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CAN THE EFFECTIVE POTENTIAL DETERMINE WHETHER A GIVEN THEORY EXHIBITS CONFINEMENT ?

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ABSTRACT

Recently it was pointed out that a classical field theory, whose Lagrangian has a term $\phi^2 \ln \phi^2$ exhibits bags and confinement. We conjecture that, when dealing with a quantum field theory, an inspection of the logarithmic terms of its effective potential may decide whether it exhibits (or not) confinement. Inspiring ourselves on classical field theory we are able to establish rules of confinement. These rules are then applied, with success, in various models whose effective potentials we know (in the one loop approximation).

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1. INTRODUCTION

Recently[1,2] I. Ventura and G.C. Marques observed that a certain class of classical field theories - which were called logarithmic theories[1-5] - exhibit the phenomenon of confinement. The relativistic logarithmic theory is defined by the following Lagrangian density

$$\mathcal{L}(\vec{x},t) = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) \quad , \tag{1.1}$$

where ϕ is a charged scalar field and the potential $V(\phi)$ is given by

$$V(\phi) = \left(\frac{1}{\lambda^2} + \frac{1}{\ell^2}\right)|\phi|^2 - \frac{1}{\ell^2}|\phi|^2 \ln\left(|\phi|^2 a^{d-1}\right) ; \tag{1.2}$$

a, ℓ and λ being dimensional parameters and d the number of spatial dimensions.

The effect of confinement - which will be discussed below - is caused just by the logarithmic part of the potential[2], namely by the term

$$- \frac{1}{\ell^2}|\phi|^2 \ln\left(|\phi|^2 a^{d-1}\right) \quad . \tag{1.3}$$

On the other hand it is known[6-9] that the effective potential of certain bosonic theories - when computed in the one loop approximation - also contains logarithmic terms which are similar, though not exactly equal, to (3).

As in the classical theory defined by the Lagrangian (1.1), confinement is due uniquely to the logarithmic piece of the potential, it is natural to conjecture that a <u>quantum</u> field theory whose effective potential displays a term like (1.3) should also exhibit confinement - maybe an inspection of the logarithmic terms that appear on the effective potential of a given theory could determine whether it exhibits (or not) confinement. The aim of this paper is to show, with example, that this seems really to be the case. We will establish some criteria that decide whether or not a particular logarithmic term of an effective potential leads to confinement.

Coleman and Weinberg[6] have shown that, in a given field theory, the singularities of the effective potential at the origin of the classical field space come from the infrared divergences of the theory. We know, however, that confinement is generated just by those infrared divergences. Then, it is reasonable to expect that it has much to do with the singularities (logarithmic terms) of the effective potential.

Let us first review the reasons why terms of the type(1.3) imply in confinement within the classical theory. In reference (2) it is shown that in the relativistic logarithmic theory it is impossible to have a plane wave solution for the field ϕ describing states of finite charge and finite energy. Due to the logarithmic singularity (1.3) one cannot excite the vacuum ($\phi = 0$)[10] of the theory. This means that the basic field - by means of which one builds the Lagrangian (1) - is not directly associated to particles that can be seen at freedom. Or, in other words, the "elementary particle" associated to ϕ is confined.

Well, but we must also show <u>where</u> the "elementary particle" is confined to. The classical equation of motion of the relativistic logarithmic theory displays a large class of gaussian soliton-like solutions[3-5]. Among these solitons the simplest ones are[4].

$$\phi_\omega(\vec{x},t) = A(\omega)\exp\left(-i\omega t - \frac{\vec{x}^2}{2\ell^2}\right) \quad . \tag{1.4}$$

In reference (4) it was shown that by adding a small "quantum" fluctuation η about the classical soliton(1.4),in such a way that

$$\phi = \phi_\omega + e^{-i\omega t}\eta \quad , \tag{1.5}$$

the fluctuation should obey the following linearized equation

$$\left(\partial_t^2 - 2i\omega\partial_t - \partial_{\vec{x}}^2 + \frac{\vec{x}^2}{\ell^4} - \frac{d+1}{\ell^2}\right)\eta = \frac{1}{\ell^2}\eta^* . \tag{1.6}$$

The above equation is very similar to that of an harmonic oscillator. This means that our fluctuation is confined to live near the soliton, subjected to an harmonic force. Then, the confinement picture is perfect: The "quantum" excitations can only manifest thenselves in the presence of a soliton. This soliton works as a bag for trapping the "elementary particle" - that is represented by the excitation.

2. PHENOMENOLOGY OF THE LOGARITHMIC TERMS

In this section we will study what is the effect (concerning to confinement) of certain logarithmic terms,that usually occur on the effective potential of quantum field theories containing bosons, when they are put on the potential of a classical field theory. We will treat a classical theory that describes at least one charged scalar meson whose field is ϕ.

Let us define our theory inside a box of volume V (with periodic boundary conditions). Consider the classical state where the charged meson has momentum $\vec{p}$ and charge +1 [11]. This state is described by the wave function

$$\phi_{\vec{p},+1}(\vec{x},t) = \frac{1}{\sqrt{2V\omega}} \exp i(\vec{p}\vec{x} - \omega t) \tag{2.1}$$

If in the thermodynamic limit ($V \to \infty$) the energy $E(\vec{p},+1)$ of the state (2.1) is finite, one concludes that associated to the field ϕ there exist a particle of mass $\sqrt{E^2(\vec{p},+1) - \vec{p}^2}$ and charge +1. Otherwise, if the energy is infinite, one must infer that (since states of infinite energy have no physical meaning) there are no particles, which can be seen at freedom, associated to the field ϕ, i.e: the particles described by ϕ must be confined.

(a) The logarithmic term that implies classical confinement:

Suppose the potential of our classical theory displays a term like (1.3). Its contribution to the energy should be

$$\frac{1}{\ell^2} \ln\left(\frac{2\omega V}{a^{d-1}}\right) \tag{2.2}$$

Of course as V goes to ∞ the above expression diverges, and one must conclude that logarithmic terms of type (1.3) lead to confinement, as the state (p, +1) does not exist.

(b) Logarithmic terms that do not imply classical confinement:

By doing computations analogous to that presented above, the reader may verify by himself that - in the thermody namic limit - the contribution to the energy of each one of the following logarithmic terms is convergent (so, they are supposed not to imply confinement):

$$\pm\ |\phi|^{2+n} \ln |\phi|^2 \qquad (\text{with } n>0) \tag{2.3.a}$$

$$\pm\ |\phi|^{n} \ln (a^2+|\phi|^2) \ (\text{with } n>0 \text{ and } a^2>0) \tag{2.3.b}$$

(c) A logarithmic term that gives a divergent contribution to the energy but does not lead to confinement of the boson:

$$+\ |\phi|^2 \ln |\phi|^2 \tag{2.4}$$

This is just a term of the type (1.3) except for the sign which is changed. Its contribution to the energy is divergent ($-\infty$) but it does not imply confinement. The reason for this is that in a theory where the bosonic potential contains a piece like (2.4), ϕ = 0 is no more the vacuum (because the second derivative of the potential is negative at the origin), and any discussion about confinement around a state which is not the vacuum is meaningless. Hence, what we have here is a case of spontaneous symmetry breaking - not of confinement.

Now we have enough "experimental" data about the implications of the logarithmic terms (in classical field theories) to propose without proof some rules concerning the role of the logarithmic terms that appear in the effective potentials of quantum field theories. By analogy with our classical results we will assume that:

rule (1): "If $V(\varphi_c)$ - the effective potential associated to a bosonic field ϕ on a quantum field theory - contains a term of type (1.3), the elementary particle described by ϕ is confined."

rule (2): "If the logarithmic terms of $V(\varphi_c)$ are all of the types (2.3) or (2.4), the elementary particle is not confined".

3. USING THE RULES

Now let us apply (and test) our rules on field theoretical models where the effective potentials associated to certain bosons are known. In what follows all the effective potentials that will be presented were computed in the one loop approximation.

Let us start with three models that exhibit confinement

(c.1) Bidimensional electrodynamics of massless scalar mesons:

This theory is defined by the following Lagrangian density

$$\mathcal{L}(\vec{x},t)=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}(\partial_\mu-ieA_\mu)\phi^*(\partial_\mu+ieA_\mu)\phi \tag{3.1}$$

where ϕ is a charged scalar boson and A_μ is the photon.

Since this theory is the bosonic counterpart of the Schwinger electrodynamics[12] (where the fermion is confined) we should expect that the charged meson be confined. Let us look at the effective potential associated to ϕ (computed in the Landau Gauge):

$$V_1(\varphi_c) = -\frac{3e^2}{8\pi}\varphi_c^2 \ln \varphi_c^2 + \text{const.}\, \varphi_c^2 \quad , \tag{3.2}$$

In fact, one can see that the first term of $V_1(\varphi_c)$ is of the type (1.3). Then, according to our rule (1), the charged meson should really be confined.

This result provides an important support in favor of our rules for searching confinement.

(c.2) Bidimensional electrodynamics of massive scalar mesons.

Here the effective potential associated to the massive charged meson also displays a term of the type (1.3)[13]. So this meson must also be confined. Since this model is the bosonic analogue of the massive Schwinger Model[14], our conclusion agrees with the results of reference (14). There it was shown that the fermion of the massive Schwinger Model is confined.

(c.3) Bidimensional massless $\lambda\phi^4$ theory:

Its effective potential is

$$V_2(\varphi_c) = -\frac{\lambda}{16\pi}\varphi_c^2 \ln \varphi_c^2 + \frac{\lambda}{4!}\varphi_c^4 + \text{const.}\, \varphi_c^2 \, . \tag{3.3}$$

One observes again the confining logarithmic term. This means that the elementary boson should be confined.

As examples of theories where confinement does not occur we give the following:

(n.c.1) Bidimensional massive $\lambda\phi^4$ theory.

In this model the effective potential is

$$V_3(\varphi_c) = -\frac{\lambda}{16\pi}\varphi_c^2 \ln\left(\varphi_c^2 + \frac{2m^2}{\lambda}\right) + \frac{\lambda}{4!}\varphi_c^4 + \text{const.}\, \varphi_c^2 \, , \tag{3.4}$$

We see that the logarithmic term is of the type (2.3b). Hence, according to rule (2), the boson is not confined.

(n.c.2) Bidimensional massless Yukawa theory with a scalar meson. The interaction Lagrangian of this model is given by

$$-G\bar{\psi}\psi\phi \quad . \tag{3.5}$$

The effective potential associated to the meson field contains a logarithmic part of the form

$$+G\varphi_c^2 \ln \varphi_c^2 \quad . \tag{3.6}$$

The above term is of the type (2.4) (note the plus sign). Then, as discussed in section 2, the meson should not be confined because $\varphi_c = 0$ is not the vacuum. Instead, the meson must acquire a mass through a spontaneous symmetry breaking mechanism.

(n.c.3) Four-dimensional massless $\lambda\phi^4$ Theory:

Here the logarithmic term of the effective potential is[6]

$$\varphi_c^4 \ln \varphi_c^2 \quad . \tag{3.7}$$

It is of type (2.3.a). So, from our rule (2) we conclude that there is no confinement in the $\lambda\phi^4$ four-dimensional theory - a result that is in accordance with the current feeling among physicists.

We think that the six examples discussed in this section have furnished sufficient support to our rules to give then a certain respectability.

4. CONCLUSIONS

Before concluding we want to list some proposals for further investigation and also to rise some conjectures.

- Of course our rules require a proof in the context of Quantum Field Theory. One must show that, if the effective potential associated to a certain bosonic field ϕ displays a term like (1.3), the elementary particle described by ϕ cannot appear in the asymptotic states with which the Fock space of the theory is built.

- The rules that we have proposed in this paper are suitable for a description of bosonic confinement. If we want to study confinement in more realistic theories (where the quarks are fermions), it is necessary to know what kind of logarithmic terms of the <u>fermionic</u> effective potential would lead to confinement. Besides that we need to attack the hard task of computing fermionic effective potential.

- Carrying the analogy with the classical logarithmic theory further one can imagine that in some cases the logarithmic terms of the effective potential may also be related to the existence of bags(that would trap the confined particle). We think that some light can be focused into this question if one looks for soliton-like solutions of the equations of motion derived from the <u>effective action</u> (neglecting,of course,terms with high powers on the derivatives), instead of treating the equations that comes from the original non quantized action.

Finally we want to say that it is quite stimulating the possibility of having a method to verify confinement which deals directly with local theories and that is based upon the well known perturbation theory.

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FOOTNOTES and REFERENCES

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(10) V(0) is not the absolute minimum of $V(\phi)$. The reader who wonders whether this fact could lead to any trouble may add to $V(\phi)$ a term $\lambda|\phi|^4$ - with a sufficien-

tly large λ - in order to guarantee that $\phi=0$ is really the absolute vacuum. This procedure will not introduce any essential change in the confinement phenomenon under consideration.

(11) In a classical theory the charge has a continuous spectrum. We have put Q=1 for reasons of simplicity. Had we choosen any other finite value for Q our conclusions should be the same.

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(13) In order to obtain the various effective potential mentioned in this paper the reader may use the methods of references (6), (8) or (9).

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