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SUPERCONDUCTIVITY

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## Di-Antiquarks condensation in Color Superconductivity

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Some consequences of a classical vector field (chromo-electromagnetic field) coupled to quarks, which undergo to superfluid and/or superconductive states with diquark / diantiquark condensation, are investigated. For this, one scalar field exchange is considered in the lines investigated by Pisarski and Rischke [1] in the mean field approach. Some effects and possible consequences are discussed.

### Introduction

In the high energy density regime of the phase diagram of matter, hadrons are not expected to remain as confined bound states due to deconfinement and chiral symmetry restoration. In particular at low temperatures, several configurations of diquarks (condensates) are expected to appear in different channels (such as scalar and vector), due to attractive color interaction (in the triplet channel) although color superconductivity might occur without a GAP [2,3]. Nambu-Jona-Lasinio (NJL) and gluon exchange models capture many features of the dense quark-gluon environment.

In principle diquark condensate would differ from the diantiquark condensate for a phase, although the distance of antiquarks from the quark Fermi surface can make the di-antiquark to disappear [1,4]. Usual calculations do not consider however antiquarks, that are produced at high energies, which can either annihilate or form bound/stationary states with quarks [5]. On the other hand from the phase of confined quarks and gluons to the energies where deconfinement takes place, gluons can assume an enormous variety of configurations [6]. These two aspects are considered in this work.

It is pointed out in this communication that particular configurations of (classical) vector fields [5-7] can produce different effects including modifications of the fermion masses, chemical potentials inducing diquark/di-antidiquark condensates at low temperatures. Results are in agreement with other independent works concerning diquark condensation at low energy densities [8]. The model considers the different composite condensates of quarks/antiquarks are due to a scalar field exchange (which can represent a phonon) in the mean field approximation. Possible related effects are discussed for Astrophysics and hadrons [9,5].

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## Quark condensation due to one scalar boson exchange

Consider that the exchange of a scalar boson ( $\phi$ ) generates condensates in the framework developed by Pisarski and Rischke [1]. With an additional coupling to a classical vector field the partition function will be given by:

$$Z = N \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi\mathcal{D}A_\mu \exp \int i \left\{ \bar{\psi}_i (i\gamma_\mu D_A^\mu - \gamma_0\mu + M_i^*) \psi_i + \mathcal{L}_\phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}, \quad (1)$$

where  $\mathcal{L}_\phi$  is the Lagrangian density of the massive scalar field, the effective mass for each quark with label  $i$  (its quantum numbers) is  $M_i^* = m_i - g\phi$ , a covariant derivative,  $D_\mu$ , can also be used for the scalar field, if directly coupled to the vector field.  $\mu$  is the chemical potential which can be redefined with the classical component  $\lambda^a A_0^a \equiv A_0$  (where  $\lambda_a$  are the GellMann color matrices) such that the energy eigenvalues and equations for each component of quarks/antiquarks are differently modified. Integrating out the scalar the scalar field an effective action is obtained which contains fourth order quark-antiquark term. The shifts of the two-point functions in the mean field approach are like:  $\psi\psi \rightarrow \psi\psi - \langle \psi\psi \rangle$  and  $(\bar{\psi}\bar{\psi})^\dagger \rightarrow (\bar{\psi}\bar{\psi})^\dagger - \langle \bar{\psi}\bar{\psi} \rangle^\dagger$ ; and the same for the  $\psi\bar{\psi}$  functions. In the limit of  $M_s \rightarrow \infty$  a NJL model is recovered. A more complete calculation of the properties of the model due to the vector field coupling will be shown elsewhere.

The action can be rewritten with doublets of quarks/antiquarks, using Nambu-Gorkov spinors [4,1], as:

$$I = \int \frac{d\mathbf{x}_i}{2} \left( \bar{\Psi} A_1(\mathbf{x}_1, \mathbf{x}_2) \Psi + \bar{\Psi} B_1(\mathbf{x}_1, \mathbf{x}_2) \Psi_C + \bar{\Psi}_C B_2(\mathbf{x}_1, \mathbf{x}_2) \Psi + \bar{\Psi}_C A_2(\mathbf{x}_1, \mathbf{x}_2) \Psi_C \right), \quad (2)$$

where the matrices  $A_i, B_i$  are respectively proportional to the quark-antiquark and diquark/di-antiquark GAPs. Considering that there is no singularity or topological configuration a Fourier transformation is performed and the GAP equations are obtained [1,10]. In the GAP equations the following "dressed propagator" emerges:

$$(G_{\bar{q}q}^\pm)^{-1}(k) = ((G_0^\pm)^{-1} - \Delta^\mp G_0^\mp \Delta^\pm)^{-1}. \quad (3)$$

where  $(\Delta^\pm)$  are the diquark/di-antiquark GAPs, the bare propagator is  $G_0^\pm$ . The total quark masses are given by:  $\tilde{M} = m_q - g_a \int_{p_b} D(p_b - p) \langle \bar{q}q \rangle^+$ , where  $D(k, M)$  is the scalar field propagator. They mix states of chiralities and duplicate the number of possible different di-quark/antiquark condensates Decomposing the GAP into states of helicity/chirality for the quarks/antiquarks,  $\Delta(k) = \sum_{(c=r,l),(h=\pm)}^{(e=\pm)} \phi_{c,h}^e$ , the separated (coupled) equations are obtained. The corresponding quark/antiquark dispersion relations are given by:

$$(\epsilon_a^\pm)^2 [\phi_{r,+}] = (\sqrt{\tilde{p}^2 + \tilde{M}^2} \mp \mu + gA_0)^2 + \sum_\alpha^{i,j} \mathcal{B}_\alpha^{i,j} |\phi_{\alpha^{i,j}}|^2, \quad (4)$$

where there is a combination of each of the GAPs in the last term. The conditions in which the component  $A_0$  assumes a particular value will not be shown elsewhere, although it might be associated to usual superconductivity [11]. In a first analysis the quark masses (including  $\langle \bar{q}q \rangle$ ) are neglected yielding only four possible states of defined chirality.

Results were obtained for massless quarks when GAP equations are of following form:

$$\phi_{r,+}^+ = \phi_{l,-}^+ (F_0^+( \phi_{l,-} ) - F_1^+( \phi_{l,-} )) + \phi_{l,+}^- (F_0( \phi_{l,+} + F_1( \phi_{l,+} ))), \quad (5)$$

where the remaining integrals in these expressions can be solved analytically and they are of the following types:

$$(F_0^\pm, F_1^\pm) = \frac{g^2}{2} \int \frac{dp_0}{2\pi} \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{(k-p)^2 - \tilde{M}^2} \frac{(1, \hat{k}\hat{p})}{p_0^2 - (\epsilon^\pm[\phi])^2}. \quad (6)$$

It will be considered nearly zero exchanged momentum, yielding constant  $(\Delta^\pm, \phi^\pm)$ .

### External chromo-electromagnetic fields

The vector field Euler Lagrange equation can be written as:  $D_\mu F_a^{\mu\nu} = (\bar{\psi}(\mathbf{x})\gamma^\nu\Gamma\psi(\mathbf{y}))_a$ , where  $\Gamma$  can contain color, flavor and spin, and the dynamics of the vector field, can be strongly modified according to the coupling to quarks. In particular at very high energy densities (chemical potential) the behavior of chromo(electro)magnetic fields is also quite relevant [3,8,12].

In the cases considered by Ebert and collaborators [8] the chromomagnetic (constant or not) field chosen were those such as: (1)  $A_i^a = Hx_i\delta_{\mu,2}\delta_3, a$  ( an abelian field); (2)  $A_i^a = \sqrt{H/g'}\delta_{i,a}$  ( $i = 1, 2, 3$ ) else  $A_i^a = 0$ , leading to  $H_i^a = \delta_i^a H$  (3)  $A_1^1 = A_2^2 = \sqrt{H/g}\delta_{i,a}$  yielding  $H_i^a = \delta_{i,a}H$ . It is interesting to calculate, as those authors do, the corresponding energy spectra (for equal chemical potential).

In each of these cases, whenever the diquark condensate is formed there is a corresponding limit in which a di-antiquark condensate might appear. This is not necessarily when  $\mu \rightarrow -\mu$  because of the (external) vector field. The energy spectra for quarks and antiquarks in the chromo(electro)magnetic field can have the same modulus and different signs for the free fermion case ( $\mu = 0$ ) in some cases. This yields the same value for the GAP of diquark and antidiquark condensates, unless for a phase. On the other hand, other contributions, mainly from the temporal component of vector fields, can modify the relative strength of the attractive interaction / chemical potential at lower energies such as those shown above (contributing to an effective chemical potential for each component of quarks/antiquarks) [5,9]. However the effective chemical potential is not the Lagrange multiplier. The solutions for the (classical) field are the key issues in these cases.

### Possible consequences

Whereas the energy needed to a quark to propagate in the background of a (scalar) quark-antiquark should be infinitely large due to the confinement, (colored) quarks/antiquarks propagate in a color-superconductive state either neutral or not [13]. There must have a maximum amount of energy for which this quark/antiquark ( $q_i/\bar{q}_i$ ) will not disturb/break the condensate in which it propagates,  $E_{max}^{q_i} > \Delta_{j,k}$ . The interaction of a quark with  $\langle \bar{q}q \rangle$  can produce a local quark-antiquark annihilation, in which case instabilities would probably arise. Otherwise it would give rise to quark-antiquark bound states. In this processes meson production could be relevant if energy density is close to the confinement region. These mechanisms can be of relevance for the structure of dense stars their structure (if quarks and antiquarks configurations can coexist or not) and dynamics including energy realise and cooling [3]. They will be investigated deeper elsewhere.

Finally, it can be considered the formation of di-antiquark codensates in finite size systems completely akin to has been found for diquark condensation [14] associated to a boxes of length of nearly 3 fm. This size is almost small enough as to permit asking whether

the different gluon configurations/degrees of freedom can trigger diquark/di-antiquark condensation inside hadrons, whose typical volume scale can be of the order of  $4 \text{ fm}^3$  (for a "mean diameter" of a nucleon of the order  $d \simeq 2 \text{ fm}$ ). Consequences of hadron production in (intermediary and) high energies (low temperatures) could be considerable. Antiquarks could appear when di-antiquarks condensates are broken, mainly as the energy density approaches the (deconfinement/chiral restoration) phase transition point. Lattice calculations can provide a valuable framework for investigating eventual condensation inside hadrons. Cosmological consequences would also arise if one considers hidden antimatter inside hadrons [5].

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