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Higgs Field, Higgs Mechanism and the Boson of Higgs

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Higgs Field, Higgs Mechanism and the Boson of Higgs

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Abstract. We present to graduate and posgraduate students of physics a brief and didactic analysis about the Higgs field, Higgs mechanism and the boson of Higgs.

I) Introduction.

This paper was written to graduate and postgraduate students of physics to analyze the theme “boson of Higgs”[1] to-day very popular, appearing in newspapers, magazines and television. This boson, according to the current theory of elementary particles (Standard Model[2]) is the single particle that was not yet observed. It was predicted by Peter Higgs[(*)] in 1964 but up to 2008 there were no technological conditions to verify its existence when the LHC (“Large Hadron Collider”) begin to operate at the CERN. The maximum value for the boson of Higgs mass is expected[1] to be ≈126 GeV. In fact, the uncharged particle named boson of Higgs, with spin zero, is the quantum of a scalar field ϕ defined as “Higgs field” (remember that the photon is the quantum of the electromagnetic field). It is assumed that the Higgs field is analogous to an old-fashioned “aether” which pervades all space-time. It acts like a continuous background medium even at short distances. It would play a fundamental role since it seems to represent the key to explain the origin of the mass of other elementary particles. As will be shown in what follows the interaction of ϕ with other fields is able to give mass to elementary particles. Let us call generically the mechanism of mass creation by “Higgs mechanism” (this name was given originally to explain the mass creation of gauge bosons in the electro-weak interaction). In order to understand the Higgs mechanism it is first necessary to understand the meaning of “symmetries”, “spontaneous symmetry breaking” and “gauge theory”. Note that our intention is to present only a brief analysis of these topics, as rigorously as possible, since they are mathematically very complex and vast. Only a few

(*) The boson of Higgs prediction is usually attributed to Higgs, however, it was also done, e.g., by F.Englert, R.Brout, G.S.Guralnik, C.R.Hagen and T.Kibble.

references, articles and books will be cited. Probably the main contribution of this paper will be to indicate a way that can be followed by the students to better understand the subjects. In Section 1 we make some comments about Symmetries in Physics. In Section 2 we analyze the Spontaneous Symmetry Breaking (SSB) in Physics. In Section 3 is presented the fundamental aspects of the Theory of Gauge which is today accepted as an amazing approach that would be able to explain the internal structures of all elementary particles. In Section 4 we present the electromagnetic gauge theory. In Section 5 we discuss the mass problem of the gauge field quantum. In Section 6 we consider the Higgs particle in external electromagnetic field and submitted to a very peculiar self-interaction potential described by a Lagrangian with a U(1) gauge symmetry. We show that the Higgs mechanism is generated by a spontaneous symmetry breaking of the U(1) gauge invariance giving origin to two *massive gauge bosons*: one vetorial gauge boson and one scalar gauge boson named "boson of Higgs". In Section 7 we briefly comment the unified electroweak gauge model and the the Higgs mechanism proposed by Weinberg-Salam that explains the masses of the vectorial bosons $W^{\pm}$ and Z^{o}. Finally, in Section 8 we analyze the Goldstone theorem and the massless Goldstone bosons.

1) Symmetries in Physics.

A review of symmetries in physics can be found in Ref.[3] where are shown many current references, articles and books. In physics, *symmetry* includes all features of a physical system that exhibit the property of *symmetry*, that is, under certain *transformations*; aspects of these systems are "unchanged", according to a particular observation. Symmetry of a physical system is a physical or mathematical feature of the system (observed or intrinsic) that is "preserved" under some change or transformation.[3] Transformations may be continuous or discrete. Symmetries may be broadly classified as *global* or *local*. A *global symmetry* is one that holds at all points of spacetime, whereas a *local symmetry* is one that has a different symmetry transformation at different points of spacetime. Let us see some examples of symmetries seen in basic courses.[4,5] Newton's second law $\mathbf{F} = md^2\mathbf{r}/dt^2$ is invariant by time-reversal $t \to -t$. A spherical system, like an atom in the ground state, is invariant by a rotation around any fixed axis. An isosceles triangle parallel to a plane mirror is invariant by a mirror reflection $\mathbf{r} \to -\mathbf{r}$. The orbit of particle describing a helix around a z-axis has a cylindrical symmetry around the z-axis. An electric field due to a infinite straight wire exhibit cylindrical symmetry because the electric field strength at a given distance *r* from the electrically charged wire has the same magnitude at each point on the surface of a cylinder (whose axis is the wire) with radius *r*.

According to the special relativity[4,5] the physical laws are invariant under a Lorentz transformation between two inertial systems S and S´. The transformation depends only on the relative velocity between S and S´ not on their positions in space-time. They can be infinitesimally close together or at opposite ends of the universe; the Lorentz transformation is still the same. Thus Lorentz transformation defines a "global symmetry". Maxwell equations are invariant[6-8] by a global Lorentz transformation and by a "gauge transformation". In Section 3 we show that the electromagnetic field is invariant by a local U(1) gauge transformation and that, due to this invariance the electromagnetic forces must have an infinite range since the quantum of the field, the photons, are massless particles.

In general relativity[7] the description of relative motion is much more complicated because the references frames are in a gravitational field. The essential difference between special and general relativity is that the inertial reference frame can only be defined "locally" at a single point in the gravitational field. How the individual measurements in S and S´ are related to each other? Clearly, one cannot perform ordinary Lorentz transformations between S and S´. Einstein solved this problem of relating nearby free falling frames by defining a new mathematical relation known as a "connection" in a Riemannian spacetime. With this "connection" a physical quantity A_μ in S can be related to A'_ν in S´ using the Christoffel symbols $\Gamma^\mu{}_{\nu\lambda}$ that are components of the "connection". This procedure is a "local gauge" transformation as will be seen in Section 3.

Continuous symmetries and discrete symmetries are frequently amenable to mathematical formulation such as *group representations* and can exploited to simplify many problems.[8,9] *Continuous symmetries* that are described by continuous or smooth functions are characterized by invariance following a continuous change in the geometry of the system.

In this paper we will analyze essentially the Lorentz and the "gauge" transformations. The Lorentz transformations are represented by the "Lorentz symmetry group of special relativity" and the gauge transformations are represented by the "gauge symmetry group".[8,9]

Finally, the symmetry properties of a physical system are intimately related to the conservation laws characterizing the system. For continuous symmetry of a physical system a precise description of this relation is given by the Noether's theorem.[10] This theorem states that each symmetry of a physical system implies that some physical property of that system is conserved. Conversely**,** each conserved quantity has a corresponding symmetry.

2) Spontaneous Symmetry Breaking.

To understand the Higgs mechanism, that is, how (gauge) bosons acquire mass it is essential to understand the meaning of *Spontaneous Symmetry Breaking* (SSP). We verify in the literature [8,11] that there are at least two *slightly different* processes of SSB. The first one occurs in the Heisenberg ferromagnet[12] which is formed by spins **S** at Bravais lattice points obeying the Hamiltonian $H = -(J/2)\Sigma_{ij}\, \mathbf{S}_i\cdot\mathbf{S}_j - g\, \mu_B\, \Sigma_j\, \mathbf{B}\cdot\mathbf{S}_j$ where J is the exchange energy, g the Lande´s g-factor, μ_B the Bohr magneton and **B** the internal magnetic field which includes the external field plus any “molecular” field. The Hamiltonian H is invariant by rotations and the excited states $|\,\Psi >$ of H are also invariant by rotations. However, in the ground state $|\,0 >$ of the system the spins becomes aligned along a given direction. The spherical symmetry is broken and as the alignment direction is arbitrary the “vacuum” $|\,0 >$ is infinitely degenerate. To study the magnetic system properties we need to choose one of these vacuums and so the symmetry is broken. This is an example of “spontaneous symmetry breaking” (SSB) that is defined as follows:

Spontaneous symmetry breaking occurs when the Hamiltonian (or Lagrangian) is invariant by a symmetry group and the vacuum $|\,0 >$ *is not invariant by this symmetry group.*

The second kind of SSB is observed in the Meissner Effect[8, 13,14] which occurs when a superconductor is submitted to an external magnetic field. To understand this kind of SSB we remark that the symmetries of a system are defined by the Hamiltonian symmetries, consequently, its solutions must also have the same symmetries. If an actual measurement shows one solution with symmetry different from the predicted ones we say that there is a SSB defined as follows:[8]

“Spontaneous symmetry breaking is a spontaneous process by which a system in a symmetrical state ends up in an asymmetrical state”.

To exemplify this last SSB let us consider the superconductor which was explained by Bardeen et al.[15] using the quantum field theory.[16] However, the superconductivity can be described in a more simple way using only electromagnetism and a quantum mechanical nonrelativistic model.[8,13] The coherent electronic current (or “charged condensate”) in the superconductor, formed by Cooper pairs, behaves like a single free quantum-mechanical particle. The coherent system of Cooper pair electrons can be described by the Schrödinger wavefunction [8]

$$\Psi(\mathbf{x},t) =(N/2)^{1/2} \exp[-2ie\varphi(\mathbf{x}.t)/\hbar c] \qquad (2.1),$$

where (N/2) is the density of Cooper pairs assumed constant and φ is the 4-component of the vector potential $A_\mu = (\mathbf{A}, i\varphi)$. The electronic current density **J** is given by[16]

$$\mathbf{J} = (ie\hbar/2m)\,(\Psi^*\,\mathrm{grad}\Psi - \Psi\,\mathrm{grad}\Psi^*) - (2e^2/mc)\,|\Psi|^2\,\mathbf{A} \qquad (2.2).$$

Substituting (2.1) into (2.2) gives

$$\mathbf{J} = (Ne^2/mc)\,(\mathrm{grad}\varphi - \mathbf{A}) \qquad (2.3)$$

Since, due to charge conservation $\partial\rho/\partial t = \mathrm{div}\mathbf{J} = 0$ we see from (2.3) that we must have $\mathrm{div}(\mathrm{grad}\varphi) = \nabla^2(\varphi) = \mathrm{div}\mathbf{A}$. Adopting the Coulomb gauge[6] that is, putting $\mathrm{div}\mathbf{A} = 0$ we see that $\nabla^2(\varphi) = 0$. We permit us to assume that gradφ = constant. Choosing gradφ = 0 we get from (2.3)

$$\mathbf{J} = -(Ne^2/mc)\,\mathbf{A} \qquad (2.4),$$

which is called *London equation*.[13] Assuming that the superconductor is submitted to a magnetic field **B**, using Maxwell's equation $\mathrm{rot}(\mathbf{B}) = (4\pi/c)\,\mathbf{J}$ and taking into account that $\mathrm{rot}(\mathrm{rot}(\mathbf{B})) = \mathrm{grad}(\mathrm{div}(\mathbf{B})) - \nabla^2(\mathbf{B})$, $\mathrm{div}(\mathbf{B}) = 0$ and that $\mathbf{B} = \mathrm{rot}\mathbf{A}$ we get the following relation:[8]

$$\nabla^2\mathbf{B} = (4\pi Ne^2/mc^2)\,\mathbf{B} \qquad (2.5).$$

Solving (2.5) we verify[8,13] that the magnetic field decreases exponentially with a penetration depth $\lambda = (mc^2/4\pi Ne^2)^{1/2}$ called "London penetration depth". The value of λ is typically 30-50 nm. The interaction of an external magnetic field with the condensate results in the "Meissner Effect". That is, when the static magnetic field penetrates into the superconductor it induces a flow of Cooper pair current. This induced current generates its own magnetic field which cancels internally, not completely, the external field: the external field penetrates only a small distance into the superconductor. Thus, inside the superconductor the electromagnetic forces somehow become short-range breaking the U(1) gauge invariance of the electromagnetism. The existence of the superconductor phase and its interaction with the magnetic field was assured by an Hamiltonian and by Maxwell equations that are manifestly U(1) gauge invariant. In conclusion: the U(1) symmetry of initial state is violated in the final state.[(1,1a)] This occurs due to the Cooper pair current: it is a *self-coherent system* which is necessary to break the gauge symmetry.

Finally, let us show a case of "simple symmetry breaking", that is, which is not a SSB. Let us consider an isolated hydrogen atom described by the Hamiltonian H_o. In the ground state the atom has a spherical symmetry. If the atom is submitted to an external magnetic field **B** the Hamiltonian now would be given by $H = H_o - \boldsymbol{\mu} \cdot \mathbf{B}$, where **μ** is the magnetic moment of the atom. In this condition the spherical symmetry is broken by the magnetic field **B** which selects out a particular directional in space. The atomic symmetry becomes only cylindrical around the **B** direction. Of course, we do not have SSB because H_o and H have completely different symmetries.

3) Gauge Theory

Brief review articles about the gauge theory can be seen elsewhere.[8,17,18] The earliest field theory having a gauge symmetry was Maxwell's formulation of electrodynamics in 1864. In that context[6,7] the gauge transformations, were given by,

$$\mathbf{A}' \rightarrow \mathbf{A} + \mathrm{grad}\,\lambda \quad \text{and} \quad \varphi' \rightarrow \varphi - (1/c)(\partial\lambda/\partial t) \qquad (3.1),$$

where $A_\mu = (\mathbf{A}, i\varphi)$ is the 4-vector potential and $\lambda = \lambda(\mathbf{x}, t)$ is an arbitrary functions of the space **x** and time t. They were interpreted only as a useful mathematical device for simplifying many calculations in electrodynamics. The importance of this symmetry remained unnoticed in earlier similar formulations. Later in 1919 Hermann Weyl, in attempt to unify general relativity and electromagnetism, conjectured incorrectly an invariance under which a change of scale or “gauge” might to be a local symmetry of general relativity.[8] After the development of quantum mechanics the “old gauge theory” was modified by replacing the *scale factor* by a complex quantity and turned the scale transformation into a change of phase (a U(1) local gauge symmetry, as will be shown below). This explained the effect of the electromagnetic field on the wave function of a charged quantum mechanical particle. The success of this *electromagnetic gauge theory* was recognized and divulgated by Wolfgang Pauli in the 1940s.

In 1954, Chen Ning Yang and Robert Mills proposed that the strong nuclear interaction should be described by a field theory with local gauge invariance analogous the electromagnetism. Generalizing the gauge invariance of electromagnetism, they attempted to construct a theory based on the action of the (non-Abelian) SU(2) isotopic-spin group. Although the Yang-Mills theory failed to explain the strong interaction, it established the foundations for modern gauge theory. They showed for the first time that local gauge symmetry was a powerful fundamental principle that could provide a new insight to investigate the old idea of existence of internal degrees of freedom of the elementary particles. As will be shown below the Yang-Mills theory unifies these internal degrees of freedom in a non-trivial way with the dynamical motion in space-time, Yang and Mills discovered a new type of geometry in physics (the "fiber bundle spaces").[8,9,17,18]

The gauge theory found a successful application in the quantum field theory of the weak force resulting in its unification with electromagnetism in the electroweak interaction. Gauge theory became as important as the successful field theories explaining the dynamics of elementary particles. Historically, these ideas appeared first stated in the context of classical electromagnetism and later in general relativity. However, the modern importance of gauge symmetries appeared first in the relativistic quantum mechanics of electrons and quantum electrodynamics. Today, gauge theories are useful in condensed matter, nuclear and high energy physics among other subfields.[18]

"Gauge theory" is a type of field theory[17–20] in which the Lagrangian is invariant under a continuous group of local transformations. It is assumed[8,9,19] that any particle or system which is localized in a small volume and that carries internal quantum numbers (like isotopic spin) is considered to have a direction in the internal symmetry space. This internal direction can be arbitrarily chosen at each point in space-time. The internal symmetry space is described, in the general case, by a non-Abelian symmetry group G of dimension N. When a particle described by a wavefunction $\Psi(x)$ moves in a space-time submitted to an external field ("gauge field") their internal states rotates by local angles $\theta^k(x)$. The parameters $\theta^k(x)$ or "rotation angles" are continuous functions of x and represent the internal degrees of freedom of the particle. The dependence of $\theta^k(x)$ with x permits the "connection" of the internal degrees of freedom with the external field in different points of the space-time. In a first approximation the wavefunction $\Psi(x)$ can be written as [8,9]

$\Psi(x) = \sum_\alpha \psi_\alpha(x)u_\alpha$ where u_α form a set of a "basis vectors" in the internal space and $\psi_\alpha(x)$ is then a component of $\Psi(x)$ in the basis u_α. It can be shown[8,9] that due to the gauge field the state $\Psi(x)$ is modified according to a local symmetry transformation $U(x)\Psi(x)$ being $U(x)$ an unitary operator given by

$$U(x)=\exp[-ig(\sum_k \theta^k(x)F_k)] \qquad (3.2),$$

where g is the particle "charge" or a general "coupling constant", F_k the generators of the non-Abelian internal symmetry group G with dimension N which satisfy the usual comutation relations $[F_i, F_j]= ic_{ijk}F_k$; the constants c_{ijk} depend on the particular G group and $k =1,2,\ldots,N$. The G groups are named "gauge groups". We verify that for each force field or "gauge field" is associated a given gauge group.

At this point it is important to remember that in the framework of the gauge theory the ordinary derivative operation ∂_μ is generalized[8,9] and is given by D_μ, named "gauge covariant derivative":

$$D_\mu \psi_\beta = \Sigma_\alpha \left[\delta_{\beta\alpha} \partial_\mu - iq(A_\mu)_{\beta\alpha} \right] \psi_\alpha \qquad (3.3),$$

where A_μ is an external ("gauge") field also named "gauge potencial" and $(A_\mu)_{\beta\alpha}$ is the "connection operador" defined by

$$(A_\mu)_{\beta\alpha} = \Sigma_k (\partial_\mu \theta^k)(F_k)_{\beta\alpha} \qquad (3.4).$$

Note that the (3.3) gauge covariante derivative D_μ describes the changes in both the external and internal parts of $\Psi(x)$. The "gauge field" or "gauge potential" A_μ transforms in a non-covariant manner,

$$A_\mu{}' = UA_\mu U^{-1} - (i/q)(\partial_\mu U)\, U^{-1} \qquad (3.5).$$

So, according to the modern gauge theory all physical systems must obey a "local internal gauge symmetry" whose properties have been defined above. This implies that the Lagrangian of the systems must be invariant by a local symmetry gauge transformation. A general feature of the Lagrangian which is familiar from classical physics is that it must contain terms which describe the difference between the kinetic and potential energies of the system. This leads to the usual equations of motion. The new requirement is that the energy terms must be invariant under local non-Abelian gauge transformations. Local gauge invariance is not a simple constraint to impose on a Lagrangian. Complication arises from the different behavior of particle and gauge fields under gauge transformations.

When such a theory is quantized, the quanta of the gauge fields are called "gauge bosons" that in the gauge theory context must have mass zero, as will be discussed in Section 5.

As commented above, gauge theories are important as the successful field theories explaining the dynamics of elementary particles.[8,18–20] Quantum electrodynamics is an *Abelian gauge theory* with the symmetry group U(1) (see Section (3.1)) and has one gauge field, the electromagnetic vector potential A_μ, with the *photon* being the "gauge boson". The electroweak interaction, which is the result of the unification of the electromagnetic and weak interactions, is described by a *non-Abelian* SU(2)xU(1) group. The massless bosons ("gauge bosons") of this unified theory mix after *spontaneous symmetry breaking* (SSB) to produce 3 massive weak bosons (W^+, W^- and Z^o) and the photon. The strong interaction which is described by the "quantum chromodynamics" (QCD) is an SU(3) *non-Abelian* gauge theory where the gauge bosons are named *gluons*. The standard model[2] *combines* the unified electroweak interaction with the strong interaction through the SU(2)xU(1)xSU(3) symmetry group. Nowdays the strong interaction is not unified with electroweak interaction, but from the observed running of the coupling constants it is believed they all converge to a single value at very high energies.[18–20]

4) The Electromagnetic Gauge Theory.

In the gauge formalism the Lagrangian density *L* for the familiar interaction between the electron field ψ and the electromagnetic potential A_μ can be written as[8]

$$L = i\,\bar{\psi}\gamma^\mu D_\mu\psi - (1/4)F^{\mu\nu}F_{\mu\nu} - m\,\bar{\psi}\psi \qquad (4.1).$$

The first term of (4.1) involving the covariant derivative D_μ gives the kinetic energy of the electron. The second term is the familiar form for the energy density contained in the electromagnetic field[6-8] and the last term gives the mass of the electron. It can be easily verified that each term of *L* is separately gauge invariant. The gauge field A_μ is the electromagnetic 4-vector potencial $A_\mu = (\mathbf{A}, i\varphi)$ and the photon is the "gauge boson".

Taking into account (3.1) and (3.5) we verify[8] that the operador U(x) can be written as U(x)= exp[iλ(x)]. Consequently, the gauge transformation of the electric field is given by ψ(x)´= exp[iλ(x)] ψ(x). This shows that the electromagnetism is an *Abelian* gauge theory described by U(1) group.

Applying the Euler-Lagrange equations to (3.1.1) yields the Dirac and Maxwell equations:

$$i\,\gamma^\mu D_\mu\psi = m\,\psi \qquad (4.2)$$

$$\partial_\mu F_{\mu\nu} = j_\nu \qquad (4.3)$$

$$j_\nu = q\,\bar{\psi}\gamma_\nu\psi \qquad (4.4),$$

where $D_\mu = \partial_\mu - iqA_\mu$ and $F_{\mu\nu} = \partial_\mu A^\nu - \partial^\nu A_\mu$. The Dirac equation (4.2) comes from the kinetic energy term and the electron mass term. The field energy term in *L* gives rise to the left side of the Maxwell equations (4.3) while the current j_ν (4.4) comes from the kinetic energy term. The Lagrangian (4.1) has all necessary terms to give the equations of motion and no more; it provides the most reasonable starting point for a general non-Abelian gauge theory. It is reasonable to expect that all classes of theoretical models which do not resemble electromagnetism would be excluded[8] even if they might satisfy the general properties for a gauge symmetry presented above. However, the only way to verify whether a proposed model is reasonable or not is by testing the theoretical predictions against experiments.

The interaction of the electron with the external field occurs through the gauge field A_μ. According to the Maxwell equations (4.3) the field A_μ in the vaccum obeys the equation

$$\partial_i\partial^j A_\mu = \nabla^2(A_\mu) - (1/c^2)\partial_t^{\,2}A_\mu = \Box A_\mu = 0 \qquad (4.5),$$

where $\Box$ is the Dalembertian operador. Eq.(4.5) shows that A_μ propagates with velocity c in the vacuum. In the framework of the quantum field theory[16,17,21] this implies that the quanta of the vector gauge field A_μ are massless particles. They are the famous bosons named photons. The interaction between the vector gauge field A_μ and the charges is mediated by bosons with spin 0.

(5) The Mass Problem of the Gauge Quantum Field.

As commented above, it is reasonable to expect that all classes of theoretical gauge models which do not resemble electromagnetism would be excluded.[8] However, only experiments will be able determine whether a proposed model is reasonable or not. Based on the Electromagnetic gauge theory Yang and Mills[8,18-20] proposed a general *non-Abelian gauge theory* replacing the fields in the Lagrangian (4.1) with new fields that carry the desired internal quantum numbers. This theory failed to describe the strong interaction because it could not explain, for instance, the short range of the nuclear force.

We have said in Section 3 that the quantum of the gauge field must be massless. It was shown by (4.5) that it is valid for the gauge electromagnetic theory. Since a rigorous demonstration of this statement for the general case is very difficult we will be satisfied only giving a reasonable explanation for this fact. If the Lagrangian of a general gauge theory must resemble *L* defined by (4.1) we see that similarly to the electron mass term $m\overline{\psi}\psi$ it is missing a mass term $M^2A_\mu A^\mu$ for the gauge field A_μ where M is the mass of the gauge field particle. However, this term is not gauge invariant due to the last term that arises from the gauge transformation defined by (3.5): $A_\mu' = UA_\mu U^{-1} - (i/q)(\partial_\mu U)U^{-1}$.This implies that M must be zero (see Appendix A).

The problem of the zero mass gauge field was solved around 1967 through the spontaneous symmetry breaking in the unified electroweak theory[8] proposed almost simultaneously by Weinberg[22] and Salam.[23] The idea of unifying the weak and electromagnetic interactions into a single gauge theory had been suggested much earlier by Schwinger[24] and Glashow.[25]

As commented in Section 3, the electroweak interaction, which is the result of the unification of the electromagnetic and weak interactions, is described by a *non-Abelian* gauge group SU(2)xU(1). In this unified theory the massless gauge bosons, after *spontaneous symmetry breaking*, are mixed producing three massive weak bosons (W^+, W^- and Z^o) and the photon (see Section 7). The spontaneous symmetry breaking of the gauge invariance responsible for the generation of the massive gauge bosons was named "Higgs mechanism".[8]

One of the most promising developments of the modern gauge symmetry is the "quantum chromodynamics" (QCD) created to explain the strong interactions. This theory is based on a new hypothetical quantum number called "color" which is carried by the quarks. The gauge symmetry of the QCD, also named "color gauge theory", is dictated by the non-Abelian SU(3) group.[8] The massless gauge bosons are named "gluons".

(6) The Spontaneous Symmetry Breaking and the Boson of Higgs.

Now we show how the spontaneous symmetry breaking of a system described by a U(1) local gauge invariant Lagrangian is responsible for the creation of two massive gauge bosons: one *vetorial gauge boson* and one *scalar gauge boson* named "boson of Higgs". It will be done in two steps (6.1) and (6.2).

(6.1) *Mexican Hat Potential and the Spontaneous Symmetry Breaking.*

In place of the Cooper pairs one postulates[8,21] the existence of a new fundamental spin-0 field ϕ ("Higgs field") such that its interaction with an external electromagnetic field gauge field A_μ is described by the following *Higgs Lagrangian* density *L* which is invariant by the U(1) local gauge group (see Appendix B),

$$L = (D^\mu\phi)^*D_\mu\phi - (1/4)\,F^{\mu\nu}\,F_{\mu\nu} - V(\phi) \qquad (6.1),$$

where the first term is the kinetic energy of the Higgs field, the second term gives the energy density of the electromagnetic gauge field and, finally, V(ϕ) is the potential energy of the Higgs field. In the gauge theory V(ϕ) which is interpreted as a kind of "self-interaction" of the Higgs field is given, for instance, by[11]

$$V(\phi) = -\,10\,|\phi|^2 + |\phi|^4 \qquad (6.2).$$

The Higgs field is uncharged but may carry quantum numbers like weak isotopic spin. Remember that the electromagnetic field A_μ acts on the internal states of the boson. Using the Lagrangian (6.1) in the Euler-Lagrange equations will yield the Yang-Mills equations[8] again and the Klein-Gordon equation for the scalar field ϕ (Appendix A).

In figure 1 is shown the "Mexican hat" potential $V(\phi)$ as a function of ϕ_x and ϕ_y, since $|\phi| = (\phi_x^2 + \phi_y^2)^{1/2}$, where the vertical axis measures the potential energy.

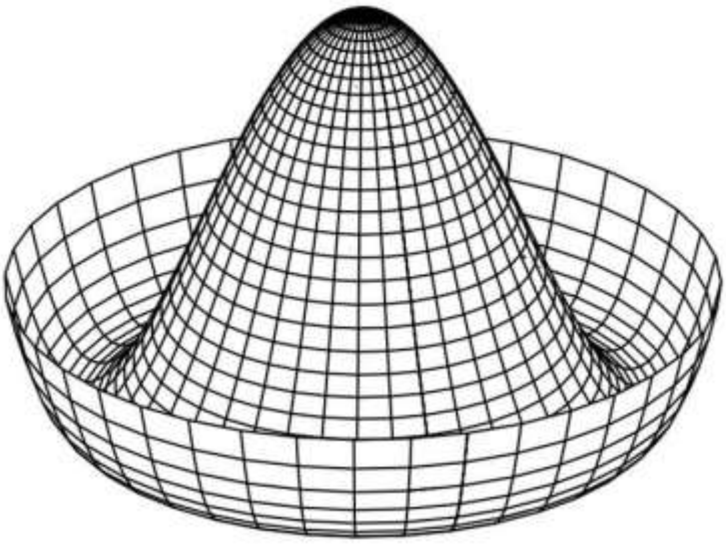

Figure 1. Graph of Mexican hat[11] potential $V(\phi) = -10\,|\phi|^2 + |\phi|^4$. The potential $V(\phi)$ is measured along the vertical z-axis. Along the *x* and *y*-axes are measured ϕ_x and ϕ_y , respectively. The origin of the coordinates are taken at the point of maximum $\phi = 0$. The points of minima are along the circle in horizontal plane (*x*,*y*) given by (6.5) $\phi_o^{(2)}(x) = \sqrt{5}\exp(i\theta)$ where θ, between 0 and 2π, is measured around the circle.

It is easily verified (see Appendix B) that the Lagrangian function (6.1) is gauge invariant by a local U(1) phase transformation $\phi \rightarrow \exp[i\alpha(x)]\phi$, that is, it is invariant by the U(1) gauge group. Note that in this case, according to (3.5), the transformed $A_\mu{}' = A_\mu + (1/q)\partial_\mu\alpha$.

The ground state ("vacuum") of the system occurs for the minimum values of $V(\phi)$ that can be calculated solving the equation $\partial V(\phi)/\partial\phi = 0$. That is,

$$\partial V(\phi)/\partial\phi = \pm\, 2|\phi|\,(-10 + 2|\phi|^2) = 0 \qquad (6.3),$$

from which we obtain $|\phi| = 0$ and $|\phi|^2 = -5$, that is,

$$\phi_o^{(1)}(x) = 0 \qquad (6.4),$$

which is an unstable vacuum (see Fig.1) and an infinite number of possible stable vacuum states given by

$$\phi_o^{(2)}(x) = \sqrt{5}\exp(i\theta), \qquad (6.5),$$

where θ is any real number between 0 and 2π. So, we see that does not exist a single vacuum but infinity of vacuum physically equivalent. The state

$\phi_o^{(1)} = 0$ is invariant by the U(1) local gauge group but $\phi_o^{(2)}$ is not invariant. This shows a SSB since the Lagrangian (6.1) is invariant by the U(1) local gauge symmetry but some vacuum states are not invariant by this symmetry transformation. Thus, $\phi_o^{(2)}$ breaks the gauge symmetry.

In the literature[8,11] the "Mexican hat potential" is generically indicated by

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4 \qquad (6.6)$$

and is called "$\lambda |\phi|^4$ potential". In this case the $\phi_o^{(2)}$ vacuum states (ground states) are given by the coherent phase of the Higgs field

$$\phi_o^{(2)} = (-\mu^2/\lambda)^{1/2}\exp[i\theta(x)] \qquad (6.7).$$

Note that the shape of potential $V(\phi)$ changes dramatically when the sign of the parameter μ^2 is changed. Now we are in condition to define clearly the properties of the Higgs model.

6.2) *Creation of Massive Gauge Bosons.*

It can be shown that[8,22] that due the SSB of the vacuum generated by the self-interaction potential $V(\phi)$ the vectorial gauge field A_μ is substituted by a new "broken" *vectorial gauge field* B_μ given by

$$B_\mu = A_\mu - (1/e)\, \partial_\mu\phi \qquad (6.8),$$

which clearly takes into account both fields A_μ and ϕ. To see, in a first approximation, how the Higgs ground state breaks the gauge symmetry we only need to look at the kinetic energy (K.E.) term of the Lagrangian (6.1):[8]

$$\text{K.E.} = |D_\mu\phi_o|^2 \qquad (6.9)$$

which contains the minimal coupling interaction between the gauge field and the degenerate ground state.

Since $D_\mu = \partial_\mu - ieA_\mu$ and $B_\mu = A_\mu - (1/e)\, \partial_\mu\phi$ we verify that

$$|D_\mu\phi_o|^2 = e^2 |B_\mu|^2 |\phi_o|^2 \qquad (6.10),$$

which has just the desired form for *vectorial gauge field mass* term (Appendix A). Thus, the mass value m_v of the *vectorial gauge boson* is given by

$$m_v = e \sqrt{|\phi_o|^2} = e\, (-\mu^2/\lambda)^{1/2} \qquad (6.11).$$

Note that the mass of the *vectorial gauge boson* (6.11) only arises from the interaction with the ground state. The mass value cannot be estimated from (6.11) because the parameters μ and λ are unknown.
To obtain the mass of the boson of Higgs, that is, the *scalar gauge boson* associated with the scalar field ϕ it is necessary to perform a more accurate calculation [21] putting, for instance,

$$\phi = \phi_o + \varphi_1 + i\,\varphi_2 \qquad (6.12),$$

where φ_1 and φ_2 are real fields that at the ground state obey the condition $<\varphi_1> = <\varphi_2> = 0$ in order to have the *vacuum expectation value* $<\phi> = \phi_o$ Eq.(6.12) can also be written as

$$\phi = (1/\sqrt{2})(v + \chi)\exp(i\Theta/v) \qquad (6.13),$$

where $v = (-2\mu^2/\lambda)^{1/2}$, $\Theta(x)$ and $\chi(x)$ are real fields that at the ground state obey the condition $<\Theta(x)> = <\chi(x)> = 0$. Substituting (6.13) in (6.1) we verify, taking into account the average values $<\Theta(x)> = <\chi(x)> = 0$ and neglecting constant terms:

$$L = \partial_\mu\chi\partial^\mu\chi - (1/4)\,F^{\mu\nu}F_{\mu\nu} + e^2\,|A_\mu|^2\,(-\mu^2/\lambda) + (1/2)|\mu|^2\,\chi^2 + \ldots \qquad (6.14),$$

showing that there appear *two massive gauge bosons*: one vetorial gauge boson with mass $m_v = e\,(-\mu^2/\lambda)^{1/2}$ (see 6.11) and one scalar gauge boson ("boson of Higgs") with mass $m_H = (1/\sqrt{2})|\mu|$.

We remark that the boson mass terms only arises from the interaction with the ground state. If the scalar field is not the ground state $\phi_o^{(2)} = (-\mu^2/\lambda)^{1/2}\exp[i\theta(x)]$, given by (6.7), the phase angle is not coherent and one obtains only the ordinary kinetic energy of a free particle. It is important to note that the Higgs field alone cannot create the mass of the Higgs boson. This is seen in Section 8 where we have shown that in absence of the gauge field A_μ only the massless Goldstone's boson appear.

(7) Massive Gauge Bosons of the Electroweak Unified Theory.

In Section 6 we saw how the symmetry of the gauge fields can be broken through the interaction of the gauge fields with the self-coherent ground state ϕ_o of the Higgs field. In the transformation of the electromagnetic and weak interactions into a unified SU(2)xU(1) gauge theory [17,20,26] ("Weinberg-Salam model") there is an "entanglement" between the original weak and the electromagnetic gauge fields.[8] The original electromagnetic field $A_\mu(x)$, defined by (3.4), and the original weak field $W_\mu(x)$ now become new fields[8] $A_\mu^{+}(x)$ and $W_\mu^{3}(x)$, respectively, related by the following equations

$$A_\mu = (g\, A_\mu^{+} - q\, W_\mu^{3})/(g^2 + q^2)^{1/2} \qquad (8.1)$$

and

$$Z_\mu^{o} = (q\, A_\mu^{+} + g\, W_\mu^{3})/(g^2 + q^2)^{1/2} \qquad (8.2),$$

where Z_μ^{o} is a new neutral weak field, g is the new weak isotopic charge and q is the weak "hypercharge". The true electric charge e is now given by $e = qg/(g^2 + q^2)^{1/2}$. The photon continues to be the gauge boson of the new field A_μ^{+}. To complete the Weinberg-Salam model we must break the gauge symmetry of the weak interaction and generate masses for the W_μ^{3} and Z_μ^{o} gauge fields. In Section (6.2) we saw how the symmetry of these gauge fields can be broken through the interaction of the gauge fields with the self-coherent ground state ϕ_o of a Higgs field. Taking into account this interaction we can calculate the kinetic energies of the W_μ^{3} and Z_μ^{o} fields using, for instance, the "broken" forms of the gauge potentials similarly to that given by (6.8), that is, $B_\mu = A_\mu - (1/q)\, \partial_\mu \phi_o$. It can be shown that the new forms of the W_μ^{3} and Z_μ^{o} kinetic energies are given by[8]

$$(\, g^2\, |W_\mu^{3}|^2 + g_o^{2}\, |\, Z_\mu^{o}\, |^2\,)\, |\phi_o|^2 \qquad (8.3)$$

where $g_o = (g^2 + q^2)^{1/2}$ and we recognize the mass term values M_W and $M_Z{}^{o}$ for the fields W_μ^{3} and Z_μ^{o}, respectively,

$$M_W = g\, \sqrt{|\phi_o|^2} \quad \text{and} \quad M_Z{}^{o} = g_o\, \sqrt{|\phi_o|^2} \qquad (8.4).$$

The observed masses of the vectorial bosons (8.4) are $M_W \approx 80.4$ Gev/c^2 for the charged $W^{\pm}$ bosons and $M_Z{}^{o} \approx 91.2$ Gev/c^2 for the neutral Z^{o} boson, in good agreement with theoretical predictions. There appears no scalar boson of Higgs: the $W^{\pm}$ and Z^{o} bosons have gained weight "eating the Higgs field".[8] On the basis of the experimental successful tests of the unified theory, the Nobel prize in physics was awarded to Glashow, Weinberg and Salam in 1979.

(8) Goldstone´s Theorem and Goldstone´s Boson.

Goldstone´s theorem states that if a theory has an exact symmetry, such as a gauge symmetry, which is not a symmetry of the vacuum, then the theory must contain a massless gauge quantum.[8] In the case of the Higgs field (See Section 6) the Goldstone´s theorem implies that there is a massless scalar field other than ϕ itself. To see this let us turn off the electromagnetic field in (6.1) getting,

$$L = \partial^\mu \phi^* \partial_\mu \phi - \mu^2 |\phi|^2 - \lambda |\phi|^4 \qquad (8.1).$$

Looking at the kinetic energy term at the ground state:

$$|\partial_\mu \phi_o^{(2)}|^2 = | \partial_\mu [v \exp(i\theta(x))]|^2 = |\phi_o^{(2)}|^2 \partial_\mu \theta \partial^\mu \theta \qquad (8.2).$$

Putting (8.2) in (8.1) we get

$$L = |\phi_o^{(2)}|^2 \partial_\mu \theta \partial^\mu \theta - \mu^2 |\phi_o^{(2)}|^2 - \lambda |\phi_o^{(2)}|^4 = |v|^2 \{\partial_\mu \theta \partial^\mu \theta - \mu^2 - \lambda\} \qquad (8.3).$$

From (8.3) using the Euler-Lagrange equations we obtain (see Appendix A) the Klein-Gordon equation for a zero mass particle,

$$\partial_\mu \theta \partial^\mu \theta = 0 \qquad (8.4),$$

interpreting the angle $\theta(x)$ a new scalar field. This shows the existence of massless gauge boson associated with the scalar field $\theta(x)$, called Goldstone´s boson. This boson propagates through the ground state medium. According to Moriyasu[8] this wave is not a mathematical fiction. Such waves actually are observed in condensed matter systems. In the superconductor these waves are known as plasma oscillations.[8] For ferromagnet systems the waves are the Bloch spin waves or magnons.[12] The Goldstone waves for broken gauge symmetry are absorbed by the external gauge field shown in (6.1). The presence of these waves was undesirable in elementary particles physics because they represent massless scalar particles which have never been observed. The Higgs mechanism showed how the Goldstone waves could be eliminated.

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Appendix A. Klein-Gordon Lagrangian for Scalar Field.

The Lagragian density L for a complex scalar field φ of a particle with rest mass M in an external electromagnetic field A_μ is given by[16,17,21]

$$L = (1/2)\,(D_\mu\varphi)^* D^\mu \varphi + (1/2) M^2 \varphi^* \varphi \qquad (A.1),$$

where $D_\mu = \partial_\mu - iqA_\mu$. It is important to note that for a particle with mass M the Lagragian contains a term $M^2\varphi^*\varphi$. Similar mass term appears for a vectorial field Ψ_μ, that is, we have $M^2\Psi_\mu{}^*\Psi_\mu$.[17] Using the Euler-Lagrange equations [16,17,21]

$$\partial_\mu (\partial L/\partial\varphi_{i,\mu}) - \partial L/\partial\varphi_i = 0, \text{ where } \varphi_i = \varphi, \varphi^* \text{ and } \varphi_{i,\mu} = \partial\varphi_i/\partial_\mu$$

we get the Klein-Gordon wave equation

$$D_\mu\varphi^* D^\mu\varphi = M^2\varphi^*\varphi \qquad (A.2).$$

From (A.2) when M = 0 we get

$$D_\mu\varphi^* D^\mu\varphi = 0 \qquad (A.3).$$

Appendix B. Local U(1) Gauge Symmetry of the Higgs Lagrangian.

Let us show that the Higgs Lagrangian (6.1) given by

$$L = (D^\mu\phi)^* D_\mu\phi - (1/4)\, F^{\mu\nu} F_{\mu\nu} - \mu^2 |\phi|^2 - \lambda |\phi|^4 \qquad (B.1)$$

is invariant by the local U(1) phase transformation

$$\begin{aligned} &\phi' = U(x)\phi = \exp[i\alpha(x)]\, \phi \qquad \text{and, putting } \alpha_\mu(x) = \partial_\mu\alpha(x). \\ &A_\mu{}' = UA_\mu U^{-1} - (i/e)\, (\partial_\mu U)U^{-1} = A_\mu + (1/e)\, \alpha_\mu(x). \end{aligned} \qquad (B.2)$$

The terms $F^{\mu\nu} F_{\mu\nu}$, $\mu^2 |\phi|^2$ and $\lambda |\phi|^4$ are clearly invariant by the gauge transformation. Let us analyze the kinetic term $(D^\mu\phi)^* D_\mu\phi$. So,

$$D_\mu{}'\phi' = (\partial_\mu - ieA_\mu{}')(\exp(i\alpha)\phi) = (\partial_\mu - ieA_\mu - i\, \alpha_\mu)(\exp(i\alpha)\phi) \qquad \text{and}$$

$$(D_\mu{}'\phi')^* = (\partial_\mu - ieA_\mu{}')^* \exp(-i\alpha)\phi^* = (\partial_\mu + i\, eA_\mu + i\, \alpha_\mu)(\exp(-i\alpha)\phi^*). \text{ So,}$$

$$(D_\mu{}'\phi')^* (D_\mu{}'\phi') = (\partial_\mu - ieA_\mu{}')^* \exp(-i\alpha)\phi^* x\, (\partial_\mu - ieA_\mu{}') \exp(i\alpha)\phi$$

$$=\{(\partial_\mu + i\, e[A_\mu + (i/e)\alpha_\mu])(\exp(-i\alpha)\phi^*) \;x \{(\partial_\mu - ie[A_\mu + (i/e)\alpha_\mu])(\exp(i\alpha)\phi)$$

$$= (\partial_\mu + i\, eA_\mu + i\, \alpha_\mu)(\exp(-i\alpha)\phi^*) \;x\; (\partial_\mu - ieA_\mu - i\, \alpha_\mu])(\exp(i\alpha)\phi)$$

$$= \{\exp(-i\alpha)\, \partial_\mu\phi^* - i\, \exp(-i\alpha)\alpha_\mu\phi^* + ieA_\mu \exp(-i\alpha)\phi^* + i\, \alpha_\mu \exp(-i\alpha)\phi^*\}$$

$$x\; \{\exp(i\alpha)\, \partial_\mu\phi + i\, \exp(i\alpha)\alpha_\mu\phi - ieA_\mu \exp(i\alpha)\phi - i\, \alpha_\mu \exp(i\alpha)\phi \}$$

$$= \exp(-i\alpha)\,(\partial_\mu + ieA_\mu)\phi^* \ \text{x}\ \exp(i\alpha)\,(\partial_\mu - ieA_\mu)\phi$$

$$= (\partial_\mu + ieA_\mu)\phi^* \ \text{x}\ (\partial_\mu - ieA_\mu)\phi \ = \ (D^\mu\phi)^* D_\mu\phi.$$

Since, $(D_\mu'\phi')^*(D_\mu'\phi') = (D^\mu\phi)^*(D_\mu\phi)$ we verify that $L'(\phi', D_\mu'\phi') = L(\phi, D_\mu\phi)$, that is, Higgs Lagrangian $L(\phi, D_\mu\phi)$ given by (B.1) is invariant by a local U(1) phase transformation defined by (B.2).

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