energy coefficients. In the zero frequence limit the imaginary part of the response function disappears.

In the limit of low momenta, $q << 2k_F$, the $w = 0$ limit of the Lindhard functions are simplified, as shown in the Appendix. The polarizabilities of symmetric nuclear can be approximatedly written in the following form:

$$
\mathcal{A}_{s,t} = \mathcal{A}_{s,t}(T, \rho) + A_{s,t}^{(1)}(T, \rho) q + A_{s,t}^{(2)}(T, \rho) q^2, \tag{35}
$$

where $A_{s,t}(T, \rho)$ is the usual symmetry energy coefficient in the channel (s, t) [10, 11] and $A_{s,t}^{(i)}(T, \rho)$ are functions of the Skyrme force parameters (combined in the functions $V_0^{s,t}$ and $V_1^{s,t}$), ρ and T.

4.1 Results for Skyrme interactions in the 4 channels

In this section the generalized polarizabilities are investigated numerically for the Skyrme forces SKM and SLy(b) for the four channels of the particle-hole interaction as functions of the exchanged momentum at the normal density ρ_0. For this, the chemical potential was adjusted to mantain a constant stable nuclear density $\rho(T) = \rho_0$. As a consequence the results are not very strongly dependent on T. In Figure 1 the neutron-proton polarizability is shown as a function of the (exchanged) momentum for temperatures $T = 0, 4$ and 7 MeV for Skyrme force SLyb and at $T = 0$ MeV for the force SKM. Both forces produce widely accepted values for the symmetry energy coefficient, $\mathcal{A}_{0,1}(q = 0, T = 0) \simeq 32$ MeV. There is a general behavior (for both forces at any of the temperatures) of decreasing $\mathcal{A}_{0,1}$ with increasing exchanged momentum up to $q \simeq 500$ MeV. This corresponds to nearly twice the nucleon momentum at the Fermi surface. However $\mathcal{A}_{0,1}$ does not reach negative values. The behavior of decreasing values of $\mathcal{A}^{0,1}$ for increasing momenta is in agreement with other analysis for the momentum dependence of the symmetry energy [71]. The symmetry energy coefficient has, according to expressions (34,35), a linear/quadratic behavior for very small exchanged momenta q. This is followed by an abrupt change of behavior at $q \simeq 500$ MeV. For higher q the generalized s.e.c. ($\mathcal{A}_{0,1}$) increases nearly linearly at different temperatures. For the s.e.c. $\mathcal{A}_{0,1}$ to become negative the Skyrme parameters t_0 and t_3 should result in larger values of $\overline{V}_0$ than those of SLyb (this variable is still smaller for the force SKM) and/or different values for t_1, t_2.

In Figure 2 the spin-isospin generalized polarizability dependence on exchanged momentum between neutrons and protons with spin up and down is investigated for the same cases of Figure 1. There is again a quite defined change of behavior at $q \simeq 500$ MeV. The generalized spin-isospin s.e.c. remains nearly constant with increasing q up to $q \simeq 500$MeV. The force SKM yields smoother variations than SLyb like in the n-p channel. Above the saturation density, $\mathcal{A}_{1,1}$ decreases for most forces eventually reaching a negative value [21] . Finite temperature effects are larger for higher values of q.

In Figure 3 the spin generalized polarizability, $\mathcal{A}_{1,0}$, is shown for the same cases of the previous figures. The turning point present in the isospin-dependent channels, investigated in figures 1 and 2, is the same ($q \simeq 500$ MeV). This is due to the form of the Lindhard functions. However the behavior is completely different for each of the forces that already have very different predictions of $\mathcal{A}_{1,0}(q = 0, T = 0)$. SKM yields a nearly constant behavior followed by a strong increase of $\mathcal{A}_{1,0}(q)$ for very large q whereas SLyb decreases to a local minimum at $q = q_c \simeq 500$ MeV. The behavior resulted by the use of SKM force shows qualitative agreement with the results by Kaiser within Chiral Perturbation Theory [13]. For the force SLyb the spin symmetry energy coefficient may decrease still more for large values of q, at zero temperature, eventually it may become negative at larger densities. The instability associated to $\mathcal{A}_{1,0} < 0$ is the one towards a ferromagnetic alignment which has been found in several works with several Skyrme forces and relativistic models at higher densities

[14, 21, 20, 12, 16, 15]. However this transition is absent in several calculations. The most well known calculations in which the ferromagnetic alignment is not found are those ones based on NN interactions with different methods [17, 19, 18, 13]. However there are particular Skyrme forces which do not provide this ferromagnetic phase for nuclear matter: those parametrizations with the inclusion of NN tensor Skyrme-type force by Liu *et al* [72] or using SLyb at low momentum as seen in figure 3, for higher densities and n-p asymmetries - seen in the second of the references [21]. The effect of the momentum dependence, however, is the decrease of $\mathcal{A}_{1,0}(q)$. Whereas the functional density formalism with Skyrme forces and the relativistic (mean-field) models with nucleon-mesons couplings are effective models for the nuclear many body problem the NN based calculations are subject to approximative methods which may not capture all the relevant degrees of freedom appropriatedly in each part of the nuclear phase diagram. At finite temperatures $\mathcal{A}_{1,0}(q,T)$ does not vary significantly.

In the Figure 4 the scalar polarizability, $\mathcal{A}_{0,0}$ as defined in expression (34) is plotted. It shows a continuous increase with momentum without the turning point at $q \simeq$ 500MeV. This parameter, a dipolar incompressibility, is proportional to the nuclear matter incompressibility, like it is shown in the Appendix.

In all examples shown above, the increase of temperature is more relevant for larger q and the increase of the nuclear temperature always yields larger $\mathcal{A}_{s,t}$. Usually it is not expected a large variation of the static symmetry energy with the temperature from microscopic calculations in finite nuclei [73, 74]. Modifications of the chemical potential at high temperatures can lead to stronger dependences on T.

To understand the behavior of the generalized polarizabilities the total densities (N^q, ρ^q, M^q), as defined above, are shown in figure 5 as functions of q for the parameters of force SLyb. They are obtained from the zero-frequence of the generalized Lindhard functions and they generate the behavior of expressions (35) seen in figures 1 to 4. Whereas Π_0 and Π_2 present a smooth behavior towards to zero with the increase of q, the momentum density, $|\vec{j}| \propto M$, has a dramatic changement at $q \simeq 2k_F$. This prevents the polarizabilities $\mathcal{A}_{s,t}$ to become negative for the forces investigated in this work, in particular the neutron-proton $\mathcal{A}_{0,1}$ and spin $\mathcal{A}_{1,0}$ ones.

4.2 Other considerations

From the stability analysis of section 2, the results shown in figures 1 to 4, mainly for the force SLyb, suggest that nuclear matter is close to undergo phase transitions around $q \simeq 500$ MeV, i.e., when the exchanged momentum q is nearly twice the momentum at the Fermi surface, k_F. The q-dependence of the Lindhard function yield $M_i(q)$, as well as $N_i(q), \rho_i(q)$ for $\omega = 0$, which prevents nuclear matter to undergo phase transitions.

This analysis was done for zero energy with $\mathcal{A}_{s,t}$ only as a function of exchanged momentum. The frequence dependence of the polarizabilities was analysed associatedly to the exchanged momentum for the dipolar collective motions where zero-sound like excitations were found [10, 11]. In nearly symmetric nuclear matter they disappear at temperatures of the order of $T \simeq 7$ MeV [75, 76] (and higher temperatures for non zero asymmetries). Their disappearance may occur with the liquid-gas phase transition [77, 78]. The increase of the giant dipole isovector resonance width stops so that the corresponding energy is probably being used for changing the phase of the system

These results can also be expected to yield consequences for the Supernovae mechanism and proto-neutron or "neutron" stars with their dynamical behavior involving energy and momentum dependence of $\mathcal{A}_{s,t}$. The symmetry energies contribute, among other ways, by means of $\mathcal{A}_{0,0}$, $\mathcal{A}_{1,0}$ (due to the coupling to neutrinos) and $\mathcal{A}_{0,1}$, $\mathcal{A}_{1,1}$ for the different neutron-proton densities and the other related effects [79, 24]. The neutronization of a proto-neutron star in the quasi-static phase of the supernova can be partially suppressed due to the eventual increase of the symmetry energy coefficient although the momentum dependence shown in Figure 1 presents the opposite trend of decreasing $\mathcal{A}_{0,1}$ up to $q \simeq 2k_F$. This second (dynamical) effect seemingly would facilitate the neutronization and it should compete with the former. On the other hand the spin symmetry energy is strongly dependent on the used Skyrme interaction. Although the (continuous) increase of $\mathcal{A}_{1,0}$ seems to be rather in agreement with other works [17, 13] new developments are needed and they include the need of new parametrization of effective forces focusing on spin-dependent observables from nuclei and nuclear matter.

5 Summary

In this paper the nuclear matter symmetry energy terms were investigated as generalized polarizabilities. Stability conditions with respect to neutron-proton fluctuations were derived being complementary to others usually investigated [59, 57]. A differential equation for the simultaneous dependence of the generalized symmetry energy coefficients on the neutron-proton asymmetry and on the total nuclear density was proposed with equation (17). For this, no considerations about the microscopic reasons for the resulting stability density with a given n-p asymmetry were raised. The stability density is, in this case, to be a function of the n-p asymmetry as it should be in a general formulation. Some solutions for this equation were given. This procedure is interesting for finite nuclei as well. These results may be of interest for the investigations of the role of the symmetry energy on observables in Radioactive Ions which are being prepared and done mainly for the RIA and GSI machines. At different densities the isospin dependence of the symmetry energy may be different from the usual one. Finally, within the framework of the linear response of non relativistic nuclear matter with Skyrme forces, the q-dependence (exchanged momentum between the components of nuclear matter, eg. neutrons and protons) of the coefficients $\mathcal{A}_{s,t}$ was investigated. For low momenta the n-p symmetry energy decreases (linearly and quadratically) until $q_c \simeq 500$ MeV in agreement with earlier investigations of the symmetry energy potential [71]. In this range of momentum transfer q phase transition(s) may take place if other conditions are present, such as different (higher/lower) nuclear densities. This indication can be seen in the other symmetry energies, the spin-dependent ones, which however depend strongly on the particular Skyrme effective interaction. The results in the spin-dependent channels show no defined sign of such ferromagnetic phase for the SLyb force at the saturation density. However the decrease of $\mathcal{A}_{1,0}(q)$ with increasing transferred momentum may favor such phase transition in different conditions of densities and n-p asymmetries. The scalar coefficient $\mathcal{A}_{0,0}(q)$, the dipolar incompressibility, has continously larger values with the increase of exchanged momentum.

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Appendix: Skyrme force parametrization, functions V_i, relation between K_∞ and $\mathcal{A}_{0,0}(q=0)$

In this appendix we exhibit the functions V_i (for expression (34) with parametrization of Skyrme forces SKM, SLyb and others [52] given by:

$$\begin{aligned} v_{12} = \;& t_0(1+x_0P_\sigma)\delta(\mathbf{r_1}-\mathbf{r_2}) + \frac{t_1}{2}(1+x_1P_\sigma)\left[\delta(\mathbf{r_1}-\mathbf{r_2})k^2 + k'^2\delta(\mathbf{r_1}-\mathbf{r_2})\right] + \\ & t_2\,(1+x_2P_\sigma)\,\mathbf{k}'.\delta(\mathbf{r_1}-\mathbf{r_2})\mathbf{k} + \frac{t_3}{6}(1+x_3P_\sigma)(a_1(\rho_1+\rho_2)^\gamma + a_2\rho^\alpha)\delta(\mathbf{r_1}-\mathbf{r_2}), \end{aligned} \tag{A.1}$$

where P_σ is the spin exchange operator. The parameters for the forces SLyb and SKM are given respectively in references [46, 47].

From the linear response calculation for a time dependent Hartree Fock frame, in the lines discussed in [10, 53], we can write the corresponding functions $V^0_{s,t}$ and $V^1_{s,t}$ in each channel for the more general calculation in asymmetric nuclear matter:

$$
\begin{aligned}
\overline{V_0^{0,1}} &= \left(-\frac{t_0}{2}\left(x_0+\frac{1}{2}\right)-\frac{t_3}{12}\left[a_2\left(x_3+\frac{1}{2}\right)+a_1\left(1+\frac{x_3}{2}\right)-\frac{1}{4}(1-x_3)(\alpha+2)(\alpha+1)\right]\rho^\alpha+\right.\\
&\left.-\frac{q^2}{16}(3t_1(1+2x_1)+t_2(1+2x_2))\right)(1+bc)+V_2^{0,1},\\
V_1^{0,1} &= \frac{1}{16}(t_2(1+2x_2)-t_1(1+2x_1)),\\
V_2^{0,1} &= t_3\left[a_2(\frac{1}{2}+x_3)\alpha\rho^{\alpha-1}(c\rho_n+(c-1)\rho_p)+\right.\\
&\left.+a_1\left((1+\frac{x_3}{2})\alpha\rho^{\alpha-1}(c\rho_n+\rho_p(c-1))+2(1-x_3)(\alpha+2)(\alpha+1)(c\rho_n^\alpha+\rho_p^\alpha(c-1))\frac{1}{16}\right)\right]\frac{1}{12},\\
\overline{V_0^{0,0}} &= \left(3\frac{t_0}{4}+(\alpha+1)(\alpha+2)t_3\rho^\alpha\left[a_1(1+\frac{x_3}{2})\left(\frac{1+b}{2+b}\right)^2\frac{1}{16}+a_2(1+\frac{x_3}{2})\frac{1}{12}\right]+\right.\\
&\left.+q^2(9\frac{t_1}{32}-(5+4x_2)\frac{t_2}{32})\right)(1+bc)+V_2^{0,0},\\
V_1^{0,0} &= 3\frac{t_1}{16}+(5+4x_2)\frac{t_2}{16},\\
V_2^{0,0} &= \frac{t_3}{12}\left\{(x_3+.5)(c\rho_n+(c-1)\rho_p\rho^{(\alpha-1)})+\right.\\
&\left.+a_1\alpha(1-x_3)\left(\frac{(2\rho)^\alpha}{(2+b)^{\alpha+2}}+2\frac{((1+b)^2\rho)^\alpha}{(2+b)^{\alpha+2}}\right)\frac{1}{2}-a_2\frac{(1+(1+b)^2\rho^\alpha)}{(2+b)^2}\right\}\\
\overline{V_0^{1,0}} &= \left(-.5t_0(x_0+.5)-\frac{t_3}{12}\rho^\alpha(.5+x_3)-\frac{q^2}{8}(t_2*(x_2+.5)+3t_1(.5+x_1))\right.\\
&\left.+\frac{a_1}{12}t_3x_3\rho^\alpha(2+\alpha)+\frac{a_2}{24}t_3\rho^\alpha(2x_3-1)\right)(1+b.c)+V_2^{1,0},\\
V_{1,0}^1 &= \frac{1}{8}(t_2(x_2+.5)-t_1(x_1+.5))\\
V_2^{1,0} &= \frac{t_3}{12}(.5+x_3)\rho_n\rho_p^{(\alpha-1)}\alpha).c+t_3(.5+x3)\rho_p\rho^{(\alpha-1)}\frac{\alpha}{12}(c-1),\\
\overline{V_0^{1,1}}(q^2) &= \left(-\frac{t_0}{4}-\frac{t_3}{24}\rho^\alpha-\frac{a_1}{48}t_3((2\rho_n)^\alpha+(2\rho_p)^\alpha)-\frac{a_2}{24}t_3\rho^\alpha+\right.\\
&\left.+q^2(-3\frac{t_1}{32}-\frac{t_2}{32}))(1+b.c)\right)+V_2,\\
V_1^{1,1} &= -\frac{t_1}{16}+\frac{t_2}{16},\\
V_2^{1,1} &= \frac{\alpha}{24}t_3\rho^{(\alpha-1)}(\rho_n.c-\rho_p(1-c))-a_1t_3(2+\alpha)((2\rho_n)^\alpha c+(2\rho_p)^\alpha(c-1))\frac{1}{12}+\\
&+a_2t_3\alpha\rho^{(\alpha-1)}(-\rho_n\frac{c}{2}-(c-1)\frac{\rho_p}{2})(\rho_n.c-\rho_p(1-c))\frac{1}{24},
\end{aligned}
\tag{A.2}
$$

where ρ_n, ρ_p and ρ are the proton, neutron and total densities of asymmetric nuclear matter, a, b, c are the asymmetry coefficients defined in section 4.

For the longwavelength limit of $\mathcal{A}_{0,0}(b=0,\rho,q^2=0)$ the following relation is obtained in terms of the incompressililiyt modulus [11]:

$$\frac{1}{9}K_\infty = \mathcal{A}_{0,0}-\frac{4}{5}T_F+2V_1k_F^2\rho_0-\frac{3}{4}t_3\rho_0^{\alpha+1}. \tag{A.3}$$

They have different relevant terms such as the one proportional to the nucleon kinetic energy at the Fermi surface, T_F, and a term from the density dependence of the Skyrme forces proportional to t_3. This can be seen, in general, by remembering that the calculation of $\mathcal{A}_{0,0}(\rho, q=0)$ was done with the quadratic form for the binding energy in the presence of an external perturbation which induces density fluctuations of expression (3). It is rewritten below:

$$H = H_0 + \mathcal{A}_{0,0}(\rho)\frac{(\delta\rho_{0,0})^2}{\rho} + \epsilon'\delta\rho. \tag{A.4}$$

Terms containing $(\delta\rho)^n$, for $n \neq 2$, in H were neglected and would correspond to the terms which yield the usual K_∞. This more general parametrization will be considered in a forthcoming work.

The general structure of the zero frequence (real) generalized Lindhard functions Π_{2N} at zero temperature can be written as:

$$\begin{aligned}
\Pi_0(T=0) &= \frac{M^* k_F}{\pi^2}\left(-1 + \frac{a}{q}(1 - \frac{q^2}{2k_F^2})Ln\left|\frac{q-2k_F}{q+2k_F}\right|\right),\\
\Pi_2(T=0) &= \frac{M^* k_F^3}{2\pi^2}\left(-3 + 3q^2b^2 + \frac{b}{q}(1-3cq^2)(1 - \frac{q^2}{2k_F^2})Ln\left|\frac{q-2k_F}{q+2k_F}\right|\right),\\
\Pi_4(T=0) &= \frac{M^* k_F^5}{\pi^2}\left(a_4 + b_4 q + c_4 q^2 + d_4 q^3 + e_4 q^4 + \frac{1}{3q}(1 - \frac{q^6}{(2k_F)^6})Ln\left|\frac{q-2k_F}{q+2k_F}\right|\right),
\end{aligned} \tag{A.5}$$

where a_i, b_i, c_i, d_i, e_i dependend on k_F and on M^*.

Figure captions

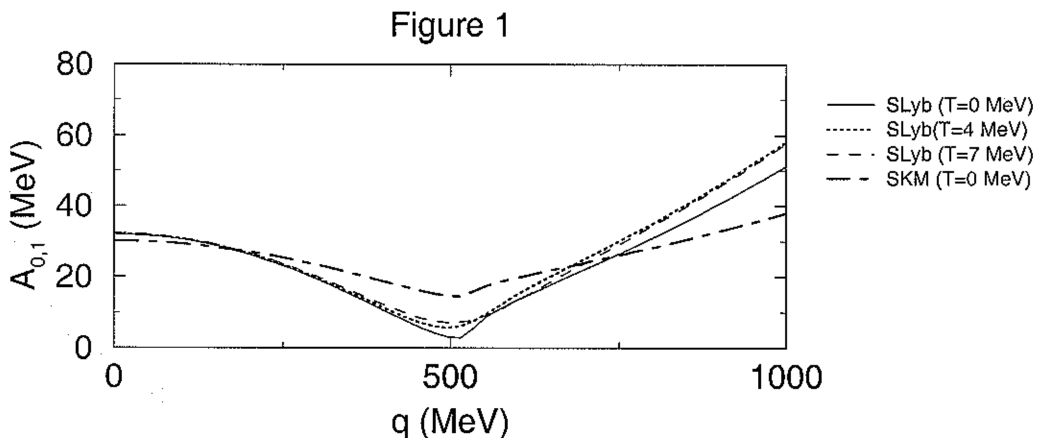
Figure 1 Neutron-proton symmetry energy coefficient $A_{0,1} = \rho/(2\Pi_R^{0,1})$ of symmetric nuclear matter as a function of the momentum transfer between neutrons and protons, q(MeV), for interactions SLyb for $T = 0, 4, 7$ MeV and SKM (T=0).

Figure 2 Spin symmetry energy coefficient $A_{1,0} = \rho/(2\Pi_R^{1,0})$ of symmetric nuclear matter as a function of the momentum transfer between neutrons and protons, q (MeV), for interaction SLyb for $T = 0, 4, 7$ MeV and SKM (T=0).

Figure 3 Spin-isospin symmetry energy coefficient $A_{1,1} = \rho/(2\Pi_R^{1,1})$ of symmetric nuclear matter as a function of the momentum transfer between neutrons and protons, q (MeV), for interaction SLyb for $T = 0, 4, 7$ MeV and SKM (T=0).

Figure 4 Scalar symmetry energy coefficient $A_{0,0} = \rho/(2\Pi_R^{0,0})$ of symmetric nuclear matter as a function of the momentum transfer between neutrons and protons, q (MeV), for interaction SLyb for $T = 0, 4, 7$ MeV and SKM (T=0).

Figure 5 The densities N, ρ, M as functions of the transfered momentum between neutrons and protons for the force SLyb. They are nearly independent of the force unless for the values of m^* and k_F.



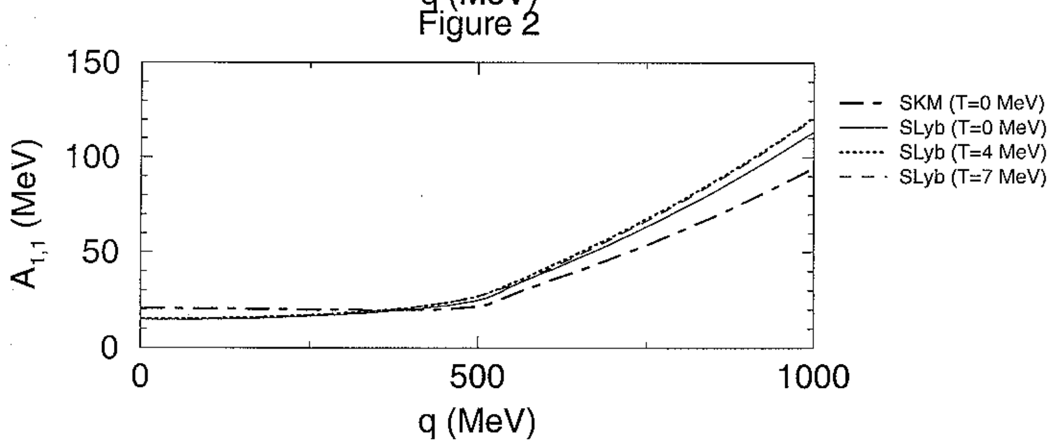


Figure 3

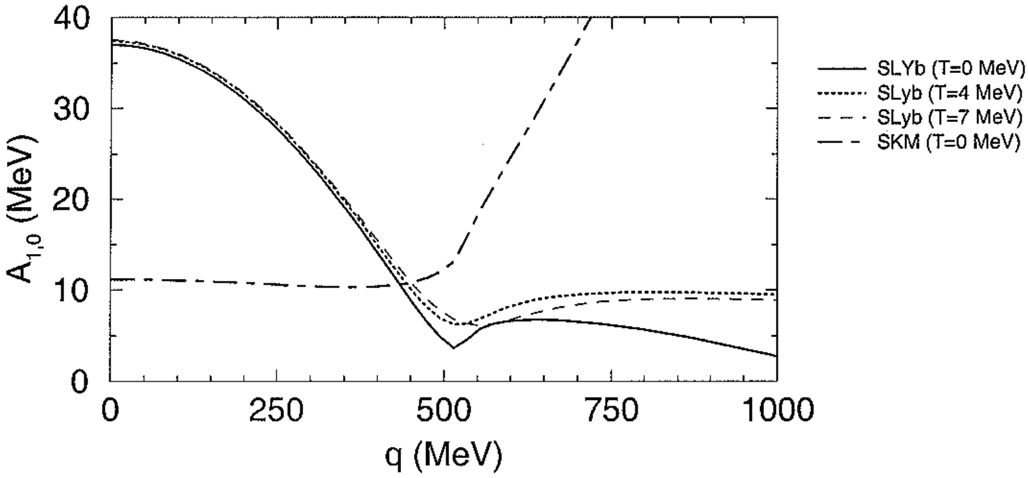


Figure 4

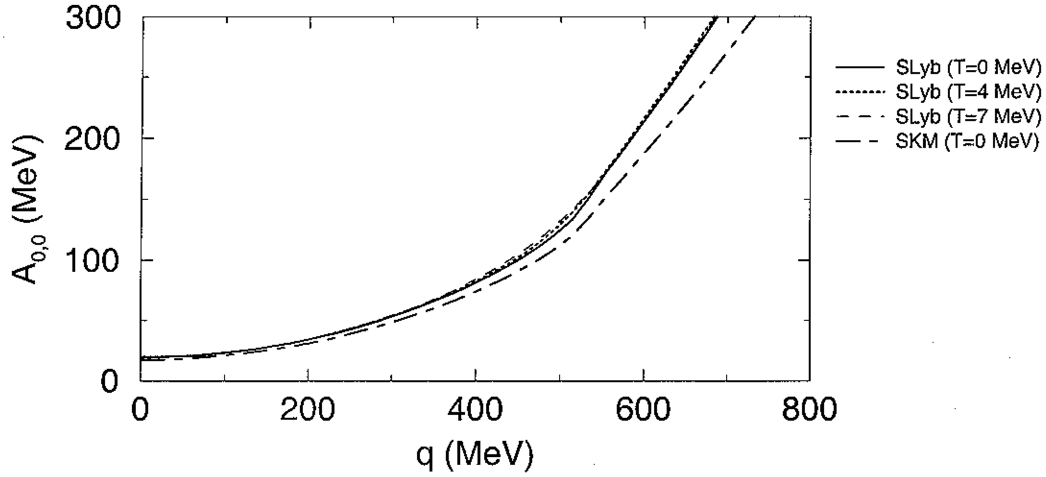


Figure 5