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LINEAR SIGMA MODEL AT FINITE BARYONIC DENSITY:  
SYMMETRY BREAKINGS AND ANTIMATTER

**Braghin, F. L.**

*Instituto de Física, Universidade de São Paulo, Caixa Postal  
66318 –CEP, 05315-970 São Paulo, S.P, Brasil*

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UNIVERSIDADE DE SÃO PAULO  
Instituto de Física  
Cidade Universitária  
Caixa Postal 66.318  
05315-970 - São Paulo - Brasil

# Linear Sigma Model at finite baryonic density: symmetry breakings and antimatter

Fábio L. Braghin

Instituto de Física, Universidade de São Paulo; C.P. 66.318; CEP 05315-970; São Paulo, Brazil.

The linear sigma model at finite baryonic density with a massive vector field is investigated considering that all the bosonic fields develop non zero expected classical values (eventually associated to condensates) corresponding to dynamical symmetry breakings which might occur in the QCD phase diagram. A modified equation for the classical field of the vector field is proposed with its respective solution. Some in medium properties for some hadrons are investigated within particular prescriptions.

## INTRODUCTION

The equation of state (e.o.s.) of high energy density matter has been continuously investigated in the last decades aiming the understanding of strong interactions. For this great attention is being given to experimental results which have been obtained in heavy ions collisions in BNL, CERN and GSI. The phase diagram of QCD is usually expected to contain condensates at different densities. Besides lattice calculations, effective models are developed from first principles such that the relevant properties, and eventually symmetries, are respected for the energy range of interest. With different approaches these models at finite energy density (temperature and/or chemical potential) have been extensively studied. In the vacuum, the lightest strong interacting particles are known to respect, approximatedly at least, chiral symmetry  $SU_L(2) \times SU_R(2)$  which is expected to be spontaneously broken down to  $SU(2)$ . The emergence of a scalar (quark-antiquark) condensate expected to occur in the ground state from a SSB may have relevant effects in a realistic calculation, it rearranges the properties of the theory [1-3]. As a consequence the *in medium* hadronic properties which depend on this condensate (which can be the order parameter or proportionally to it [4]) are expected to vary strongly with energy density, in particular the rho vector meson mass/widty has been considered as a possible signature of the chiral symmetry restoration via dilepton emission [5, 6] although there is recent analysis which provides a measure of the rho vector meson spectral function and is incompatible with energy shift [7].

In this work the  $O(N = 3)$  Linear Sigma Model (LSM) at finite baryonic density,  $\rho_B$ , is investigated with a massive classical vector field corresponding nearly to the usually called mean field. All the mesons in the model are considered to develop classical counterparts, including the pion [8, 9]. It seems to have a renewed interest in this condensation [10].

The exact field equations and the stability equation are truncated to allow for analytical solutions by considering particular prescriptions for the stability condition. These truncations are based on the following considerations: (1) the effective potential of spin zero bosons keeps nearly the

same form of tree level calculation, (2) each component of the system, i.e. baryons/ spin zero bosons/ spin one fields, have different conditions of stability stationarity. Hopefully this assumption might go along the observation of (elliptic) flow from relativistic heavy ions collisions [11]. Furthermore the corresponding (dynamical) equation for each component is satisfied. The complete numerical investigation of the results for nuclear matter and finite nuclei will be presented elsewhere [8, 18].

## THE LINEAR SIGMA MODEL AT FINITE $\rho_B$

The Lagrangian density of linear sigma model (LSM) with baryons,  $N_i(\mathbf{x})$ , sigma and pions,  $(\sigma, \pi)$ , covariantly coupled to two vector fields,  $V_\mu, V_\mu^v$ , is given by [12]:

$$\begin{aligned} \mathcal{L} = & \bar{N}_i(\mathbf{x}) (i\gamma_\mu D^\mu - g_S(\sigma + i\gamma_5 \tau \cdot \pi)) N_i(\mathbf{x}) \\ & + \frac{1}{2} (D_\mu \sigma \cdot D^\mu \sigma + D_\mu \pi \cdot D^\mu \pi) + c\sigma + \frac{1}{2} m_V^2 V_\mu V^\mu + \\ & - \frac{1}{4} F_{\mu\nu}^v F_{\nu\mu}^v - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (\sigma^2 + \pi^2 - v^2)^2, \end{aligned} \quad (1)$$

where the covariant derivatives are given in [12]. The other terms and parameters are standard [8]. The introduction of a chemical potential, with an extra term  $\delta\mathcal{L} = -\bar{N}\gamma_0\mu_{chem}N$ , is nearly equivalent to a shift of the classical temporal component of the vector field  $V_0$  coupled to the nucleons, it is a dynamical degree of freedom (d.o.f) and will be treated as such. Since the condensates (such as  $\bar{\sigma}$ ) depend (strongly) on the density so (most of) the hadronic masses do. Part of the baryon masses are considered to come from the the coupling to the scalar mesonic field and part from an explicit mass term for the baryons in the Lagrangian due to the other contributions (eg. gluon d.o.f):  $M^* = M \pm g_S\bar{\sigma}$ . The real balance between these two contributions will not be discussed here and it is not really relevant for the results. The mass variation of vector mesons in the medium has been associated to the tendence of restoration of chiral symmetry [13] in spite of controversies [7].

The spin zero fields will be treated in the framework of the variational Gaussian approach with a truncation [3, 8]. With a truncation of the effective potential of sigma and pion the total energy density,  $\mathcal{H}^{tot}$ , can be

written as given in [8], keeping the same form of the tree level effective potential. The equations for the sigma and pion are found accordingly.

The total energy density and e.o.s. are written in terms of the four variational parameters for the fields  $\sigma$ ,  $\vec{\pi}$ , plus baryonic densities and vector field variables. To investigate the behavior the (classical) vector field, the total energy density is varied with respect to  $V_0$ , which is not quantized. This yields a sort of variational equation for this quantity. It is given by:

$$\frac{\partial \mathcal{H}}{\partial V_0} = 0 \rightarrow g_V \left( \rho_B + V_0 \frac{\partial \rho_B}{\partial V_0} \right) - \tilde{m}_V^2 V_0 = 0. \quad (2)$$

$V_0$  is found either by writing a "reasonable" (or exact) expression for  $\rho_B$ , as given below, or it is treated like a variational parameter to determine  $\rho_B = \rho_B[V_0]$ . For this derivation,  $\tilde{m}_V$  was kept constant.

The stability condition for the ground state, with binding energy  $E_0/A = \mathcal{H}/\rho_B < 0$ , can be written as  $\frac{\partial \mathcal{H}}{\partial \rho_B} = 0 \rightarrow \frac{\partial \mathcal{H}}{\partial \rho_B} = \frac{\mathcal{H}}{\rho_B} \Big|_{\rho_B = \rho_0} < 0$ , and  $\frac{\partial^2 \mathcal{H}}{\partial \rho_B^2} \Big|_{\rho_B = \rho_0} > 0$ , where  $\rho_0$  is the stability density. The expression for the energy density is separated into three parts such that each component of the hadronic matter satisfies the stability equation above separately such that the variational equations of each of the components satisfy the respective equation. The reliability of this factorization is not proven although we have provided above and below arguments for being reasonable. The resulting equations (prescriptions) are the following:

$$\begin{aligned} (i) \quad \frac{\partial E_f}{\partial \rho_B} &= \frac{E_f}{\rho_B}; & (ii) \quad \frac{\partial \mathcal{H}_V}{\partial \rho_B} &= \frac{\mathcal{H}_V}{\rho_B}; \\ (iii) \quad \frac{\partial(\tilde{\sigma}^2 + \vec{\pi}^2 - v^2)}{\partial \rho_B} &= \frac{(\tilde{\sigma}^2 + \vec{\pi}^2 - v^2)}{2\rho_B}, \end{aligned} \quad (3)$$

In this second expression there are the energy density terms with contributions of the vector field. The complete set of solutions for the equations are investigated elsewhere. Below, some aspects associated to the in medium properties of hadrons are considered.

### Densities, masses and symmetry restoration

The baryon fields, which depend on the bosonic fields through the coupled Dirac equation, is quantized in terms of creation and annihilation operators. The baryonic degrees of freedom sum up into the following densities: baryonic ( $\rho_B$ ), scalar ( $\rho_S$ ) and pseudo-scalar ( $\rho_{ps}$ ) densities ( $\rho_B$ ,  $\rho_S$  and  $\rho_{ps}$ ). These quantities will not be explicitly evaluated here although they are partially used [8, 14, 15]. The energy density due to fermions (antifermions) ( $E_f, E_{\bar{f}}$ ) and the density of baryons (antibaryons) ( $E_B, E_{\bar{B}}$ ) can be written, in the leading order,

in terms of their momenta at the Fermi surface,  $k_F$ , and of the classical vector field as [14, 15]:

$$\begin{aligned} E_{(f,\bar{f})} &\simeq \frac{\gamma}{(2\pi)^3} \int^{k_F} d^3 k \left( \frac{2E_{(+,-)}^i (M_i^* + E_{(+,-)}^i) + V_0(V_0 - 2E_{(+,-)}^i)}{2(M_i^* + E_{(+,-)}^i)} \right) \\ \rho_{B,\bar{B}} &\simeq \frac{\gamma}{(2\pi)^3} \int^{k_F} d^3 k \left( \frac{(M_i^* + E_{(+,-)}^i)^2 - k^2}{2M_i^* (M_i^* + E_{(+,-)}^i)} \right) \end{aligned} \quad (4)$$

In these expressions  $E_{\pm}$  are the eigenvalues of the corresponding in medium Dirac equation; they are investigated elsewhere [14, 15]. These expressions still correspond to an approximation.

Instead of writing explicitly  $E_f[\rho_B]$ , as it is usually possible and straightforward for a Fermi gas/liquid, we find a (schematic) solution for the dependence of  $E_f$  on the baryonic density ( $E_f = E_f[\rho_B]$ ) from the first of the differential equations (3 - (i)). This is numerically nearly in agreement with that resulting from the usual expression from the solution of the free Dirac equation in the range of densities not far from  $\rho_0$ . A solution for the above prescription (3)-(i) is given by:  $E_f = K \frac{\rho_B}{9} \text{Ln} \left( \frac{\rho_B}{\rho_0} \right) + B\rho_B - K \frac{\rho_B^2}{9\rho_0}$ , where  $B$  is a constant fixed to reproduce  $\rho_f$  according to the usual expression ( $B \simeq 3.8 \text{ fm}^{-1}$  for the values adopted in section 4) and  $K$  is the usual incompressibility modulus. The deviation from prescription (3-(i)) was found to be small and independent of  $\rho_B$ .

The scalar and pseudoscalar densities which appear in the equations of  $\sigma, \vec{\pi}$  can be expanded in terms of the scalar and pseudoscalar condensates for example as:

$$\begin{aligned} \rho_S &= \rho_S^{(0)} + \frac{\tilde{\sigma}}{\tilde{\sigma}_{vac}} \rho_S^{(1)} + \frac{\tilde{\sigma}^2}{\tilde{\sigma}_{vac}^2} \rho_S^{(2)} + \frac{\tilde{\sigma}^3}{\tilde{\sigma}_{vac}^3} \rho_S^{(3)} + o(|\tilde{\pi}|), \\ \rho_{PS} &= |\tilde{\pi}| \rho_{PS}^{(1)} + |\tilde{\pi}^2| \rho_{PS}^{(2)} + |\tilde{\pi}^3| \rho_{PS}^{(3)} + o(\tilde{\sigma}), \end{aligned} \quad (5)$$

where the coefficients  $\rho_{S,PS}^{(j)}$  are such that  $\rho_S = \rho_{ps} = 0$  when  $\tilde{\sigma} = \tilde{\sigma}_{vac}$  and  $\tilde{\pi} = 0$  and other terms are indicated by  $o(|\tilde{\pi}|)$  and  $o(\tilde{\sigma})$ .

Comparing these expansions to the variational equations of each field [8] it follows that:  $\rho_S^{(2)} = \rho_{PS}^{(2)} = 0$ . The other parameters  $\rho_{s,ps}^{(i)}$  will be shown elsewhere. By substituting these expressions into the condensate variational expressions [8] it is seen that the terms proportional to  $\tilde{\sigma}$  and  $\tilde{\pi}$  yield contributions to the *in medium* masses of sigma and pions from the finite fermionic density in expressions. The contributions are given by  $\rho_S^{(1)}$  and  $\rho_{PS}^{(1)}$ . These terms appears numerically through the derivatives of the fermionic density,  $\partial \rho_f / \partial \tilde{\sigma} \propto \rho_S$  and  $\partial \rho_f / \partial |\tilde{\pi}_a| \propto \rho_{PS}$ . The terms proportional to  $\tilde{\sigma}^3$  and  $\tilde{\pi}^3$  can produce (effective) contributions for the coupling constant, i.e.,  $\lambda \rightarrow \lambda^* = \lambda \pm \frac{\rho_S^{(3)}}{\tilde{\sigma}_{vac}^3} \simeq \lambda \pm \rho_{PS}^{(3)}$ . These two corrections for the in medium effective coupling constant may also be different from each other, leading to different interactions of the pion and sigma in the baryonic medium, breaking chiral symmetry further or not.

From the second equation in (3) there appears a solution which can define a symmetry radius in the medium:

$$(\bar{\sigma}^2 + \bar{\pi}^2 - v^2) = \tilde{C}\sqrt{\rho_B}. \quad (6)$$

In this expression  $\tilde{C}$  is a constant to be determined from the parameters of the model. Therefore in the vacuum:  $\bar{\sigma}^2 = v^2 = f_\pi^2$  as discussed above. Modifications in the equation (3-ii) will produce different dependences on the baryonic density. A more general solution, corresponding to different modifications of that equation, might be written as:  $(\bar{\sigma}^2 + \bar{\pi}^2 - v^2) = (\tilde{D} + \tilde{C}\rho_B^c)^\gamma$  where  $\tilde{D}, \tilde{C}, c, \gamma$  are constants to be related to the parameters of the model, as it is done below for  $\tilde{C}$ .

Different ways of calculating the "symmetry radius"  $\tilde{C}$  are shown in the following. To investigate the consistency of the expression for the symmetry radius  $\tilde{C}$  for densities different from  $\rho_0$  it is imposed that the equations are satisfied simultaneously with expression for any density  $\rho_f(\rho_B)$ . Considering the prescription for the fermionic density (3 (i)) at  $\rho_B = \rho_0$ , the variational equations for the sigma and pions (which have the same form) yield:

$$\tilde{C} \simeq \pm \sqrt{\frac{8\rho_f(\rho_0)}{\lambda\rho_0}}. \quad (7)$$

Any other fermionic density which satisfy stability condition (3 (i)) will produce the same result. For  $\lambda = 20$  and  $\rho_0 = 0.15\text{fm}^{-3}$  the radius  $\tilde{C}$  is given by:  $\tilde{C} \simeq \pm 0.41\text{fm}^{-\frac{1}{2}}$ . For a different relation from prescription (3 (i)), for  $\partial\rho_f/\partial\rho_B$ , there appears modification(s) in these expressions.

Another way of calculating  $\tilde{C}$  is found assuming that this symmetry radius is nearly valid for a large range of baryonic densities including those close to that of the restoration of chiral symmetry ( $\rho_c$ ). This is a crude approximation because, for example, heavier hadrons and eventually quarks and gluons d.o.f. are expected to be relevant for high energy densities. At this (critical) density  $\bar{\sigma} = \bar{\pi} \rightarrow 0$ . Three values of  $\rho_B$  and  $\bar{\sigma}^2 + \bar{\pi}^2$  are considered to find the  $\tilde{C}$ , which is a "free parameter",  $\rho = 0$ ,  $\rho_0$  and  $\rho_c$ . This critical density can be reparametrized as  $\rho_c = u\rho_0$ . In this point:  $\tilde{C} = \mp \bar{v}^2 \sqrt{\frac{1}{u\rho_0}}$ . At the saturation density the expression for  $\tilde{C}$  can be written as:

$$\bar{\sigma}^2 + \bar{\pi}^2 \simeq \bar{v}^2 \left(1 \pm \sqrt{\frac{1}{u}}\right) = (f_\pi^0)^2 \left(1 \pm \sqrt{\frac{\rho_B}{\rho_c}}\right). \quad (8)$$

Four values are considered: (i)  $u = 2$ , (ii)  $u = 3$ , (iii)  $u = 3.5$  and (iv)  $u = 4$ . For  $\bar{\sigma}^2 + \bar{\pi}^2 < \bar{v}^2$  at  $\rho_B = \rho_0$  the it follows respectively:

$$\begin{aligned} \sqrt{\bar{\sigma}^2 + \bar{\pi}^2} \Big|_{\rho_0} &\simeq 0.54\bar{v} \quad (i), & \sqrt{\bar{\sigma}^2 + \bar{\pi}^2} \Big|_{\rho_0} &\simeq 0.65\bar{v} \quad (ii), \\ \sqrt{\bar{\sigma}^2 + \bar{\pi}^2} \Big|_{\rho_0} &\simeq 0.68\bar{v} \quad (iii), & \sqrt{\bar{\sigma}^2 + \bar{\pi}^2} \Big|_{\rho_0} &\simeq 0.71\bar{v} \quad (iv). \end{aligned} \quad (9)$$

The numerical solutions for  $\bar{\sigma}^2 + \bar{\pi}^2 > \bar{v}^2$  are not presented. Since the squared value  $\bar{\pi}^2$  is a scalar which appear very often in the expressions, it may be that  $f_\pi^* \simeq \sqrt{\bar{\sigma}^2 + \bar{\pi}^2}$ . These expressions may be therefore useful for relating descriptions of different ranges of the matter phase diagram, for densities not too close to those of the deconfinement/restoration of chiral symmetry.

The values obtained for  $\tilde{C}$  from estimates (9) are respectively given by:

$$\begin{aligned} \tilde{C} &\simeq \pm 0.41\text{fm}^{-\frac{1}{2}} \quad (i), & \tilde{C} &\simeq \pm 0.33\text{fm}^{-\frac{1}{2}} \quad (ii), \\ \tilde{C} &\simeq \pm 0.30\text{fm}^{-\frac{1}{2}} \quad (iii), & \tilde{C} &\simeq \pm 0.28\text{fm}^{-\frac{1}{2}} \quad (iv). \end{aligned} \quad (10)$$

These values, in particular (i) for  $u = 2$ , are in fair agreement with the other values calculated differently.

A third way of estimating  $\tilde{C}$  is shown by considering the GAP equations and the behavior of the meson masses in the medium. With them, the symmetry radius can be written as:  $\tilde{C}\sqrt{\rho_B} = \frac{1}{4\lambda}((\mu_T^*)^2 - (\mu_T^{vac})^2)$ , where  $(\mu_T^*)^2 = (\mu_S^*)^2 + (\mu_P^*)^2$  at a given density  $\rho_B$  with other contributions, from the classical vector field, set to zero. In these expressions the coupling  $\lambda$  was also kept constant (and positive) although, as argued above, *in medium* effects can be expected. Chiral symmetry restoration can be implemented with both behaviors - as long as the masses tend to be equal at higher energy densities. For  $\rho_0 = 0.15\text{fm}^{-3}$  and  $\tilde{C} \simeq -0.15$  the above expression yields approximated values  $(\mu_T^*)^2(\rho_0) \simeq (1 \pm 0.53)\mu_T^2(\rho_B = 0)$ . The sigma and pion masses (eventually their sum) may increase (or decrease) at finite densities.

These three ways of calculating  $\tilde{C}$  provide crude (but interesting) estimations for investigating *in medium* hadron properties which, however, do not consider other degrees of freedom (quarks, gluons, heavier flavors, higher order interactions and approximations). This will be presented elsewhere.

## Matter and antimatter

From expressions (4) some effects of the coupling of the baryons to a (classical) vector field can be obtained even if back-effects are not considered here. Eventual corrections to usual properties of a medium with matter and antimatter can be calculated [14, 15] which can annihilate in the zones of contact between domains as it is expected to have happened in the early Universe [16]. However it is believed that bound states of matter-antimatter can survive such as to form scalar quark-antiquark condensate which would break chiral symmetry spontaneously and this would have occurred without their mutual annihilation and before processes causing the eventual (total) baryon-antibaryon asymmetry. Therefore this can also suggest that other types of matter-antimatter bound states might have been formed even in the hadronic phase

[15, 16]. In the case that pseudoscalar condensation could also have occurred (such as for the pions), corresponding to a mechanism for CP violation which can be proportionally amplified in the dense medium [17]. In the early Universe, the Sakharov's scenario for generation of matter antimatter asymmetry [16] could be favored.

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