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Nuclear matter symmetry energies as generalized polarizabilities

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Abstract

The symmetry energy of nuclear matter are investigated as generalized "screening functions" - not only the neutron-proton (n-p) symmetry energy but also the spin dependent ones. They depend on the nuclear densities, neutron-proton asymmetry, temperature and exchanged energy and momentum (ω, q). The dependence of the symmetry energy on q is briefly investigated for Skyrme-type forces. Very simplified forms of the polarizabilities are proposed for very low exchanged momentum. Spin dependent symmetry energies exhibit much stronger differences in behavior with q for different Skyrme force. A standard expression for the generalized polarizabilities yields a differential equation for the simultaneous dependence of the symmetry energy on the n-p asymmetry and on the density.

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1 Introduction

The nuclear symmetry energy corresponds to the amount of energy needed to modifiy the n-p (density) asymmetry in the nuclear medium (either finite or infinite), either for neutron/proton or spin up/spin down nucleons, and their combinations. The first two (n-p and spin) can be written in the quadratic

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approximation in mass formulae [1] as:

$$E/A = H_0(A,Z)/A + a_\tau(N-Z)^2/A^2 + a_\sigma(n_{up} - n_{down})^2/A^2, \tag{1}$$

where N, Z, A, n_s hold for neutron, proton, mass and nucleon (with defined spin) numbers, being that for equal volumes (as in infinite nuclear matter) [2] these numbers are replaced by the densities. The same idea for the total nucleon density is associated to the incompressibility Therefore the response of a nuclear system (finite or infinite) to an external perturbation which changes nucleon densities (or numbers) yields information on the symmetry energy [3]. The linear response can thus be used to calculate symmetry energies as generalized nuclear polarizabilities. From this, their dependences on several parameters (total density ρ, temperature T, exchanged energy and momentum q, ω, neutron-proton asymmetry b, spin up-spin down asymmetry in a spin-polarized medium) are obtained at once simultaneously. The n-p symmetry energy is quite well described by a quadratic term in the n-p asymmetry for the binding energy (either $N - Z$ or $\rho_n - \rho_p$ assuming very close - if not equal - neutron and proton densities in nuclei [2]) with small corrections of different power(s) [1]. The density dependence of the symmetry energy is deeply studied nowadays, also in experimental facilities [5, 6], and it becomes relevant to understand to what extent this component of the nuclear forces can change. Besides that the symmetry energies are also of particular relevance for astrophysical dense objects and their dynamics [4].

This work is based in [8]. It shows results for the neutron-proton and spin dependent polarizabilities of nuclear matter at zero temperature at the saturation density for Skyrme forces. It also presents constraints for the simultaneous dependence of the polarizabilities on the nuclear density and on the n-p (density) asymmetry.

2 Generalized nuclear matter polarizabities

Consider that the inclusion of a time dependent external source V_i of (small) amplitude ϵ induces small fluctuations of the nucleon densities with quantum numbers (s,t) (where $(1,0)$ is for spin up-spin down and $(0,1)$ for neutron-proton). The Hartree Fock equation to be solved in the linear approximation [3, 9] is given by:

$$i\hbar\partial_t \rho_i = [\rho_i, H_i + V_i], \tag{2}$$

where H_i is the Hartree Fock energy density for each kind of nucleon (neutron-proton, spin up-down), of density ρ_i. It was written in terms of Skyrme-type forces [7].

The external source induces density fluctuations over the (static) ground state. Therefore $\mathcal{A}_{s,t}(\rho_{(s,t)_1} - \rho_{(s,t)_2})^2$ is considered with the corresponding symmetry coefficients: ($\mathcal{A}_{1,0}$ the spin one, $\mathcal{A}_{0,1}$ the neutron-proton one). The total density fluctuation is denoted by $\beta = \delta\rho_{(s,t)_1} - \delta\rho_{(s,t)_2}$, for these two cases. β were assumed to depend on the neutron-proton (or spin-up/spin-down) asymmetries by means of prescriptions which are not discussed here [10].

In symmetric nuclear matter in the limit of zero frequence the momentum-dependent retarded response function (polarizability) in the channel s,t can written as:

$$\Pi^R_{s,t}(q) \equiv -\frac{\rho_q}{2\mathcal{A}_{s,t}(q)} = -\frac{\rho_q}{\frac{\rho^q}{N^q}\left\{1 + 2\overline{V_0^{s,t}}N^q + 6V_1^{s,t}M^*\rho^q + (V_1^{s,t})^2(M^*)^2\left(9{\rho_q}^2 - 4M_qN_q\right)\right\}}, \tag{3}$$

Where $\overline{V_0}(q^2)$ and V_1 are functions of the Skyrme forces parameters for each of the (s,t) channel and N^q, ρ^q, M^q are the zero frequence limit of the generalized Lindhard functions defined in [9] which reduce to the usual densities. The usual nuclear matter incompressibility modulus K_∞ is related to $\mathcal{A}_{0,0}(q^2 = 0)$ as shown in [8].

This calculation will yield very general dependence of the binding energy (through its parameters) as a whole on the the exchanged momentum and energy, density, temperature and n-p asymmetry. Following this method it is possible to modify the energy needed for adding (one or more) new neutron(s)/proton(s) - for the binding energy - on a specific system within different conditions..... This will be shown elsewhere.........

3 Momentum dependence of symmetry energy

In Figure 1 the n-p polarizability function $\mathcal{A}_{0,1}$ is shown as a function of the exchanged mometum q for the Skyrme forces SLyB and SKM [11] at zero temperature at the normal density ρ_0. The results are compared to a simple parametrization valid for low q:

$$A^{approx}_{s,t} \simeq \mathcal{A}_{s,t}(q = 0) - A^{q_2}_{s,t}\, q^2, \tag{4}$$

where $A^0_{s,t}$ is the zero momentum limit, the usual symmetry energy coefficient in the channel s,t. The decrease of $\mathcal{A}_{0,1}(q)$ with increasing q is in agreement with other results [12] and it has direct consequences for the dynamics of astrophysical objects such as Supernovae since the symmetry energy is lowered [8, 4, 9]. The parametrization above (4) fits well the results for $\mathcal{A}_{s,t}^{q_2} \simeq .0107\text{MeV}^{-1}$ for not very high momentum exchange. However $\mathcal{A}_{0,1}$ reaches a minimum close to zero when $q \simeq 2k_F$. Close to this point the neutronization of matter can be favored and it may indeed reach negative values at different densities.

In Figure 2 the spin up-spin down $A_{1,0}$ is shown for the same parameters of Figure 1. While for the force SLyb the spin polarizability shows a more defined trend of decrease of $\mathcal{A}_{1,0}$ with the increase of the mometum exchanged until nearly $q \simeq 2k_F$ (where a ferromagnetic phase transition could be favored) for the SKM force this polarizability is nearly constant until this value of q. Whereas in the first case the spin-polarizability is well adjusted either by expression (4) for the force SKM the spin polarizability is well adjusted by the following expression:

$$A_{s,t}^{approx} \simeq \mathcal{A}_{s,t}(q=0) + A_{s,t}^{q_1}\, q, \tag{5}$$

where $A_{s,t}^{q_1}$ is simply a slope of the dependence with q, being either positive or negative. This is in good agreement with other works [13] for which the ferromagnetic phase is not expected to occur. At different densities these conclusions may change.....

4 Simultaneous dependence of $\mathcal{A}_{s,t}$ on the n-p density asymmetry and on the total density

In this section an equation which associate the dependence of the polarizabilities on the density and on the nucleonic asymmetries is shown. It is assumed that the saturation (or ground state) density depends on the n-p asymmetry, although the dynamical origin for this is not taken into account. The general and usual form of the polarizability is given by:

$$\Pi_{s,t} = -\frac{\rho}{2\mathcal{A}_{s,t}}. \tag{6}$$

The derivative of the polarizability $\Pi_{s,t}$ with respect to b yields the following equation [8]:

$$\left\{ \left(\frac{1}{\rho} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial \rho} \right) \frac{\partial \rho}{\partial b} - \frac{1}{\mathcal{A}_{s,t}} \frac{\partial \mathcal{A}_{s,t}}{\partial b} \right\} = -f(b), \tag{7}$$

where $f(b)\beta = -\delta\beta/\delta b$ can be given by a prescription for the dependence of the densities fluctuations on the n-p asymmetry [9]. Some solutions for this equation as well as the consistency with several current parametrizations were partially investigated in [8]. In particular for the neutron-proton symmetry energy: for $\partial\rho/\partial b = 0$ the development done for the polarizabilities in [9] follows. For $\partial\mathcal{A}/\partial b = 0$ (leading to the parabolic approximation of the symmetry energy) some results for the (general) dependence of the density on the n-p asymmetry were analysed in [8]. However it is well known that other (smaller) terms besides the quadratic one appear in mass formulae including terms breaking isospin symmetry [1].

For a given parametrization for $\mathcal{A}_{s,t}(\rho)$ and another for $\mathcal{A}_{s,t}(b)$ the consistency with the equation of state can be verified through $\rho(b)$ when the expression (6) holds.

5 Summary

The nuclear matter symmetry energies, n-p and spin dependent ones, were described as generalized polarizabilities. From the linear response with a particular nucleon-nucleon effective interaction, Skyrme force, it is obtained that they depend simultaneously on the density, n-p asymmetry (and eventually spin asymmetry for polarized systems), temperature, momentum and energy exchange. This indicates the variation of the energy needed to modify the nucleon densities (numbers) in specific conditions....... Particular approximations for the limit of very low momentum were proposed in agreement with results from different authors. The behavior of the symmetry energy terms with the nuclear densities and asymmetries can be associated by assuming the usual expression for the polarizability to be very general irrespectively to the value of these parameters (densities and asymmetries).

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Figure Captions

Figure 1 Neutron-proton symmetry energy coefficient $A_{0,1} = \rho/(2\Pi_R^{0,1})$ of symmetric nuclear matter as a function of the exchanged momentum between neutrons and protons, q(MeV), for interactions SLyb and SKM. The approximation given by expression (4) is also shown.

Figure 2 Spin symmetry energy coefficient $A_{1,0} = \rho/(2\Pi_R^{1,0})$ of symmetric nuclear matter as a function of the exchanged momentum, q (MeV), between spin up and spin down nucleons for interaction SLyb and SKM. Result from expression (5) is also shown.

Figure 1

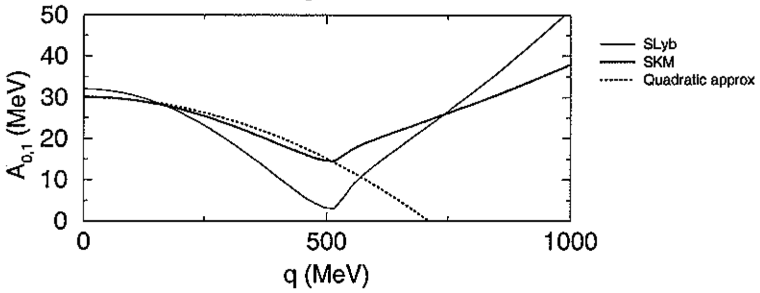

Figure 2