Two and four-level systems in magnetic fields restricted in time

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Abstract

We describe some new exact solutions for two- and four-level systems. In all the cases, external fields have a restricted behavior in time. First, we consider two types of new solutions for one-spin equation, one of them is in a external magnetic field that acts during a finite time interval. A new solution for two interacting spins is found in the case when the field difference between the external fields in each spin vary adiabatically, vanishing on the time infinity. The latter system can be identified with a quantum gate realized by two coupled quantum dots. The probability of the Swap operation for such a gate can be explicitly expressed in terms of special functions. Using the obtained expressions, we construct plots for the Swap operation for some parameters of the external magnetic field and interaction function.

1 Introduction

Finite-level systems have always played an important role in quantum physics. In particular, two-level systems possess a wide range of applications, for example, in the semi-classical theory of laser beams [1], optical resonance [2], and nuclear induction experiments [3], and so on. The best known physical system that could be identified with a two-level system is a fixed spin-one-half object interacting with a magnetic field. Four-level systems can be used to describe two interacting one-half spins, e.g., the valence electrons in two coupled semiconductor quantum dots [4]. The most detailed theoretical study of quantum mechanical equations for two and four-level systems, and their exact solutions,

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are presented in [5, 6]. Recently, two- and four-level systems attract even more attention due to their relationship to the problem of quantum computation [7]. In this problem, the computation is performed by manipulation with so-called one- and two-qubit gates [10]. The one-qubit gate can be identified with a two-level system and two-qubit gates can be identified with a four-level system. For these reasons, two- and four-level systems are crucial elements of possible quantum computers. For physical applications, it is very important to have explicit exact solutions of two- and four-level system equations. In [11] exact solutions of a four-level system are used to describe the theoretical construction of a universal quantum XOR gate using two-coupled quantum dots. This work shows how the exact solutions can be used to establish all the necessary conditions on the external fields needed for the implementation of the gate.

In the present work, we describe some new exact solutions for two- and four-level systems that were not represented in our previous works [5, 6]. These solutions are found for external fields that have a restricted behavior in time, for example, the first solution for two-level system in external field that acts along a finite time interval. In Section 2 we consider two types of new solutions for a two-level system, and in Section 3 a new solution for a four-level system. In the latter case the field difference between the external fields in each spin vary adiabatically (vanishes with time). The latter system can be identified with a quantum gate realized by two coupled quantum dots. The probability of the Swap operation for such a gate can be explicitly expressed in terms of special functions. Using the obtained expressions, we construct plots for the Swap operation for some parameters of the external magnetic field and interaction functions.

2 Two-level systems

2.1 General

We recall to the reader that two-level systems are described by the so-called spin equation

$$i\frac{dv}{dt} = (\boldsymbol{\sigma}\mathbf{F})v, \quad \boldsymbol{\sigma} = (\sigma_1,\, \sigma_2,\, \sigma_3), \quad \mathbf{F} = (F_1,\, F_2,\, F_3), \tag{1}$$

where $v = v(t)$ is a two–component spinor, $\sigma_k (k = 1, 2, 3)$ are Pauli matrices, $F_k = F_k(t)$ are components of external field strength, see [5]. The general solution of the spin equation reads

$$v(t) = R(t)v^0, \tag{2}$$

where 2×2 matrix $R(t)$ obeys the same spin equation

$$i\frac{dR(t)}{dt} = (\boldsymbol{\sigma}\mathbf{F})R(t), \tag{3}$$

and v^0 is an arbitrary constant spinor. If $R(t = t_0) = I$, where I is 2×2 unity matrix, then $R(t) = U(t)$, where $U(t)$ is the evolution operator of the spin equation. In the general case, the evolution operator is constructed by the help of the matrix $R(t)$ as follows

$$U(t) = R(t)R_0^{-1}, \quad R_0 = R(t = t_0), \; U(t = t_0) = I \; . \tag{4}$$

The matrix $R(t)$ can always be represented in the form

$$R(t) = Ip_0 - i(\boldsymbol{\sigma}\,\mathbf{p}), \quad \mathbf{p} = (p_1,\, p_2,\, p_3), \quad p_s = p_s(t), \; s = 0, 1, 2, 3 \; , \tag{5}$$

The functions $p_s(t)$ obey the set of equations

$$\dot{p}_0 + (\mathbf{pF}) = 0, \quad \dot{\mathbf{p}} + [\mathbf{p} \times \mathbf{F}] - p_0\mathbf{F} = 0, \tag{6}$$

which follows from (3). Equations (6) imply that $\Delta = \det R(t) = p_0^2 + \mathbf{p}^2$ is an integral of motion.

Let us suppose that spin equation (3) is self-adjoint, which means that the external field $\mathbf{F}$ is real. In such a case we can chose the functions $p_s(t)$ to be real. Without loss of generality, in such a case, we can set $\Delta = 1$, which means that $R(t)$ is nonsingular. Under the condition $\Delta = 1$, the functions $p_s(t)$ can be expressed via three real parameters $\alpha = \alpha(t)$, $\theta = \theta(t)$, and $\varphi = \varphi(t)$ as follows

$$\begin{aligned} p_0 &= \cos\frac{\varphi - \alpha}{2}\cos\frac{\theta}{2}, \quad p_1 = -\sin\frac{\varphi + \alpha}{2}\sin\frac{\theta}{2}, \\ p_2 &= \cos\frac{\varphi + \alpha}{2}\sin\frac{\theta}{2}, \quad p_3 = \sin\frac{\varphi - \alpha}{2}\cos\frac{\theta}{2}. \end{aligned} \tag{7}$$

2.2 Exact solutions for some restricted in time external fields

I. Let the external field be zero at $|t| \geq T$, where T is a constant, and for $|t| < T$ reads

$$\begin{aligned} F_1(t) &= \frac{\pi}{8T}[(\alpha_0 - \alpha_1)\sin\theta\cos\varphi + (\theta_0 - \theta_1)\sin\varphi]\cos\frac{\pi t}{2T}, \\ F_2(t) &= \frac{\pi}{8T}[(\alpha_0 - \alpha_1)\sin\theta\sin\varphi - (\theta_0 - \theta_1)\cos\varphi]\cos\frac{\pi t}{2T}, \\ F_3(t) &= \frac{\pi}{8T}[(\alpha_0 - \alpha_1)\cos\theta + \varphi_1 - \varphi_0]\cos\frac{\pi t}{2T}, \end{aligned} \tag{8}$$

Here $\alpha_0, \alpha_1, \theta_0, \theta_1, \varphi_0$, and φ_1 are arbitrary constants, and the functions $\theta = \theta(t)$, $\varphi = \varphi(t)$ have the forms

$$\begin{aligned} &\theta(t) = \theta_0 \;, \quad \varphi(t) = \varphi_0, \quad t \leq -T, \\ &\theta(t) = \frac{\theta_1 - \theta_0}{2}\sin\frac{\pi t}{2T} + \frac{\theta_1 + \theta_0}{2} \;, \quad |t| < T, \\ &\varphi(t) = \frac{\varphi_1 - \varphi_0}{2}\sin\frac{\pi t}{2T} + \frac{\varphi_1 + \varphi_0}{2} \;, \quad |t| < T, \\ &\theta(t) = \theta_1 \;, \quad \varphi(t) = \varphi_1 \;, \quad t \geq T. \end{aligned}$$

The external field under consideration is not zero only on a finite interval $|t| < T$ and is continuous for all t.

For such an external field there exist an exact solution of equation (3). It is given by expression (5), where the functions $p_s(t)$ have the form (7) with

$$\alpha(t) = \alpha_0, \quad t \le -T; \quad \alpha(t) = \alpha_1, \quad t \ge T;$$
$$\alpha(t) = \frac{\alpha_1 - \alpha_0}{2} \sin \frac{\pi t}{2T} + \frac{\alpha_1 + \alpha_0}{2}, \quad |t| < T.$$

II. Let the external field have the form

$$\begin{aligned}
F_1(t) = &-\frac{\theta_0}{T_1}\left[1 - 2\left(\frac{t}{T_1}\right)^2\right] \exp\left[-\left(\frac{t}{T_1}\right)^2\right] \sin\varphi \\
&- \frac{\alpha_0}{T_3}\left[1 - 2\left(\frac{t}{T_3}\right)^2\right] \exp\left[-\left(\frac{t}{T_3}\right)^2\right] \sin\theta\cos\varphi \ , \\
F_2(t) = &\frac{\theta_0}{T_1}\left[1 - 2\left(\frac{t}{T_1}\right)^2\right] \exp\left[-\left(\frac{t}{T_1}\right)^2\right] \cos\varphi \\
&- \frac{\alpha_0}{T_3}\left[1 - 2\left(\frac{t}{T_3}\right)^2\right] \exp\left[-\left(\frac{t}{T_3}\right)^2\right] \sin\theta\sin\varphi \ , \\
F_3(t) = &\frac{\varphi_0}{T_2}\left[1 - 2\left(\frac{t}{T_2}\right)^2\right] \exp\left[-\left(\frac{t}{T_2}\right)^2\right] \\
&- \frac{\alpha_0}{T_3}\left[1 - 2\left(\frac{t}{T_3}\right)^2\right] \exp\left[-\left(\frac{t}{T_3}\right)^2\right] \cos\theta \ ,
\end{aligned} \tag{9}$$

where the functions $\theta = \theta(t)$ and $\varphi = \varphi(t)$ are defined as

$$\theta(t) = \frac{\theta_0 t}{T_1} \exp\left[-\left(\frac{t}{T_1}\right)^2\right] + \theta_1 \ , \quad \varphi(t) = \frac{\varphi_0 t}{T_2} \exp\left[-\left(\frac{t}{T_2}\right)^2\right] + \varphi_1 \ , \tag{10}$$

and θ_0, θ_1, φ_0, φ_1, α_0, and T_k, $k = 1, 2, 3)$ are arbitrary constants.

The external field under consideration vanishes at $t \to \pm\infty$.

For such an external field there exists an exact solution of equation (3). It is given by expression (5), where the functions $p_s(t)$ have the form (7) with

$$\alpha(t) = \frac{\alpha_0 t}{T_3} \exp\left[-\left(\frac{t}{T_3}\right)^2\right] + \alpha_1, \quad \alpha_1 = \text{const.}$$

3 Four-level systems

3.1 General

We write the Schrödinger equation for a four-level system in the following form[1] (see [6]):

$$i\frac{d\Psi}{dt} = \hat{H}\left(\mathbf{G},\mathbf{F},J\right)\Psi\,,$$
$$\hat{H} = (\boldsymbol{\rho}\cdot\mathbf{G}) + (\boldsymbol{\Sigma}\cdot\mathbf{F}) + \frac{J}{2}\left(\boldsymbol{\Sigma}\cdot\boldsymbol{\rho}\right). \tag{11}$$

Here Ψ is a four-component column; in the general case the interaction function J, as well as, the external fields (two three vectors $\mathbf{G}$ and $\mathbf{F}$) are time-dependent; and 4×4 matrices $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ have the forms

$$\boldsymbol{\Sigma} = I\otimes\boldsymbol{\sigma}\,,\ \boldsymbol{\rho} = \boldsymbol{\sigma}\otimes I\,, (\boldsymbol{\Sigma}\cdot\boldsymbol{\rho}) = \boldsymbol{\sigma}\otimes\boldsymbol{\sigma} = \sum_{i=1}^{3}\sigma_i\otimes\sigma_i\,,$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, and I is the 2×2 identity matrices. The Hamiltonian matrix reads

$$\hat{H} = \begin{pmatrix} F_3+G_3+\frac{J}{2} & F_1-iF_2 & G_1-iG_2 & 0 \\ F_1+iF_2 & G_3-F_3-\frac{J}{2} & J & G_1-iG_2 \\ G_1+iG_2 & J & F_3-G_3-\frac{J}{2} & F_1-iF_2 \\ 0 & G_1+iG_2 & F_1+iF_2 & \frac{J}{2}-G_3-F_3 \end{pmatrix}. \tag{12}$$

Such a model is used to describe two spins, subject to the external magnetic fields $\mathbf{F}$ and $\mathbf{G}$, and interacting with each other through a spherically symmetric Heisenberg interaction whose intensity is given by the interaction function J. In particular, this model was used to describe two coupled quantum dots [4]. In our work [6] a series of exact solution of equation (11) for different choices of the interaction function and the external fields are found for the first time.

3.2 Reduction to the two-level system case

For a special case of two spins subject to parallel external magnetic fields, which we write as

$$\mathbf{G} = (0,0,\mu_B g_1 B_1)\ ,\ \mathbf{F} = (0,0,\mu_B g_2 B_2)\ \ ,\ B_{1,2} = B_{1,2}\left(t\right)\,, \tag{13}$$

where μ_B is the Bohr magneton and g_1 and g_2 are g-factors for the corresponding spins, one can show that the evolution operator $\hat{U}\left(t\right)$ for the equation (11) can be reduced to an evolution operator $\hat{u}\left(t\right)$ for the Schrödinger equation of a

[1] $\hbar = 1$

two-level system [6]. Such a reduction is given by the equation

$$\hat{U}(t) = \exp\left(-\frac{i}{2}\left[(\Sigma_3 + \rho_3)\,\Gamma(t) + \Sigma_3\rho_3\Phi(t)\right]\right) M(t) \ ,$$
$$\Gamma(t) = \int_0^t B_+(\tau)\, d\tau \,,\ \Phi(t) = \int_0^t J(t)\, d\tau \,,\ B_+ = \mu_B\left(g_1 B_1 + g_2 B_2\right) \,,$$
$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & u_{11} & u_{12} & 0 \\ 0 & u_{21} & u_{22} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \ , \tag{14}$$

where $\hat{u}(t) = ||u_{ij}||$ obeys the Schrödinger equation for a two-level system (see [5])

$$i\frac{d\hat{u}}{dt} = (\boldsymbol{\sigma}\cdot\mathbf{K})\,\hat{u}\,,\ \ \hat{u}(0) = I\,,$$
$$\mathbf{K}(t) = (J(t), 0, B_-(t)) \ ,\ B_- = \mu_B\left(g_1 B_1 - g_2 B_2\right) \,, \tag{15}$$

Thus, in the case under consideration, the four-level system problem is reduced to solve the two-level system problem (15) with an effective magnetic field $\mathbf{K}$.

3.3 An adiabatic variation of the field difference in each spin

Consider a four-level system in which the field difference varies adiabatically with time, while the interaction function is constant. Namely, we chose

$$J = a\,,\ B_-(t) = c/\cosh\omega t\,, \tag{16}$$

where $a,\ c$ and ω are real constants. In practical application the pulse applied to the system (e.g., two coupled quantum dots) needs to be shorter than the decoherence time of the system. But such fast pulse can cause a transition of the system to higher energy levels and, consequently, its dynamic can no longer be described by the Hamiltonian (12). The $c/\cosh\omega t$ dependence is the most adequate kind of a variation to avoid this higher energy level transition [12]. In addition, it is reasonable to infer that if the only quantity that varies is B_-, and $B_+ \gg B_-$, the interaction function will remain constant [11]. With regard to the variation of B_-, there are some proposals for the application of localized magnetic fields [13] and some techniques that permit the manipulation of the g-factor by changing the size of the dots or by the application of external electromagnetic fields [14, 15].

From the previous Sect., we know that the evolution operator (14) of a four-level system with the parameters (16) is expressed via an evolution operator of a two-level system with effective field

$$\mathbf{K}(t) = (a, 0, c/\cosh\omega t) \ . \tag{17}$$

The exact solution for the evolution operator with such a field can be constructed using our previous results [5]. It has the form

$$\hat{u}(t) = \frac{1}{\left|G_1^0\right|^2 + \left|G_2^0\right|^2} \begin{pmatrix} G_1(z) & -\bar{G}_2(z) \\ G_2(z) & \bar{G}_1(z) \end{pmatrix} \begin{pmatrix} \bar{G}_1^0 & \bar{G}_2^0 \\ -G_2^0 & G_1^0 \end{pmatrix}, \quad (18)$$

where

$$\begin{aligned} & G_1(z) = i(2c+\omega) z^{\mu} (1-z)^{\nu} F(\alpha, \nu; \gamma; z) \ , \\ & G_2(z) = 2a z^{\mu+1/2} (1-z)^{\nu} F(\alpha, \nu+1; \gamma+1; z) \ , \ G_{1,2}^0 = G_{1,2}(-1) \ , \\ & z = \left(\frac{e^{\varphi}+i}{e^{\varphi}-i}\right)^2 \ , \ \varphi = \omega t, \quad \alpha = \gamma + \nu \ , \\ & \mu = \frac{c}{2\omega}, \ \nu = i\frac{|a|}{\omega} \ , \ \gamma = \frac{1}{2} + 2\mu \ . \end{aligned} \quad (19)$$

$F(\alpha, \beta, \gamma, z)$ is the Gauss hypergeometric function, and complex conjugate quantities are designated by a bar above.

Substituting (19) into (15), we obtain

$$\hat{R}(t) = \exp\left(\frac{ict}{2}\right) \begin{pmatrix} \exp\left[-i\left(ct + \Gamma(t)\right)\right] & 0 \quad 0 & 0 \\ 0 & & 0 \\ 0 & \hat{u}(t) & 0 \\ 0 & 0 \quad 0 & \exp\left[-i\left(ct - \Gamma(t)\right)\right] \end{pmatrix},$$

with $\hat{u}(t)$ give in (18).

Thus, any transition amplitude for the four-level system can be calculated with the help of the evolution operator. Let us, for example, calculate the transition amplitude between the states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, which have the form

$$|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |\downarrow\uparrow\rangle = |\downarrow\rangle \otimes |\uparrow\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \quad (20)$$

This transition between these states represents, in quantum computation, the so called Swap operation and can be experimentally measured [16]. From the general expression (14), we see that

$$\langle\uparrow\downarrow| \hat{R} |\downarrow\uparrow\rangle = \langle\uparrow| \hat{u} |\downarrow\rangle \ , \ \langle\downarrow\uparrow| \hat{R} |\uparrow\downarrow\rangle = \langle\downarrow| \hat{u} |\uparrow\rangle \ , \ |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (21)$$

Therefore, in the case of the Swap operation between the states (20), we need only to calculate matrix elements of the two-level system evolution operator. One has also to stress that in this case the Swap operation does not depend on the fields' sum B_+ .

Using (18), we calculate the probability amplitude for the Swap operation with the adiabatic variation (16),

$$|\langle\downarrow|\,\hat{u}\,|\uparrow\rangle|^2 = \frac{\left|G_2\,(z)\,\bar{G}_1^0 - \bar{G}_1\,(z)\,G_2^0\right|^2}{\left(|G_2^0|^2 + |G_1^0|^2\right)^2}$$

In order to use the adiabatic pulse to implement some quantum operations (like the Swap or the XOR gate) the duration of the pulse needs to be shorter than the dephasing time of the system. E.g. in GaAs quantum dots this time is about 10 ns [16], which correspond to $\omega \simeq 1\,\text{GHz}$. In typical experimental conditions, we have fields of about 5 T, $J = 2 \times 10^{-3}\,\text{eV}$ and, to satisfy the condition $B_+ \gg B_-$, we can set the amplitude $|B_-| = 11\,\text{mT}$. So, some characteristic values for our system are

$$\frac{|a|}{\omega} = \frac{|J|}{\hbar\omega} \simeq 3\ ,\ \ \frac{c}{\omega} = \frac{\mu_B\,|B_-|}{\hbar\omega} \simeq 1\ .$$

In Figure 1 we have plots of the probability as a function of time for the above values of the parameters. The first maximum occurs at $t = 0.5\,\text{ns}$ with a probability of $P = 90\%$. For larger time, as the $\cosh^{-1}$ approaches to zero, this probability varies as $A_1 \sin^2(at) + A_2$ where $A_i = A_i\,(\omega, a, c)$. The amplitude A_1 decreases as c increases while the shift A_2 increases. The functions A_i change significantly with ω only for $c > 10a$.

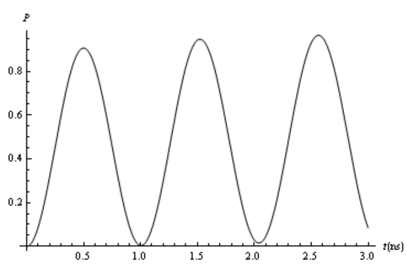

Figure 1 - Probability of the Swap operation as a function of time for $J = 2 \times 10^{-3}$ eV, $\omega = 1\,\text{GHz}$ and $B_- = 11 \times 10^{-3}\,\text{T}$.

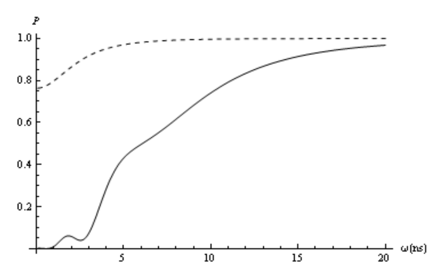

Figure 2 - Probability as a function of ω for the values $c = 1$ (dashed line) and $c = 12$ (solid line) in $t = 0.8$ns and $a = 2$.

The dependence of the probability on the parameter ω become noticeable for $c > 4a$. In Figure 2 we plot this dependence for $a/c = 2$ and $a/c = 1/6$. The parameter ω can be used to significantly attenuate the Swap transition for values of $c > 10a$.

A numerical study shows a strong dependence of the maximum values with the parameter a. This fact can be used to measure the interaction J. In Figure 3 we plot the dependence of the probability on J. The attenuation of the second maximum can be achieved by increasing the ratio c/a.

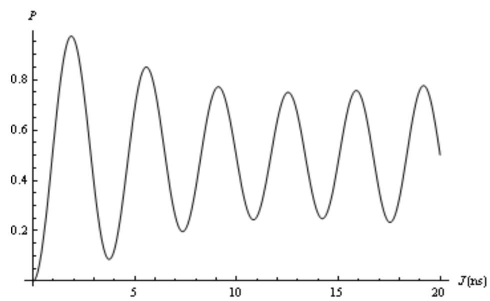

Figure 1: Figure 3 - Probability as a function of the interaction J for the $c = 30$, $t = 1\mathrm{ns}$ and $\omega = 15\,\mathrm{GHz}$.

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