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MULTI-NUCLEON TUNNELING IN THE SADDLE-SCISSION STAGE OF THE FISSION OF A HEAVY NUCLEUS

by

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ABSTRACT:

The discrepancy between the measured most probable mass split and calculation including shell effects is discussed It is suggested that during the descent from saddle to scission a number of nucleons tunnel from the light to the heavy fragment because of pairing effects. This process could explain the discrepancy.

I) INTRODUCTION AND SUMMARY

Despite the recent progress in the theoretical description of the fission process of a heavy nucleus there are still many questions that are still waiting for answers. Most of the work that has been done in the last few years has centered mainly on the first part of the evolution of a fissioning nucleus namely the transition up to the last saddle point. The region between the saddle and the final stage, the scission point, has had but a little attention. This benevolance stems from the uncertainly inherent in the dynamics of the nuclear system in the last stage of fission as a consequence of the not-so-well understood issue of nuclear dissipation (viscosity). Another important hindering factor in developing a dynaminal theory for the post last-saddle point stage is the limitation inherent in the theory used to describe the pre-last saddle point stage which is based on the quantal penetration of a one-dimensional multi-humped barrier.

Of the many facets of fission is the predominance of asymmetric fission in the transuranic region. The macro-micro theory predicts for ^{236}U an asymmetric mass split centered around two peaks corresponding to the most probable heavy and light mass fragments of A_H=136 and A_L= 100 respectively[1]. Experiment however, indicates different values for these masses: 140 and 96 respectively. The shift in the values of A_H and A_L from theory could not be accounted for unless one assumes an entirely divergent view of the nuclear configuration at the scission point. Whereas the liquid-drop model gives a scission configuration of two non-interacting fragments, what one should deal with is two interacting fragments i.e. a kind of molecular state as was demonstrated by Nörenberg[2]. As a consequence of the molecular configuration is the presence of a scission barrier. Nörenberg treatment, howevertheless, has the same shortcomings as other works as no explicit reference is made for the saddle to scission dynamics.

In the present work we construct a new model for the redistribution of the mass in the two fragments so that the correct A_H and A_L are obtained. The mechanism for this redistribution is considered to be an effective tunneling of particles from the light to the heavy fragment. We consider the nuclear system at the saddle and follow it until it scissions. We assume adiabatic motion. What suggests our model is the fact that a critical elongation of the compound nucleus is reached at which necking-in becomes favoured. We simulate the necking in by the introduction of a time-dependent barrier inside the potential well that represents the compound systems. After the elapse of a sufficient time the barrier will have developed into such a height the compound system becomes effectively two fragments with a weak force acting between them. This "Josephson junction" serves to complete the final distribution of the mass in the two fragments[3]. One should stress that the tunneling of particles through the junction can not, by itself, account for asymetric fission. The tendency toward asymetric fission is assumed to be a property of the system at the last saddle. This we simulate by placing the barrier off the center of the potential well. The idea that a Josephson type effect might account for the shift in $A_H^{ex} - A_H^{th}$ has for some time, been floating around in different seminars, coloquia etc[1]. The aim of this work is to develop this idea to the point where it could be checked numerically. Besidespredicting the right-trend of the tunneling of neutrons from the light to the heavy fragment we propose that the same effect could also account for the fact that increasing the mass of the fissioning compound nucleus will result in roughly the same A_H and only the light fragment will be affected.

If one assumes that the transfer occurs in the scission valley[2] then one gets, using a simple model, the right trend namely the transfer proceeds from the light to the heavy fragment. Of course such a model assumes, apriori, the existance of two well-defined masses at the scission valley. One could use classical statistical averaging through density matrices to account for the mass unpolaris a bility of the fragments. A parametrized form of $\rho(M)$ could be constructed from the observed mass yield shape $\rho(M)$. One needs to estimate the time that it takes the nuclear system to go from the saddle point to and through the scission barrier until it scissions. In order to get an appreciable transfer, this time should be larger than the time characteristic of the transfer of a pair of nucleons from one fragment to the other. This transfer time can be estimated to be roughly 5.6×10^{-23} sec (using the semi-empirical mass formula). The fact that the time correponding to saddle-scission transition is almost ten times greater than the above supports our contention that at least few nucleons would be tranferred even if the transfer starts in the scission valley.

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* This presupposes that friction is not so drastic as to really destroy the pairing "field" in the fragments.

Numerically one needs to solve 4-6 coupled equations. However one could still get an idea of the number of transferred pairs by analytically solving a schematized form of these coupled equations. The Josephson current can also be calculated this way although it does not contain extra information.

THE MODEL:

In order to build a more realistic model one has to start at saddle. By using time-dependent Schrodinger equation one can then construct a system of coupled equations for the transfer amplitude. The single particle energies are time-dependent and thus one has to use an expansion of the wave function of the system in terms of asymptotic basis[4] (i.e. in terms of states corresponding to the after-scission nuclear configuration). A time-dependent pairing strength acts between particles in different fragments ($V(t) \sim e^{-\gamma t^{1/2}}$) and supplies the weak contact that results in nuclear tunneling (Josephson effect). The above time dependence of V (t) is only partially true as this form was obtained assuming symmetrical fission whereas the final result is asymmetrical fission. Thus one has to correct for the time dependence (this is quite involved). Mutual internal exitation of the two prescission fragments can be simulted by introducing temperature into the BCS quantities that are needed to construct the nuclear wave function during the descent from saddle to scission. Of course the temperature dependence of the wave function could account, to a limited degree, for the presence of viscosity since viscous propagation is a nonequilibrium process and the introduction of temperature implies equilibrium. However, this approximation suffices for our purpose namely to estimate the amount of reduction in nuclear superconductivity due to internal excitation in the path from saddle to scission. The dissipation is thought not to be great[5].

The Hamiltonian of the system at saddle may be represented as follows:

$$H(t) = H_o + U(t)$$

$$H_o = \sum_{\nu,\mu} \varepsilon_{\nu\mu} a^+_\nu a_\mu + \sum_{\alpha\beta\gamma\delta} G_{\alpha\beta\gamma\delta}\, a^+_\alpha a^+_\beta a_\gamma a_\delta \quad (1)$$

U(t) is a time-dependent barrier that represents the necking-in degree of freedom where the nuclear system reaches a critical elongation ℓ_c. To follow the evolution of the system one has to solve the time-dependent Schrodinger equation

$$H(t)\,\Psi(t) = i\hbar \frac{\partial}{\partial t}\Psi(t) \quad (2)$$

One way of solving (2) is by expanding $\Psi(t)$ in the adiabatic basis:

$$\Psi(t) = \sum_\nu f_\nu(t) \exp\left[-\frac{i}{\hbar}\int^t E_\nu(t')\,dt'\right] \Phi_\nu(t)$$

$$H(t)\,\Phi_\nu(t) = E_\nu(t)\,\Phi_\nu(t)$$

The result is

$$\dot{f}_\nu(t) = -\frac{1}{\hbar}\sum_\mu \langle \Phi_\nu(t) | \dot{\Phi}_\mu(t)\rangle \times \exp\left[-\frac{i}{\hbar}\int^t \left[E_\mu(\tau) - E_\nu(\tau)\right] d\tau\right] f_\mu(t) \quad (3)$$

Assuming a linear time dependence of U(t)=ct and performing the integration one then gets

$$f_\nu(t) = \delta_{o\nu} - \frac{c}{\hbar}\int_o^t \sum_\mu \langle \Phi_\nu(t') | \frac{\partial}{\partial U} | \Phi_\mu(t')\rangle \times \exp\left[-\frac{i}{\hbar}\int^{t'} (E_\mu(\tau) - E_\nu(\tau))\,d\tau\right] \times f_\mu(t')\,dt' \quad (4)$$

It is assumed that at t=0 the system is in its ground state i.e. $\Psi(0) = \Phi_o$. In order to solve (4) one needs $\Phi_\nu(U)$ as well as the time-dependent "energies" $E_\nu(t)$. A convenient way of performing the above calculation is to use the asymptotic representation.

$$\Phi_\nu(U) = \sum_1 C_1^\nu(U)\Psi_1 + \sum_2 C_2^\nu(U)\Psi_2$$

Thus

$$G_{\nu\mu}(t) = G_{11}(U) + G_{22}(U) + G_{12}(U) + G_{21}(U) \qquad (5)$$

Where

$G_{11}(U) \equiv G_1$ = pairing force in well 1

$G_{22}(U) \equiv G_2$ = pairing force in well 2

$G_{12}(U) = G_{21}(U)$ = pairing force between particles in well 1 and those in well 2

Thus the Hamiltonian of the system becomes:

$$\begin{aligned} H(t) = & \sum_1 \varepsilon_1(t)\, b_1^\dagger b_1 + \sum_2 \varepsilon_2(t)\, b_2^\dagger b_2 \\ & - \sum_1 G_{11}(t)\, b_1^\dagger b_{-1}^\dagger\, b_1 b_{-1} \\ & - \sum_2 G_{22}(t)\, b_2^\dagger b_{-2}^\dagger\, b_2 b_{-2} \\ & - \sum_{1,2} G_{12}(t)\, b_1^\dagger b_{-1}^\dagger\, b_2 b_{-2} \\ & - \sum_{2,1} G_{21}(t)\, b_2^\dagger b_{-2}^\dagger\, b_1 b_{-1} \end{aligned} \qquad (6)$$

Where we are neglecting terms in G that break pairs i.e. we retain only the usual pairing term. The Hamiltonian (6) is a complicated one due to its explicit time-dependence both in the single particle energies as well as the pairing force strength. We shall use the simpler Hamiltonian

$$H(t) = H_1(t) + H_2(t) - V(t)\sum_{1,2}\left[b_1^+ b_{-1}^+ b_2 b_{-2} + \mathrm{H.c.}\right] \tag{7}$$

Where now $H_{1(2)}(t)$ refers to particles in well 1(2) and the term that contains V(t) is a tunneling Hamiltonian responsible for the coupling between the particles in well 1 to those in well 2. Naturally $H_1(t)$ and $H_2(t)$ are both time-dependent due to the time dependence of the single-particle energies through their dependence on V(t). In order to proceed in the solution of the dynamics governed by the Hamiltonian one has to find the time dependence of the effective tunneling potential V(t).

THE TUNNELING POTENTIAL V(t):

From definition V(t) is given by:

$$\frac{V(t)}{G} = \frac{\langle \phi_1(U)\phi_{-1}(U) \,|\, \delta(r_1 - r_2) \,|\, \phi_2(U)\, \phi_{-2}(U) \rangle}{\langle \phi_1(U)\phi_{-1}(U) \,|\, \delta(r_1 - r_1') \,|\, \phi_1(U)\phi_{-1}(U) \rangle} \tag{8}$$

Where G is the usual constant pairing force strength in either nucleus 1 or 2 and a delta potential form for the coupling Hamiltonian is assumed. In order to evaluate (8) one has to first solve for the single particle wave functions that are localyzed in wells 1 and 2 as a function of U and thus of t

To solve for the single particle wave functions that are localized in well 1 and 2 one has to first divide the set of different wave functions into a gerade (g) and ungerade(u) ones[6] Then by appropriate linear combination of the g and u functions one could achieve localization. However if the barrier is situated at a point which divides the nuclear potential well asymmetrically, as needed in fission, then the above prescription is not useful. As a matter of fact there is no known, nonperturbative, method that could be employed here save numerical calculation. Thus we shall make a very crude approximation and treat the asymmetric fission with a V(t) calculated for the symmetric fission case. Correction for the derived time-dependence due to the asymmetrically placed necking-in potential barrier can be estimated easily.

If the necking-in potential barrier (n.p.b.) is placed in the center of the total nuclear potential well then previous work[6] on heavy ion reactions indicates that the time dependence is of the form

$$\frac{V}{G} \simeq e^{-\hat{\alpha}(t)(\text{constant})}$$

$$\hat{\alpha} \cong \frac{1}{2}\sqrt{\frac{2m\,U(t)}{\hbar^2}} \tag{9}$$

$$(\text{constant}) = (\ell_c - R_1 - R_2)$$

Since U(t)=ct i.e. assuming a constant velocity in the construction of the n.p.b., one obtains a time dependence

$$\frac{V(t)}{G} \simeq e^{-\hat{\alpha}' t^{1/2}} \tag{10}$$

The quantities appearing in (10) are the parameters of the potential well of the total nuclear system. The only parameter remaining is the velocity parameter c. One could estimate this by realizing that after the elapse of a time t_f the system fissions thus the barrier $U(t_f)$ becomes roughly equal to the depth of the potential well,

$$|U(t_f)| = ct_f = |U_0|$$

Thus

$$c = \frac{U_0}{t_f} \simeq 1 \times 10^{23} (sec^{-1})\ Mev$$

An estimate for $\hat{\alpha}'$ can thus be made

$$\hat{\alpha}' \simeq 10^{11} (\ell_c - R_1 - R_2)\ sec^{-1}$$

Where ℓ_c is the critical elongation of the compound nucleus and R_1 and R_2 are the radii of the two fragments respectively. Thus $\hat{\alpha}'$ is of the order 10^{11} sec^{-1}.

THE TUNNELING PROBLEM

As has been shown above the Josephson tunneling amounts to solving the following time-dependent Schrodinger equation:

$$\left[H_1(t) + H_2(t) - V(t)\sum_{1,2}\left[b_1^{\dagger} b_{-1}^{\dagger} b_2 b_{-2} + H.C.\right]\right]\Psi(t) = i\hbar\dot{\Psi}(t) \qquad (11)$$

In the absence of V(t) the solution of (11) is that of two non-interacting BCS fluids affected by the time dependence of the single-particle energies in the two well. Thus

$$\left(H_1(t) + H_2(t)\right)\Phi_n(t) = E_n(t)\,\Phi_n(t) \qquad (12)$$

where n refers to a certain splitting of the mass of the fissioning nucleus i.e. $n = A_1 - A_2$ and $E_n(t) = E^{(1)}_{BCS}(t) + E^{(2)}_{BCS}(t)$ Here we are assuming that throughout the descent from saddle to scission until the final mass and volume splitting the pairing correlation in the two fragments is strong enough to warrant a BCS treatment.

The wave function of the system is the sum of all the different possible mass splitting of the system weighted by the usual energy factor $\exp[-\frac{i}{\hbar}\int^t E_n(t')\,dt']$

$$\Psi(t) = \sum_n f_n(t) \exp\left[-\frac{i}{\hbar}\int^t E_n(t')\,dt'\right] \Phi_n(t) \tag{13}$$

The Schrodinger equation (11) thus becomes:

$$\dot{f}_n(t) = +\frac{i}{\hbar}\sum_n f_n(t) \exp\left[-\frac{i}{\hbar}\int^t \left(E_m(t') - E_n(t')\right) dt'\right] \times \left(\Phi_n(t),\ H_I(t)\,\Phi_m(t)\right) \tag{14}$$

where we have made the approximation that ($\Phi_n(t)$, $\dot{\Phi}_m(t)$)=0 This approximation is valid as long as the single particle levels in the two wells do not change very much in a time interval of the order of the characteristic two particle transfer time.

The matrix element (Φ_n, $H_I\,\Phi_m$) can be calculated using the following observations:

$$\Phi_n = \Phi_{BCS}(A_1 + n)\,\Phi_{BCS}(A_2 - n)$$

where (15)

$$\Phi_{BCS}(A_2 \pm n) = \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{i\phi(A \pm n)}\, \Phi_{BCS}(|\Delta_A| e^{i\phi})$$

(A_2-n) system exhibits pairing rotational behaviour whereas the (A_1+n) system exhibits a pairing vibrational behaviour i.e. the nuclear state may be considered to be composed of a few pairing phonons. Thus

$$\phi(A_1+n) = \frac{B_1^+ B_2^+ \cdots B_x^+}{\sqrt{x!}} |0\rangle \tag{16}$$

where B^+ is the pairing boson creation operator i.e. it merely adds two paired neutrons when acting on the "vacuum" $|0\rangle$ i.e. the closed neutron shell.

Thus:

$$(\Phi_n, H_I \Phi_m) = V(t)\Big[\delta_{m,n-1}\sqrt{n}\,\Big|\frac{\Delta}{G}(n-1)\Big| + \delta_{m,n+1}\sqrt{n+1}\,\Big|\frac{\Delta}{G}(n+1)\Big|\Big] \tag{17}$$

Where the integral over the gauge angle is performed giving just the cronicker delta functions $\delta(m - n + 1)$ and $\delta(m-n-1)$ respectively. One should realize that the factor Δ/G which is the effective degeneracy is a function of time.

$$\dot{f}_n(t) = \frac{i}{\hbar}\Big\{f_{n+1}(t)\exp\Big[-\frac{i}{\hbar}\int^t (E_{n+1}(t') - E_n(t'))\,dt'\Big]\sqrt{n+1}\,\Big|\frac{\Delta}{G}(n+1;t)\Big| + f_{n-1}(t)\exp\Big[-\frac{i}{\hbar}\int^t \big[E_{n-1}(t') - E_n(t')\big]dt'\Big]\sqrt{n}\,\Big|\frac{\Delta}{G}(n-1;t)\Big|\Big\} \tag{18}$$

If we expand the energies in n (assuming n is small compared to A_1 and/or A_2)

$$E_{n+1}(t') - E_n(t') \simeq \frac{\partial E_n(t')}{\partial n} \equiv \lambda_2(t')$$

$$E_{n-1}(t') - E_n(t') \simeq -\frac{\partial E_n(t')}{\partial n} \equiv -\lambda_2(t') \tag{19}$$

where λ_2 (t') is the Q-value for the transfer of a pair. It should now be clear that, to within the approximation one may set the energy differences independent of time. The energies themselves are obviously time-dependent. Equation (18) is the desired tunneling equation which may be solved for fn(t) to get the probability of transffering 2n neutrons after the elapse of a long enough time namely $t=t_f$

$$P_n = \left| f_n \ (t=t_f) \right|^2 \tag{20}$$

The average number of neutrons transferred during this time, $2 < n> (t)$, reduces to a value equals to the argument of f_n if t_f is infinite.

One can also construct the Josephson current associated with the transfer $J(t) = \frac{d}{dt} \langle \Psi(t) | \hat{N}_1 | \Psi(t) \rangle$

$$J(t) = \frac{d}{dt} \sum_n n \left| f_n(t) \right|^2 + c.c.$$

or

$$J(t) = \frac{1}{2} \sum_n f_n^*(t)\, n\, \dot{f}_n(t) + c.c. \tag{21}$$

All of the above quantities can be obtained easily and directly from equation (18).

SCHEMATIC STUDY OF THE TUNNELING EQUATION:

In order to gain some insight into the nature of the solution of equation (18) we shall make a number of simplifying approximation that would serve to make equation (18) amenable to analytical solution .

We shall assume that $\frac{\Delta}{G}$ is independent of time and so is $\lambda_n(t) \rightarrow \lambda_2 \equiv \lambda$. Thus by writing for $f_n(t)$

$$f_n(t) = e^{in\lambda t/\hbar}\, g_n(t) \tag{22}$$

one arrives at the following:

$$\dot{g}_n(t) + \frac{i}{\hbar} n\lambda\, g_n(t) = \frac{i}{\hbar} V(t)\Big[\sqrt{n+1}\,\frac{\Delta}{G}(n+1)\, g_{n+1}(t) + \sqrt{n}\,\frac{\Delta}{G}(n-1)\, g_{n-1}(t)\Big]$$

$$\simeq \frac{i}{\hbar} V(t)\sqrt{n}\,\frac{\Delta}{G}(n)\big[g_{n+1}(t) + g_{n-1}(t)\big] \tag{23}$$

Writing for $g_n(t) = g(n,t) = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{in\phi}\, \widetilde{g(\phi,t)}$

approximating $V(t)\sqrt{n}\,\frac{\Delta}{G}(n) = V_e(t)$ which is independent

of n. This last approximation is quite reasonable since in all cases $n \ll A_1$ or A_2.

Thus one finally obtains the simple first order partial differential equation for the Fourier coefficients

$$\frac{\partial \widetilde{g(\phi,t)}}{\partial t} - \frac{\lambda}{\hbar}\frac{\partial \widetilde{g(\phi,t)}}{\partial \phi} = \frac{2i}{\hbar} V_e(t)\, \widetilde{g(\phi,t)} \cos\phi \tag{24}$$

The approximation involving the derivation of equation (24) implies that one is neglecting a small term in the otherwise exact equation (24). This small term is proportional to $\left(\frac{2n}{A}\sin\phi\right)$ which is certainly very small compared to the other three terms in (24).

We attempted to find an analytical solution for (24) but we failed. Leaving a numerical study of this tunneling equation for a future publication we shall indicate in the sequel some general features expected of the solution. Thus we shall start by assuming that G varries very slowly in the time interval of interested and therefore rendering $\frac{\partial G}{\partial t} \sim 0$ .One finds immediately the following approximate solution for the amplitude g_n at $t=t_f$

$$g_n(t=t_f) = J_n\left(2 \frac{V_e(t_f)}{\lambda}\right) \tag{25}$$

where J_n is the ordinary Bessel function of order n and the argument is given by $2V_e(t_f)/\lambda$

The probability for transferring 2n nucleons from the light to the heavy fragment dur ing the descent from the last saddle point to the scission point is thus obtained from the approximate solution (25)

$$P_n = \left|J_n\left(\frac{2V_e(t_f)}{\lambda}\right)\right|^2 \tag{26}$$

Therefore the average number of transferred nucleons obtains

$$\langle n \rangle = \frac{\sum_n n \left| J_n\left(\frac{2V_e(t=t_f)}{\lambda}\right)\right|^2}{\sum_n \left| J_n\left(\frac{2V_e(t=t_f)}{\lambda}\right)\right|^2} \tag{27}$$

Using an estimate value of $2\frac{V_e(t_f)}{\lambda} \simeq 0.86$ and taking in the sum over n in (27) six terms we obtain for $\langle n \rangle$ the value 0.8 i.e. only two nucleons are transferred. We feel that this result is encouraging considering the crudeness of the approximations involved and the simplifying assumptions used to derive the tunneling equation (24). The fact that one does get the right trend of the transfer should also be considered an encouraging factor that warrants further study of the model suggested in this work.

Although we have been discussing only the transfer of neutrons in the past-saddle-point stage of the fission process proton transfer could also contribute. As a matter of fact the tendency toward even-Z division is a strong indicator of the importance of proton pairing.[7] The inclusion of the isospin degree of freedom is thus important but will necessarily complicate the simple model considered here. Any attempt for quantitative discussion of this problem is bound to demand considering the transfer of both neutrons and protons.

DISCUSSION AND CONCLUSIONS:

Among the points we stressed upon in this paper we cite the most important :

1. In deriving the tunneling equation (24) the following simplyfying assumptions were made (we mention these in this section in order to indicate the points where one could make improvements):

a) In deriving the time dependence of the tunneling potential V(t) we assumed that the necking-in barrier is formed in the middle of the well although it should be placed off center to simulate asymmetric fission. We feel, however, that this approximation does not change the qualitative aspects of the problem.

b) single-particle aspects (shell effects) were not taken into account explicitly except for their effect on the pairing correlations. Considering single particle energies and wave functions in a deformed well (Nilsson orbital) will certainly modify quantitatively the results

c) We have assumed adiabaticity through ont this way we were able to expand the total wave function in an adiabatic basis Implicit in this approximation is the neglect of all viscous effects.

The general view these days, which we share, is that viscosity in the nuclear collective motion associated with the fission process is not so great and thus its effects, although important from the point of view of numerical comparison with experimental data on the conclusion of this work is mild. Of course the mere fact that pairing correlations among nucleons play an important role in determining the details of the final mass and charge division of the fissiong nucleus is an obvious indication of the less-than-drastic effect of viscosity.

2. Changing the mass of the fissioning nucleus would result in a final mass split when only the mass of the most probable light fragment is affected by this increase. This is so since the pairing correlation among the nucleons would not be affected so much thus inducing tunneling in exactly the same manner except that the light fragment is heavier. Thus during a time interval corresponding to the saddle-scission transition (this time we believe is also not affected by the increase in the mass of the fissioning nucleus)

roughly the same number of nucleon pairs will tunnel from the light to the heavy fragment (which has a most probable mass of A=132 at the saddle point in the case of ^{236}U compound system). Therefore we believe that our simple tunneling model could qualitatively explain this second feature of the mass split which is observed experimentally namely that only the peak in the mass spectrum corresponding to the light fragment is shifted to a position corresponding to a heavier mass, by an amount corresponding roughly to the increase in the mass of the fissioning nucleus itself.

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