The Uncertainty Relations from a Classical Standpoint

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Abstract

In this paper, by imposing on the finite *change operators* of classical mechanics the uncertainty principle constraints, it is shown that the uncertainty relations arise as an equation that represents a variant statement of *Newton*'s second law, thereby acquiring an operative role in physical theory. This approach suggests that when these limits are about to be violated by the motion of a particle, an *uncertainty reaction*, understood as a process whose natural paradigm is a chemical reaction, arises, producing as its outcome either the emission or the absorption of a quantum unit of radiation.

1 Uncertainty and the Laws of Newton

Let us consider the one-dimensional change $\Delta\mathcal{E}$ of the energy of a particle moving in a gas[1], corresponding to the work of a force f exerted upon it,

$$\Delta\mathcal{E} = f\Delta x, \tag{1}$$

where Δx is the displacement produced. Denoting by Δp the change of the linear momentum of the particle, equation (1) can be written in the form,

$$\Delta\mathcal{E} = \frac{\Delta p}{\Delta t}\Delta x, \tag{2}$$

where Δt is the period of time during which the momentum of the particle undergoes the change. In Newtonian mechanics the meaning of force is *always* assigned to the expression $\Delta p/\Delta t$ and, exceptionally to the derivative operation of Calculus,

$$f = \lim_{\Delta t\to 0}\frac{\Delta p}{\Delta t}. \tag{3}$$

Let us rewrite (2) in the finite form,

$$\Delta\mathcal{E}\Delta t = \Delta p\Delta x. \tag{4}$$

If the derivative (3) exists, then,

$$\lim_{\Delta t\to 0}\Delta\mathcal{E}\Delta t = 0. \tag{5}$$

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[1]As will be shown later in this paper, the existence of an environment where the particle moves and with which it interacts is a requirement imposed by the uncertainty relations. The simplest environment with thermodynamic consequences is assumed to be the gas.

To examine how the meaning of (4) is modified when a field is replaced by the uncertainty relations to describe the mutual action between particles, let us recall the *Bridgman*'s criterion[2], which requires to find out what physical processes these mathematical operations correspond to. Quantum mechanics reveals that nature imposes a lower bound on the product (4), so that the operation that gives it meaning is not (5) but, instead, the uncertainty relations,

$$\Delta\mathcal{E}\Delta t = \Delta p \Delta x \geq h. \tag{6}$$

Expression (4) represents a restatement of *Newton*'s second law, where the notion of force has been replaced by that of energy. The second law can be regarded as a *prototype* whose completion is achieved by introducing the "law of forces" that act on the particle. If it is assumed that the quotients $\Delta p/\Delta t$ and $\Delta\mathcal{E}/\Delta x$ have well defined limits for $\Delta t \to 0$ and $\Delta x \to 0$, respectivellym then (4) is converted into a second order ordinary differential equation whose solution, given its initial conditions, completely describes the motion of the particle. If, otherwise, the relations (6) are assumed, a different system of *(in)equations of motion* is obtained.

In either case, however, it is possible to assign to $\Delta p/\Delta t$ in (4) the Newtonian meaning of force and to reason, in quantum phenomena, in terms of "uncertainty forces" that act on the particles of the gas.

1.1 The Mechanical Consequences of Uncertainty

It is here intended to show that, by acknowledging the expression (6) as a variant of *Newton*'s second law, the uncertainty relations acquire an operative role in the description of motion thereby extending the scope of Newtonian mechanics to deal with certain quantum phenomena. Such acknowledgment demands for a deeper analysis of its consequences.

Since the uncertainty relations interdict the limit operation $\Delta x \to 0$, it follows that ...

§ 1 (The non-conservative nature of uncertainty)

$$\ldots \textit{the derivative} \quad \lim_{\Delta x \to 0} \frac{\Delta\mathcal{E}}{\Delta x} = f, \;\; \textit{does not exist in quantum mechanics.} \tag{7}$$

According to a well known theorem of classical mechanics,

§ 2 *The forces unleashed in the motion of a particle constrained by the uncertainty principle are* non-conservative.

Hence, it is not only hopeless to search for an approximation for this process in terms of some suitably chosen (perturbation) potential, but specially misleading, for it conceals the existence of elementary energy conversion processes and their intrinsic irreversibility, hindering the quantum nature of thermodynamic phenomena.

§ 3 (Necessary Environment)
Corollary §2 implies the necessity to admit the existence of an environment *where the particles move and with which they exchange the non-conservative amounts of energy released or acquired during those processes constrained by the uncertainty relations.*

It will be assumed that the uncertainty principle also implies that ...

§ 4 (Uncertainty Principle-Constrained Functions)
... if a continuous and differentiable function is constrained by the uncertainty relations, the latter condition prevails over the former.

[2] *"In dealing with physical situations, the operations which give meaning to our physical concepts should be the physical operations actually carried out"* [1].

Statement §4 implies also the following well known corollary:

§ 5 (The inadequacy of the limit operator)
The use of the limit *operation of differential calculus must be reviewed under the constraints of the uncertainty principle.*

This proposition has far reaching consequences, for, by denying the existence of convergence processes that, acting on the degrees of freedom of a single particle, give rise to well defined limit values, the uncertainty relations undermine the very foundations of differential calculus, thereby subverting the whole of Analytical Mechanics. Furthermore, the acknowledgment of the uncertainty-constrained version of *Newton*'s second law provides a new approach to the study of inelastic collisions.

1.2 Uncertainty Reactions

Let us consider a frontal collision between two particles in the absence of a field. In such case the distance Δx between them decreases linearly with time. It is therefore meaningful to inquire on the notion of "physical contact" under the influence of the uncertainty relations, for according to equation (6), $\Delta x \to 0$ implies $\Delta p \to \infty$. Instead of discarding this situation as unnatural, the following hypothesis is introduced:

§ 6 (Uncertainty reaction)
No matter what the laws that rule the motion of a particle are, when it is about to violate the uncertainty principle, there arises an uncertainty reaction *that compel the particles to change their states to be conformed to that principle by exchanging energy and momenta (linear and angular) with its environment.*

The picture that comes to mind is that of an *elementary chemical reaction* whose reactants and products are different systems of particles, the motion of the latter being conformed to the uncertainty principle. This image is so well established in chemistry that no further justification seems to be necessary. Being a description of the behaviour of particles in collisions, the uncertainty reaction thereby replaces the notion of collision.

1.3 The Operative Character of Uncertainty Relations

Written in the forms,

$$\Delta \mathcal{E} \geq \frac{h}{\Delta t} \qquad \text{and} \qquad \Delta p \geq \frac{h}{\Delta x}, \tag{8}$$

the uncertainty relations (6) can be readily associated, respectively, with the quantum operators of energy and momentum of formal quantum mechanics,

$$\mathcal{E} \to \frac{h}{2\pi\imath}\frac{\partial}{\partial t} \qquad \text{and} \qquad p \to \frac{h}{2\pi\imath}\frac{\partial}{\partial x}. \tag{9}$$

Besides, since inequalities (6) lead also to the forms,

$$\Delta t \geq \frac{h}{\Delta \mathcal{E}} \qquad \text{and} \qquad \Delta x \geq \frac{h}{\Delta p}, \tag{10}$$

the conjugate operators of (9),

$$t \to \frac{h}{2\pi\imath}\frac{\partial}{\partial \mathcal{E}} \qquad \text{and} \qquad x \to \frac{h}{2\pi\imath}\frac{\partial}{\partial p}, \tag{11}$$

are expected to exist in quantum theory.

The two sets (9) and (11) of operators will be shown to correspond to the two fundamental conversion processes which endow the systems of particles with the faculties to convert work into heat and *vice-versa*. The existence of operators (11) also shows that, in quantum theory, t and x cannot be treated as *independent* variables.

Accordingly, it is legitimate to consider the following hypothesis:

§ 7 *In quantum mechanics there are two distinct elementary energy conversion processes, one corresponding to the pair of operations (8) and the other, to the pair (10).*

2 Uncertainty in Classical Mechanics

Summary

In this Section we shall focus the motion of a particle in a gas, by regarding the collisions it undergoes as a motion constrained by the uncertainty relations.

2.1 Free and Polynomial Path

Let us recall *Maxwell*'s description of the motion of a particle in a gas:

§ 8 *During the great part of their course the molecules (of the gas) are not acted on by any sensible force, and therefore move in straight lines with uniform velocity. When two molecules come within a certain distance of each other, a mutual action takes place between them (...). Each molecule has its course changed, and starts a new path [2].*

We define a *free path* as the motion of a particle between two consecutive collisions. With this definition we can describe the motion of a particle in the gas according to the following image:

§ 9 (The polygonal trajectory of a particle in the gas)
The trajectory of a particle moving in a gas can be described as a random walk, which can be depicted as a polygonal line, where its sides correspond to free paths and its vertices to collisions.

2.2 Vestiges in the Definition of Action

Collision is usually understood as a two bodies problem and treated as a two dimensional motion[3]. It will be here understood as a particular kind of motion that evolves in the plane determined by two consecutive sides of the polygonal line traversed by the particle.

The *Lagrange*'s function of a particle moving in a plane in the absence of a field can be written in terms of complex variables,

$$L = m\dot{z}\dot{z}^*, \tag{12}$$

where z is the complex coordinate of a particle in the plane of motion and z^* is its conjugate. In representation (12), *Lagrange*'s function is a scalar quantity.

[3]The *Schrödinger* equation of the electron-proton system can be regarded as the result of a collision phenomenon that takes place during a transient process of hydrogen synthesis during which their motion in a Coulombian field ceases to obey the classical laws of motion, passing to obey quantum laws. It is particularly meaningful for the present approach that this equation has a non-trivial two-dimensional solution even in the absence of a field.

Let us represent these derivatives by their finite forms, $\dot{z} = \Delta z/\Delta t$ and $\dot{z}^* = \Delta z^*/\Delta t$. According to the current tenets, a finite variation of the action of the moving particle is defined by,

$$\Delta \mathsf{A} = L\Delta t = m\frac{\Delta z}{\Delta t}\frac{\Delta z^*}{\Delta t}\Delta t. \tag{13}$$

While action has usually been assumed to be a scalar quantity, equation (13) does not define a single value for $L\Delta t$, for in this case, the "*algebraical cancellation law*" is non-commutative. In fact, it can be easily seen that,

$$\left(\frac{\Delta z}{\Delta t}\Delta t\right)\frac{\Delta z^*}{\Delta t} \neq \frac{\Delta z}{\Delta t}\left(\frac{\Delta z^*}{\Delta t}\Delta t\right), \tag{14}$$

Thus defined, the action must be treated as a complex quantity. Due to the non-commutativity of the cancellation law, the complex representation of the action gives the following two conjugate values[4],

$$L\Delta t = \begin{cases} \Delta \mathsf{A} \;\; = m\frac{\Delta z}{\Delta t}\Delta z^* = p\Delta z^*, \\ \Delta \mathsf{A}^* = m\Delta z\frac{\Delta z^*}{\Delta t} = \Delta z p^*. \end{cases} \tag{15}$$

It is noteworthy the correspondence between expressions (15) and the division operation of complex algebra:

§ 10 *Let a and z be two non-zero complex numbers. The quotient a/z can be represented by the expression,*

$$\frac{a}{z} = \frac{(a\cdot z) - \imath(a\times z)}{zz^*}, \tag{16}$$

where $(a\cdot z)$ represents the scalar product and $(a\times z)$ the vectorial product of a by z. We can then rewrite this expression in the form,

$$az^* = (a\cdot z) - \imath(a\times z) = |az|(\cos\varphi - \imath\sin\varphi), \tag{17}$$

where φ is the angle between a and z.

We can put (15) in the form,

$$\begin{aligned} \Delta \mathsf{A} \;\; &= p\Delta z^* = (p\cdot\Delta z) - \imath(p\times\Delta z), \\ \Delta \mathsf{A}^* &= p^*\Delta z = (p\cdot\Delta z) + \imath(p\times\Delta z), \end{aligned} \tag{18}$$

which, in their turn, can be rewritten in the form,

$$\Delta \mathsf{A} = \Delta p\Delta q\ (\cos\varphi \mp \imath\sin\varphi). \tag{19}$$

It is noteworthy that the complex algebraic structure of $\Delta\mathsf{A}$ reveals the two distinct effects of the action of a force on the motion of a particle, *viz.*, the change of linear and angular momentum of the particle. The latter is neglected in classical mechanics, for in that theory a material point is assumed to be a geometric particle and therefore its moment of inertia is zero by definition. In spite of our ignorance about its structure, there is plenty of evidence that, as opposed to the material points of classical mechanics, quantum particles are endowed with angular momentum degree of freedom.

It is interesting to observe that in the representation of the mechanical work, the product involved is the *scalar product between two vectors*, whose multiplication rule was obtained by the disembowelment of *Hamilton*'s quaternions. In its transplant to mechanics, the original non-commutativity was lost.

[4]In the treatises on classical mechanics this bewildering fact has been tacitly neglected. Incidentally such hypothesis is valid for a particle without *spin*.

2.3 The Universal Character of the Complex Action

Equations (18) are well known results obtained from the treatment of the harmonic resonator by the methods of abstract quantum mechanics [3, 4] from which the definition of the annihilation and creation operators derive. However, the derivation of these equations in Section 2.2 makes no reference to the laws of forces involved in the motion considered, a fact that endow them with a universal character.

This character was somehow expected, for it derives from a universal law established in the theory of thermal radiation. *Planck*, regarding "*the search for the absolute as the loftiest goal of all scientific activity*" [5], was guided in his inquiry on black body radiation by *Kirchhoff*'s law of thermal radiation which says that ...

> *... in an evacuated cavity, bounded by totally reflecting walls, and containing any arbitrary number of emitting and absorbing bodies, in time a state will be reached where all bodies have the same temperature, and the radiation, in all its properties including its spectral energy distribution, depends not on the* nature of the bodies, *but solely and exclusively on the temperature [6].*

Since the spectral distribution "*depends not on the nature of the bodies*", *Planck* was justified to arbitrarily assume "*the cavity to be filled with simple linear oscillators or resonators*", for they are the simplest conceivable physical systems endowed with the faculties required to model a material substance for that purpose.

In modelling the gas it is therefore justified to merge *Bernoulli*'s with *Kirchhoff*'s pictures to form a complete image that contemplates its constitution as both material and radiant.

2.4 Vestiges in Hamilton's Equations

Hamilton's equations,

$$\frac{dq}{dt} = \left(\frac{\partial H}{\partial p}\right)_q, \qquad \frac{dp}{dt} = -\left(\frac{\partial H}{\partial q}\right)_p. \tag{20}$$

have been tacitly treated in classical mechanics as "coupled". In quantum phenomena they must be treated, instead, as independent. Equations (20), rewritten in the form of finite products, reproduce the action representation of *Newton*'s second law (4),

$$\begin{aligned} (\Delta H)_q\, \Delta t &= \quad \Delta q \Delta p, \\ (\Delta H)_p\, \Delta t &= -\Delta p \Delta q, \end{aligned} \tag{21}$$

which can be straightforwardly connected to the uncertainty relations.

Hypothesis §7 allows recognize in equations (21) the manifestation of two *different* and *independent* processes[5] by whose means the energy of the particle can change in the time interval Δt during which an state transition takes place.

Summary. The amendments here added to Newtonian mechanics do not characterize a radical departure from its foundations, for the representation (4) of *Newton*'s second law does not rule the uncertainty relations out of it. In fact, the original formulation of that law requires neither the existence of any convergence process nor that the *laws of force* be expressed in terms of differentiable functions.

By revealing the existence in the particle of an angular momentum degree of freedom (spin), the theory thus amended explains both the non commutativity of the cancellation law in (14)

[5]The opposing processes hypothesis was emphasised in the notation adopted in equation (21), where the substripts q and p were imported from the partial derivatives in (20).

and the uncoupling of *Hamilton*'s equations, thus requiring a revision both of its algebra and the conditions under which calculus can be applied. However, thus circunscribed to classical mechanics, that theory is still unable to provide the missing equation that completes the system of equations (6) to describe the motion of the particle under the constraints of the uncertainty relations. The new amendmends required to explain how Newtonian chargeless particles can interact with its environment are introduced and discussed in the next Section.

3 The Relativistic Nature of Heat

The identification of the nature of the environment whose existence was previewed in §3 is essential for the foundation of any theory which acknowledges the uncertainty relations. Every attempt to describe the energy delivered or acquired by a particle during the action of *uncertainty forces* in terms of the classical forms of kinetic or potential energy is, as previewed in corollary §7, condemned to failure. As a form of energy by itself, heat entered physics through thermodynamics. Empirical evidence has shown that material substances are endowed with the faculty to absorb and emit heat in the form of radiation whose law of exchanges [7] was established theoretically by *Prévost*. That such thermal radiation is a mere instance of electromagnetic radiation was previewed theoretically by *Maxwell* and verified by *Hertz*. Such facts about heat cannot be properly described by Newtonian mechanics, even if we consider the amendments hitherto introduced to acknowledge the uncertainty relations.

3.1 The Physical Nature of the Environment

To help us in the endeavour to unveil the nature of the environment with which the particles of a gas interact, let us concentrate on the equality in (6),

$$\Delta\mathcal{E}\Delta t = \Delta p\Delta x = h, \tag{22}$$

which, as will be shown later, establishes the limit between elastic and inelastic collisions. Let us then search for the subsidiary laws needed to form with (22) a determinate system of non-differential equations of motion.

Equations (22) have been interpreted as describing the trading of quanta between matter and radiation. As shown in Sections 2.2 and 2.4, they can be understood as a mathematical possibility raised by hypothesis §7. Besides, it was shown that the spin — a mathematical consequence of acknowledging the complex algebraic structure of the action change (15) during a collision — is a property of a particle that endows it with the susceptibility to absorb and emit angular momentum by exchanging it with an unspecified environment.

The laws required to complete the system of equations that describe an uncertainty reaction came to light in the beginning of the synthesis of quantum mechanics through an unorthodox combination of the theory of heat radiation, special relativity and quantum mechanics itself[6], as a bricolage of the discovery of the quantum nature of the black body radiation by *Planck*, the explanation of the photo-electric effect and the disclosure of its corpuscular propagation by *Einstein* and the first two postulates of *Bohr*.

These findings, together with the relativistic relationship between the energy and the momentum of the (then hypothetical) massless particle, the photon,

$$\Delta\mathcal{E} = c\Delta p, \tag{23}$$

[6]The elementary energy conversion processes turned out to be more appropriately treated as a subject of quantum-electrodynamics.

and combine with the *Einstein–Bohr* relation,

$$\Delta\mathcal{E} = \frac{hc}{\lambda}, \tag{24}$$

where λ is the wavelength of the photon whose theoretical origin stems from *Maxwell*'s electromagentic theory, completes the system of equations that describe an uncertainty reaction. The momentum of the photon, expressed in terms of *Planck*'s constant, can then be obtained,

$$\Delta p = \frac{h}{\lambda}. \tag{25}$$

System (22–24) describes, in terms of wave length of the photon[7], an uncertainty reaction (an inelastic collision) in which a quantum is exchanged between a particle and its environment. The nature of the latter is thereby revealed to be electromagnetic and the occurrence of *Planck*'s constant, introduced by the *Einstein–Bohr* relation (24) in these expressions, establishes the coupling between electrodymaics and the representation of Newtonian mechanics amended by the acknowledgment of the uncertainty relations.

3.2 Uncertainty and Collision

The formal connection between collision phenomena and the system of equations (22–24) can then be summarized in the following postulate, which completes the notion of an uncertainty reaction defined in §6:

§ 11 (Interaction matter-radiation)
The uncertainty relations define the domain in which a collision between two particles is elastic. When the motion of two colliding particles are in the way to leave this domain, they are compelled to interact with the electromagnetic field by exchanging a photon with it.

From §11 we derive the following definition of a collision,

§ 12 *A collision is the physical phenomenon whose system of non-differential equations of motion is given by (22–24).*

In classical mechanics the cause of change of the state of motion of a body is identified in *Newton*'s first law with the *action of a force*, whose effect is given by the law of force to be grafted in the second law. Likewise, the causes of change of the motion of a particle under the constraints of the uncertainty relations are, according to §6, the *uncertainty forces* that arise during its collision with another particle. Its consequences might be, besides some unpredictable side-effects, either a mere elastic collision or the absorption or emission of a unit of electromagnetic quantum by a particle[8].

3.3 Quantum Theory and Thermodynamics

According to the current tenets, heat is transferred from a body to another by a flux of photons which is known as radiation. The other known forms of heat transfer, *viz.*, convection and conduction, are supposed to be reducible to the exchange of photons between matter and radiation. This flux is explained in terms of the processes of *absorption* and *emission* of radiation by material particles, which leads to the following important epistemological conclusion:

[7]The introduction of the wave length of the photon can be treated as the specification of an initial condition of a collision.

[8]The quantum effects of a collision depend on each situation and can be interpreted either as opposed or superposed to the classical effects.

§ 13 *Quantum Mechanics and Thermodynamics are complementary theories in relation to heat: the former provides a first-principles revelation of the existence (and a theoretical description) of* elementary heat conversion processes*, which the latter* acknowledges *as a fact of experience.*

In summary, after "*smuggling*" a relativistic corpuscle into non-relativistic quantum mechanics, it can be said that ...

§ 14 *... electromagnetic radiation is an inescapable environment of every thermodynamic system*[9]*.*

Since equations (22–25) are *necessary* to completely characterize the inelastic collisions, a strictly non-relativistic quantum theory is impossible.

3.4 Uncertainty Forces and Newton's First Law

It is remarkable that once the quantum mechanical meaning of "force" is acknowledged, we can recognise, somehow embodied in the statement of *Newton*'s first law, the first postulate of *Bohr*:

§ 15 (Bohr's 1st postulate) *Every body (*particle*) continues in its state of rest* (occupying one of a set of stationary states)*, or of uniform motion in a right line, <u>unless</u> it is compelled to change that state by* (uncertainty*) forces impressed upon it.*

A paradigmatic illustration of the behaviour of a system about to violate the uncertainty principle is given by the following conceivable synthesis of the hydrogen atom:

> *In the collision of an electron with a proton caused by Coulombian attraction forces, an* uncertainty reaction *that impedes the violation of the uncertainty principle takes place, giving rise to the exchange of energy and momenta (both linear and angular) between the colliding particles and the electromagnetic field. The Coulomb law is therefore* superseded *by the* Schrödinger *equation of a particle on a Coulombian field that describes the set of the possible eigen-states of the Hydrogen atom.*

§ 16 *The work performed by an attractive Coulomb force from the infinity to the* uncertainty critical distance λ *between the particles is known to be given by the integral,*

$$W = \int_{-\infty}^{\lambda} -\frac{e^2}{r^2} dr = \frac{e^2}{\lambda}. \tag{26}$$

Assuming that the uncertainty relations establishes a critical interparticle distance λ*, below which an uncertainty reaction occurs, we obtain the energy* e^2/λ *of the photon emitted by the colliding particles during that reaction.*

§16 provides an elementary interpretation of the fine structure constant.

4 Action, Energy and Momenta

In his formulation of wave mechanics *Schrödinger* defined the wave function ψ in terms of the action developed by a particle in motion, according to the complex function,

$$\psi = a e^{\mathsf{A}/\hbar}, \tag{27}$$

[9]It is surprising that most treatises on Statistical Mechanics make no reference to the essential role played by radiation in the formation of thermodynamic equilibrium.

which, due to the multi-valuedness of the complex logarithm function[10], redefines the action in terms of the additional integer number ℓ,

$$\mathsf{A} = \hbar \left(\ln \psi \pm 2\pi\imath\ell \right) . \tag{28}$$

Equation (28) can then be interpreted as a revelation of the existence in a particle of a new degree of freedom[11]:

§ 17 (The elementary thermo-mechanical particle)
The action of the elementary quantum particle is an extension of its classical action to which one imaginary discrete component, multiple of the quantum unit of action h, is added.

Definition §17 can be judged either by its consequences or by the following reasons given by the current tenets of quantum theory.

First, let us recall that while in every elastic collision the total energy and momentum of the system of colliding particles are conserved, this is not true for the energy and momentum of each colliding particle. According to quantum mechanics, a particle can undergo no change of energy *unless* it exchanges (at least) one quantum with radiation, according to the formula ($\Delta E = h\nu$). Hence, elastic collisions can only be conceived when the changes underwent by the particle are *classical*; that is, outside the domain interdicted by the uncertainty relations.

Second, definition §17 is in conformance with the second quantization formalism where a particle is tacitly assumed to occupy a *single* quantum state, instead of conceived as decomposed in three independent *de Broglie*'s degrees of freedom (three quantum numbers). The particle thus defined combines continuous and discrete degrees of freedom, which suggests a reinterpretation of *Bohr*'s correspondence principle, for the classical and the quantum behaviours of the particles coexist in its motion, the former expressed as the real components of the momentum and the latter as the imaginary quantity ℓ. It is such coexistence that imparts to the particle the bizarre behaviour which is more or less corpuscular depending on the value of the value of the argument of the action (28) in the complex plane, which turns out to be a mere rephrasal of *Schrödinger*'s granularity criterion. It is relevant to remark at this point that, according to definition §17, the *quantum number* describes the quantum state of the particle moving in a gas, but gives no information about its dynamic (classical) state.

Statement §17 defines a *thermo-mechanical* elementary particle, which can be regarded as a Newtonian particle moving under the constraints imposed by the uncertainty principle. The present approach can be interpreted as the study of the consequences of the quantum laws to thermodynamic phenomena. Unlike atomic theory, which can be largely considered as the theory of the motion of the electron in a Coulombian field, the present approach makes no mention to electric charge. The following Sections discuss the consequences of adopting §17 as the *definition* of the *corpuscles* of the gas. We use the word "corpuscle", in opposition to "molecule", to emphasize that, to explain the thermodynamic behaviour of gases, the description of its internal structure is intentionally left outside the scope of the present theory.

4.1 The Action Change During an Uncertainty Reaction

In classical mechanics the *action* of a particle provides a complete description of its motion, for its energy, linear and angular momenta can be derived from it by means of differential operations of calculus. The linear momentum of a particle is given by the spatial derivative of

[10] Recall that definition (15) imposes a complex structure to the action function.
[11] To an elementary particle there are assigned four degrees of freedom, three of them are classic, corresponding to the linear momentum, and a single angular momentum quantum number ℓ.

the action,

$$p = \frac{d\mathsf{A}}{d\mathbf{r}} = \left\{ \frac{\partial \mathsf{A}}{\partial x}, \frac{\partial \mathsf{A}}{\partial y}, \frac{\partial \mathsf{A}}{\partial z} \right\} = \nabla \mathsf{A}.$$

If the operator ∇ is applied to the action defined by (28), the momentum turns out to be independent of the quantum degree of freedom ℓ, thus concealing every subsequent quantum effects. As already discussed in §4, since the limit operation is ruled out by the uncertainty principle, we replace the derivative by the finite product,

$$\frac{d\mathsf{A}}{dz} \Rightarrow |\Delta \mathsf{A}| = |p \cdot \Delta z|. \tag{29}$$

The results obtained in the previous Sections recommend to adopt equations (15) as the *definition* of the complex variation $\Delta \mathsf{A}$ of the action.

Quantum mechanics assigns physical meaning to $\Delta \mathsf{A}$ and $\Delta \mathsf{A}^*$ either directly from (28) or from one the following correspondences:

§ 18 (Angular momentum)
The angular momentum operator $\hat{\ell}_r$ is defined according to the correspondence[12],

$$(p \times \Delta z) \sim \pm \hat{\ell}_r. \tag{30}$$

§ 19 (Creation and annihilation of photons)
The quantities $p^* \Delta z$ and $p \Delta z^*$ correspond to the annihilation (a) and creation (a^+) operators of abstract quantum mechanics[13],

$$\begin{array}{ll} a & \sim p^* \Delta z, \\ a^+ & \sim p \Delta z^*. \end{array} \tag{31}$$

Let us denote the action of a moving particle by $\mathsf{A} = \mathsf{A}_{\mathcal{C}} + \imath \mathsf{A}_{\mathcal{Q}}$. According to (18), both §18 and §19 lead to the imaginary part of (28),

$$\Delta \mathsf{A}_{\mathcal{Q}} = \pm 2\pi\hbar \Delta \ell = \pm 2\pi\hbar. \tag{32}$$

Whatever the correspondence we adopt, the products (18) represent the variation of the action due to the exchange of a quantum between the particle and the environment[14]. The quantity $(p \cdot \Delta z)$ represents the classical action change due to the work of an uncertainty force, either on the environment by the particle, or *vice-versa*. The operations $(p \cdot \Delta z)$ and $(p \times \Delta z)$ stand for two separate effects, respectively the change of the linear momentum of the particle and the change of its intrinsic angular momentum.

Using (32) we can write the unitary variations of the action during a collision,

$$\Delta \mathsf{A} = (p \cdot \Delta z) \pm 2\pi\imath\hbar. \tag{33}$$

While equations (18) give the exact variation (32) of $\Delta \mathsf{A}_{\mathcal{Q}}$, they give no clue about the distribution of the total momentum and energy of the system *particle+photon* among the products of the interaction of the particle with the environment.

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[12]Once the collision is acknowledged as a central field phenomenon, the discrete nature of the angular momentum degree of freedom arises naturally in the solutions of *Schrödinger* equation (see, for instance, §§26, 33 and 122 in [8]).

[13]See, for instance, §41 in [3].

[14]It will shown later that such environment is the electromagnetic field.

4.2 Reinterpreting Uncertainty Relations

Substituting any of the equations (28), (30) or (31) in (19), we have[15]:

$$\Delta p \Delta q \sin \varphi = \hbar, \tag{34}$$

whose value is constrained to vary in the interval [0–1], according to the following inequalities,

$$0 \leq \sin \varphi = \frac{h}{\Delta p \Delta q} \leq 1, \tag{35}$$

The real part of the variation of the action of the moving particle corresponds to its mechanical properties (momentum and kinetic energy) and the imaginary part, to its thermal property. Hence, relations (35) describe three different situations, depending on the value of φ:

§ 20 *When* $0 < \varphi < 2\pi$ *there is neither the emission nor the absorption of a photon by the particle.*

In this case, the collision is elastic, where no uncertainty reaction occurs.

§ 21 *The conditions* $\varphi \to 0$ *and* $\varphi \to 2\pi$ *describe the imminence of a violation of the uncertainty principle by the colliding particles. As opposed to a convergence process of analysis which leads to a limit value, this situation triggers a discontinuous process (an* uncertainty reaction*), whose outcomes are either the emission or the absorption of a photon by the particle, thus conforming the values of the degrees of freedom of the particle to that principle.*

For the two latter cases there is an interesting mathematical interpretation: an uncertainty reaction can be regarded as a transition from the current sheet to its predecessor (successor) in the *Riemann* analytical continuation of the complex action, where the sheet index is the quantum number of the particle.

In summary,

§ 22 *The uncertainty relations (6) define the domain in which a collision between two particles is elastic. When the motion of the colliding particles does not satisfy that inequality, an uncertainty reaction occurs, triggering an elementary energy conversion process during which a unit of electromagnetic quantum is exchanged between the system formed by the colliding particles and radiation.*

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