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SUB-BARRIER PHOTOFISSION OF ^{238}U

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A B S T R A C T

Photofission cross sections of ^{238}U below threshold have been calculated using a double humped potential barrier parameterized by smoothly joining four parabolas, and a coulomb potential at and beyond the scission point. The primary potential well is made arbitrarily wide in order to reproduce a density of states comparable with that known to exist in the compound nucleus and which manifests itself as fine structure and spreading of the resonances in the transmission coefficient. Relative strength in the fission channel has been calculated and an attempt has been made to interpret the apparent resonance structures observed recently in photofission experiments of ^{238}U as excited states of the fission isomer. We find a set of parameters for a double humped barrier which are consistent with known spontaneous fission and isomeric fission half lives. In addition to reproducing satisfactorily the observed resonance structure near threshold, the calculation also predicts several low energy resonances in the cross sections.

INTRODUCTION

Measurements of photofission cross sections and of the angular distribution of fission fragments are an excellent means of studying the physics of low energy fission. The restricted angular momenta (mostly 1^- and 2^+ for an even-even nucleus) in the entrance channel, permit the study of the relative contributions of various fission channels at different excitation energies. Photofission is particularly useful for studying the shape of the potential barrier below the neutron threshold which, of course, cannot be studied with neutron induced fission.

Bohr and Wheeler¹⁾ and Frenkel²⁾ were the first to give a quantitative account of the fission process in terms of the classic liquid drop model. They envisaged the existence of a potential barrier between divided and coalesced states of the nucleus. The rapid rise in the fission cross section at the energy at which this barrier is surmounted is called the fission threshold, although it is not a threshold in the conventional sense because fission is possible at any energy. For sub barrier fission, the cross sections are determined by the probability of tunneling through such a barrier. The transmission coefficient through a simple barrier, is essentially a smooth increasing function of energy and therefore, in the liquid drop model, one does not expect any structure in the sub barrier cross sections. However, a resonance structure has been observed in this energy region in photofission cross section of ^{238}U , and these results are summarized in Fig.1. In this figure the solid line represents the results of Rabotnov et al.³⁾ using bremsstrahlung; this is a gross resolution measurement and suffers typically from poor knowledge of the bremsstrahlung spectrum needed in unfolding the

cross sections from the measured fission yields. The open circles represent the results of Mafra et al.⁴⁾ using discrete neutron capture γ rays. These γ rays are extremely narrow with widths of the order of few electron volts. One therefore has to exercise caution in identifying this structure with that of the previous experiment because, with this technique, one could be sensitive to the structure in the entrance channel corresponding to excitation of individual compound nuclear states. The dashed and dash-dot curves represent respectively the results of Khan and Knowles⁵⁾ and those of Anderl et al.⁶⁾ using Compton scattered capture gamma rays, a method which has an energy resolution intermediate between the methods previously mentioned. Measurements⁵⁾ with this technique exhibit evidence of resonance structure in photofission cross sections of ^{238}U at 5.2, 5.7, 6.2, 7.1 and 7.8 MeV. The crosses are the results of Dickey and Axel⁷⁾ using "tagged" bremsstrahlung, which is a method which claims better resolution than Compton scattered gamma rays. These measurements appear to confirm the resonances of ref.(5) except the lowest energy (5.2 MeV). The cross section however is significantly larger.

It is difficult to understand the existence of such structure in sub barrier photofission cross sections in terms of the simple liquid drop potential, however, Strutinsky⁸⁾ has suggested the existence of a double humped barrier in fission obtained by adding single particle effects to the collective (i.e. liquid drop) component of the nuclear potential energy. This potential barrier has at least one additional deep potential minimum called the second well, and it appears possible that the above noted resonance structure could be explained in terms of the quasi-stable states in this second well. The best evidence for the second well is the existence of long lived quasi stable states known as fission isomers.

The states we may be observing in photofission may thus be excited states corresponding to shape oscillations about the deformed configuration which is supposedly characteristic of the fission isomer.

PENETRATION THROUGH A DOUBLE HUMPED BARRIER

It has been indicated by Strutinsky and Bjørnholm⁹⁾ that most of the important features emerging from the physical picture of a double humped barrier can be understood in terms of a simple one dimensional calculation of penetrability through such a barrier. For a potential barrier as shown in fig. 2, WKB approximation can be used to calculate the transmission coefficient. Defining T_A, T_B and T as the respective transmission coefficients for the barrier A alone, barrier B alone and the entire barrier, it can be shown¹⁰⁾ that

$$T = T_A T_B / \left\{ [1 + \sqrt{1-T_A} \sqrt{1-T_B}]^2 \cos^2 \nu_2 + [1 - \sqrt{1-T_A} \sqrt{1-T_B}]^2 \sin^2 \nu_2 \right\} \quad (1)$$

where

$$T_A = \left| e^{\nu_3} + \frac{1}{4} e^{-\nu_3} \right|^{-2} \quad (2)$$

$$T_B = \left| e^{\nu_1} + \frac{1}{4} e^{-\nu_1} \right|^2 \quad (3)$$

The quantities ν'_i are the integrals in respective regions as shown in fig. 2 of the phase

$$K_1(x) = \left\{ 2\mu(E - V(x)) / \hbar^2 \right\}^{1/2} = i k_2(x) \quad (4)$$

for example

$$\nu_1 = \int_{a_3}^{a_4} k_2(x) dx, \quad \nu_2 = \int_{a_2}^{a_3} k_1(x) dx \quad (5)$$

and

$$\nu_3 = \int_{a_1}^{a_2} k_2(x) dx$$

The limits a_1, a_2, a_3, a_4 are the respective classical turning points also shown in fig. 2. It is supposed that the coordinate x has been chosen such that the corresponding effective mass μ remains constant throughout the fission process and thus may be taken to be equal to the reduced mass of the resulting fragments. Hofmann and Dietrich¹¹⁾ have shown that for a one-dimensional calculation a coordinate transformation can be used to reduce any hamiltonian with a variable mass to another with a constant effective mass. Further, it has been shown by Cramer and Nix¹²⁾ that for a constant effective mass throughout the fission process, the penetrability becomes independent of the choice of the effective mass.

A relation similar to equation (1) has been reported earlier by Ignatyuk et al.¹³⁾ and Gai et al.¹⁴⁾. Their results are slightly different because they use a further approximation for penetrability through a barrier. For example, if, instead of equations (2) and (3), we were to use

$$T_A = e^{-2\nu_3}, \quad T_B = e^{-2\nu_1} \quad (6)$$

our formula reduces to

$$T = 64 T_A T_B / [(T_A T_B + 16) \cos^2 \nu_2 + 16 (T_A + T_B)^2 \sin^2 \nu_2] \quad (7)$$

which is exactly their expression. Cramer and Nix¹²⁾ have also studied the penetration through a double humped barrier parameterized by parabolic potentials which can be solved exactly by writing the solutions in terms of the parabolic cylinder functions.

BEHAVIOUR NEAR TRANSMISSION RESONANCES

It is shown in ref.10) that the total transmission coefficient T displays a resonant behaviour with a Lorentzian shape in the immediate neighbourhood of quasi-stable states in the second well:

$$T(E_n^0 + \Delta E) = \Gamma_f \Gamma_d / [(E - E_n^0)^2 + \Gamma^2/4] \quad (8)$$

Here, Γ_f and Γ_d are the widths for the formation and for the decay of a quasi-stable state in the second well and are given by

$$\Gamma_f = \left\{ \frac{D_2}{\pi} \left[\frac{(1 - \sqrt{1 - T_A})(1 + \sqrt{1 - T_B})}{(1 + \sqrt{(1 - T_A)(1 - T_B)})} \right] \right\} \quad (9)$$

and

$$\Gamma_d = \left\{ \frac{D_2}{\pi} \left[\frac{(1 - \sqrt{1 - T_B})(1 + \sqrt{1 - T_A})}{(1 + \sqrt{(1 - T_A)(1 - T_B)})} \right] \right\} \quad (10)$$

with the total width $\Gamma = \Gamma_f + \Gamma_d$.

D_2 represents the level spacing in the second well. The resulting peaks at E_n^0 are extremely narrow as will be apparent in fig. 4.

The transmission coefficient as given by eq. (1) attains a maximum value when $\psi_2 = (2n+1)\pi/2$. We then obtain the maximum value of the transmission coefficient as

$$T_{\max.} = T_A T_B / [1 - \sqrt{(1 - T_A)(1 - T_B)}]^2 \quad (11)$$

A perfectly symmetric double humped barrier ($T_A = T_B$) produces perfect transmission, $T_{\max} = 1$. When $\psi_2 = n\pi$ the transmission coefficient attains a minimum value given by

$$T_{\min.} = T_A T_B / [1 + \sqrt{(1-T_A)(1-T_B)}]^2 \quad (12)$$

It is worth noting that although the total transmission coefficient T displays resonances corresponding to the states in the second well, T_{\max} and T_{\min} are smoothly increasing functions of energy—a behaviour very similar to the transmission coefficient of simple barrier. A study of transmission coefficients for symmetric as well as asymmetric double humped barriers has been carried out by parameterizing the potential by smoothly joining three parabolas. Details can be seen in ref.10).

CALCULATION OF FISSION LIFETIMES

The ground state spontaneous fission halflife can be written as

$$\tau_G^{sf.} = (2\pi \hbar^2 / \omega_1 T_G) \quad (13)$$

where $(2\pi/\omega_1)$ is the time required for a single assault on the barrier, $\hbar\omega_1$ being the curvature of the primary well and T_G is the transmission coefficient through the double humped barrier at the ground state energy. Again using the WKB approximation

$$T_G = \left| e^{\nu} + \frac{1}{4} e^{-\nu} \right|^{-2} \quad (14)$$

where ν is the integral across the entire barrier ;

$$\nu = \int_{a_1}^{a_4} k_2(x) dx \quad (15)$$

Decay from the isomeric state, the lowest state in the second well, will depend upon the penetrability through each of the two barriers surrounding the secondary minimum. The main competing processes determining the isomeric half-life are gamma deexcitation and spontaneous fission from the isomeric state. Nix and Walker¹⁵⁾ have recently given an upper limit for the gamma deexcitation half-life,

$$\tau_i^\gamma \leq 10^{-21.5} / T_A \quad (\text{years}), \quad (16)$$

and for the spontaneous fission half-life from the isomeric state,

$$\tau_i^{s.f.} = (2\pi \ln 2 / \omega_3 T_B) \quad (17)$$

Here T_A and T_B are the respective transmission coefficients through the barriers A and B at the isomeric energy, $\hbar\omega_3$ is the curvature of the second well. The total isomeric half-life is then given by 15)

$$\tau_i = \left[(\tau_i^\gamma)^{-1} + (\tau_i^{s.f.})^{-1} \right]^{-1} \quad (18)$$

CALCULATION OF PHOTOFISSION CROSS-SECTIONS

Low energy photofission is considered to be a two-step process: photoabsorption leading to an excited state in the primary well and subsequently tunneling through the fission barrier. The compound nucleus obtained by the absorption of a photon can also decay by gamma emission or, if suitably energetic, by neutron emission. The excited state wave function always represents an unbound state and has been appropriately normalized in the asymptotic region. Using the

quasi-classical approximation and dividing the space as illustrated in fig. 2, we take the wave function in the outgoing region V as

$$\Psi_V = N \left[(t u_+ e^{i\delta} + t^* u_- e^{-i\delta}) / 2|t| \right] \quad (19)$$

where t is the transmission amplitude for a double humped barrier, u_{\pm} are the WKB wave functions

$$u_{\pm} = (k_1(x))^{-1/2} \exp \left[\pm i \int_{a_4}^x k_1(x) dx \right] \quad (20)$$

and δ is the phase shift determined by the requirement of an exponentially decaying wave function in the region left of the primary well. This wave function has the normalization factor given by

$$|N|^2 = 2k_0 / (a_1 - a_0) \quad (21)$$

where a_0 and a_1 are the two classical turning points as shown in fig. 2 which serve to delineate the limits of the wavefunction in the primary well and k_0 is the wavenumber $k_1(x)$ in the asymptotic region,

$$k_0 = (2\mu E)^{1/2} / \hbar \quad (22)$$

Note the increase in the denominator on the right hand side of (21) when the primary well is made arbitrarily wide. It is possible to envisage the stationary wavefunctions such as eq. (19) if we suppose we are feeding back exactly the same current as is leaking through the barrier so as to create an

artificial stationary state. In the primary well (region I) of the double humped barrier simple considerations of penetration through a barrier allow us to write the corresponding wave function denoted by Ψ_I , as

$$\Psi_I = N [e^{i\delta}(u_- + \delta u_+) + \bar{e}^{i\delta}(u_+ + \delta^* u_-)] / 2|t| \quad (23)$$

where δ , t are the reflection and transmission amplitudes for the entire fission barrier. We will also need the transmission coefficient T , given by $T = |t|^2$, where the explicit expression for t is

$$t = \left[\left(\frac{1}{4} e^{-\nu_3} - e^{\nu_3} \right) \left(\frac{1}{4} e^{-\nu_1} - e^{\nu_1} \right) e^{i\nu_2} + \left(e^{\nu_3} + \frac{1}{4} e^{-\nu_3} \right) \left(e^{\nu_1} + \frac{1}{4} e^{-\nu_1} \right) e^{-i\nu_2} \right]^{-1} \quad (24)$$

We now integrate the absolute square of the wavefunction in the primary well region given by (23) between the two classical turning points a_0 and a_1 . This is obtained as

$$\int_{a_0}^{a_1} |\Psi_I|^2 dx = c \int_{a_0}^{a_1} \left[e^{-2iU(x)} e^{4i\nu_4} - e^{2iU(x)} + 2i e^{2i\nu_4} \right] / k_1(x) \cdot dx \quad (25)$$

where

$$U(x) = \int_x^{a_1} k_1(x) dx \quad (26)$$

and

$$\nu_4 = \int_{a_0}^{a_1} k_1(x) dx \quad (27)$$

and c depends upon energy and is given by

$$c = |N|^2 T / \left\{ 4 \left[\sqrt{1-T} \left(e^{-i\delta_R} - e^{i\delta_R + 4i\nu_4} \right) + i e^{2i\nu_4} (2-T) \right] \right\} \quad (28)$$

where $\delta_R = \arg(\mathcal{R})$, is the phase of the reflected component in the primary well region.

Since the probability of raising the nucleus from the groundstate to an excited fissile state is governed by the amplitude of the excited state wavefunction in the primary well we identify the integral of the modulus-squared wavefunction in the primary well, as the probability that, having excited to a primary well state by photoabsorption, the system will fission. An interesting result is obtained when the primary potential well is made arbitrarily wide. This leads to a dense spectrum of resonances corresponding to states in the primary well, superimposed upon the transmission resonances corresponding to the states in the second well. With a suitable width for the primary well these states have a density of states comparable with that known to exist in the compound nucleus, and manifest themselves as fine structure upon the comparatively broader transmission resonances. For illustrative purposes fig. 3 shows such behaviour for a primary potential well ten times wider than the second well.

DAMPING AND BROADENING OF TRANSMISSION RESONANCES

As noted above, the dense spectrum of primary well resonances manifests itself as fine structure upon the transmission resonances. This means that a state in the second well appears as an augmentation of several neighbouring primary well states spanning an energy region which define the width of the transmission resonance. Such states are similar to doorway states.

When one takes an average over a suitable energy interval containing several of these primary well resonances, to isolate the secondary structure, the results show a broadening of the

transmission resonances when compared with the simple transmission coefficient peaks; an example is shown in Fig. 4. It is worth noting here that damping (as opposed to spreading) of transmission resonances have been simulated recently^{16,17)} by adding an imaginary part to the potential in the second well. The use of complex potential artificially takes care of the leakage of probability current from fission to other degrees of freedom which manifests itself as the damping width, while our method artificially introduces the effects known as the spreading width of the resonance. The photofission cross section below the neutron threshold is related to the photoabsorption cross section by

$$\sigma_{\gamma,f}(E) = \langle \Gamma_f / (\Gamma_f + \Gamma_\gamma) \rangle \sigma_{\gamma,abs.}(E) \quad (29)$$

Γ_f and Γ_γ are the widths for fission and gamma deexcitation and are related to the transmission coefficients in their respective channels by

$$\langle \Gamma_i \rangle = \langle D \rangle T_i / 2\pi \quad (30)$$

where i represents the individual channel for deexcitation of the excited state in the primary well. Thus in order to calculate the photofission cross sections, one needs to know the photoabsorption cross section and the relative strength present in the fission channel.

PHOTO-ABSORPTION CROSS-SECTION

Reliable data on the cross section for dipole and quadrupole photoabsorption at low energies are not available. However, Axel¹⁸⁾ has estimated the total photonuclear dipole absorption

cross section of heavy elements near 7 MeV as

$$\bar{\sigma}_{\gamma, \text{dip}}(E) = 5.2 (E/7)^3 (0.01A)^{8/3} \text{ mb.}, \quad (31)$$

and below 3 MeV as:

$$\bar{\sigma}_{\gamma, \text{dip}}(E) = 3.8 (E/7)^2 (0.01A)^{7/3} \text{ mb.} \quad (32)$$

In eqs. (31) and (32) E is the photon energy in MeV and A is the mass number of the nucleus. Huizenga and Britt¹⁹⁾ have recently compared the values obtained from eq. (31) with the absorption cross sections obtained by extrapolation to lower energies from the photon cross sections of ^{232}Th and ^{238}U measured recently by Veyssiere et al.²⁰⁾, and find reasonable agreement although the extrapolated cross sections are slightly smaller than those deduced from equation (31).

The ratio of the quadrupole to dipole absorption cross section is approximately equal to 0.02 for low energy gamma rays^{19,21)}. Therefore, one might expect the photoabsorption cross section to be given by the dipole absorption cross section (eq. (31)), in the absence of any significant enhancement (e.g., a quadrupole absorption resonance) in the energy region of interest. We have therefore used equation (31) to represent the total photoabsorption cross section.

From equations (29) and (30), it is seen that in addition to photoabsorption cross section, we need to know the transmission coefficients corresponding to fission and gamma de-excitation channel. As mentioned earlier, the integral of the modulus-squared wavefunction in the primary well region has been identified as the intrinsic probability that having excited to a primary well state by photoabsorption, the system

will fission. A moving average over the fine structure of this quantity with an averaging interval of 300 keV represents the transmission coefficient in fission channel. For the transmission coefficient for gamma deexcitation for an even-even heavy nucleus a result has been given by Vandebosch and Huizenga²²⁾ and is shown in fig. 5. We find that the fission cross sections are then overestimated as shown by the dashed dot curve in fig. 6 and that we need a different behaviour for T_γ to obtain cross sections comparable to those measured in the threshold region as shown in fig. 5. Knowles and Mafra²³⁾ have recently found evidence for the existence of a peak in photoabsorption cross section of ^{238}U near 6 MeV. Also the results on angular distribution measurements³⁾ of photofission fragments of ^{238}U are known to indicate an enhancement of quadrupole fission in this energy region. Whether it is simply due to channel effects or due to any possible enhancement in quadrupole photoabsorption in this energy region is not clear.

The halflife of ^{238}U shape isomer as obtained in our calculation is equal to 191 ns which is in good agreement with the measured value of $(195 \pm 30)\text{ns}$ ²⁴⁾. The ground state spontaneous fission halflife of ^{238}U obtained in our calculation is 6.88×10^{15} yrs. which is also in good agreement with the measured value of $(6.5 \pm 0.3) \times 10^{15}$ years²⁵⁾.

The calculated photofission cross sections near threshold are compared in Fig.6 (solid line) with those measured in ref.5 (dashed line). The resonances are reproduced satisfactorily. However, the theoretical curve shows the individual peaks more distinctly than those measured. Comparison with the tagged bremsstrahlung data of ref.(7) is shown in Fig(7). To account for the larger cross section we have adopted a different value for T_γ (see Fig.5). With this modification the structure is very well reproduced with the exception of the 5.2 MeV peak. Since the

peak above 6.1 MeV is above the neutron binding energy, inclusion of a neutron emission channel in our calculation should lead to a broadening of this peak and thereby provide a better fit with that measured⁵⁾. Since the peaks shown in the cross section would correspond, in the present interpretation, to the 5th, 6th and 7th excited states of the fission isomer with a ground state at 2.6 MeV, it is natural to look for the lower excited states also; Fig.(8) shows peaks corresponding to these states. Since a sensitivity for fission in the range of 10^{-11} to 10^{-10} barns is apparently within experimental capabilities²⁶⁾, measurements in this region are interesting in order to look for this resonance structure.

We would like to comment on the observations of Bowman et al²⁶⁾ and Zhuchko et al²⁷⁾ concerning the cross section in the region well below the top of the barrier. These experiments show the presence of a "shelf" in the cross section (see Fig.(8)) meaning an abrupt decrease in the slope, near 4.2 MeV. This is attributed to delayed fission on the assumption that at these energies γ decay to the fission isomer (ground state) is competitive with decay by fission direct from the second well. Since our model contains no explicit provision for γ decay in the second well we have made some estimates based upon penetration of the double humped barrier with an absorptive part in the potential in the region of the second well. The calculation again uses WKB approximation. Using the terminology of eqs.(2)-(7) with the added feature that the phase v_2 now has added an imaginary part $i\epsilon$, expressions for the transmission coefficient for the double barrier, and for the "loss" in the second well are found to be

$$T = \left[T_A T_B / \left\{ e^{2\epsilon} + 2\sqrt{(1-T_A)(1-T_B)} \cos 2v_2 + (1-T_A)(1-T_B) e^{-2\epsilon} \right\} \right]$$

and

$$L = T \left(\frac{1}{T_B} e^{2\epsilon} - \frac{(1-T_B)}{T_B} e^{-2\epsilon} - 1 \right) \quad (34)$$

Assuming that a constant fraction κ (using the notation of ref. (27)) of the flux loss L , eventually contributes to fission from the isomeric ground state (and therefore indistinguishable from prompt fission in the experiments described) we are led to add a delayed fission contribution,

$$T' = T + \kappa L \quad (35)$$

in our calculation of transmission coefficient. We note, however, that the resonance like structure in the transmission coefficient is not diminished by this procedure because this peculiar energy dependence is exhibited by the function T , which is also a factor in the expression for L . This structure is furthermore not seriously effected by any reasonable estimate for the imaginary phase ϵ (although this factor does have a broadening effect as is to be expected). It appears that, when the excitation energy is appropriate for the transmission resonance, there is a large amplitude for the wave function in the second well which augments both fission and γ -decay equally. Thus the fission output, whether direct or delayed, is amplified at the resonance energy. In figure (9) we show a sample calculation of T and T' which displays the "shelf" of refs (26) and (27). For this calculation we have chosen an energy dependence for ϵ of the form

$$\epsilon = \left((E - E_i) / 6 \right)^2 \quad (36)$$

This dependence is rather arbitrary although, within reasonable limits, the details of the assumed behaviour are not specially significant in this calculation. We have also shown in Fig(8) our earlier fission cross section augmented by the same factor(T'/T) (dotted line). This display is indicative only, and intended to show that the residual structure should be readily observable on top of the shelf. The data taken in this region shows no structure⁽²⁶⁾, but the same experiment reveals no resonance even in the region above 5 MeV where the other measurements (discussed in the main part of this paper), show considerable evidence for structure.

The barrier parameters of a double humped potential barrier of ^{238}U yielding the above halflives and cross section in this calculation are as follows: primary well curvature, $\hbar\omega_1 = 1.20$ MeV, second well curvature $\hbar\omega_3 = 0.50$ MeV, inner barrier curvature $\hbar\omega_2 = 2.30$ MeV, outer barrier curvature $\hbar\omega_4 = 0.98$ MeV, inner barrier height $E_A = 6.50$ MeV, outer barrier height, $E_B = 6.65$ MeV, isomer excitation energy $E_i = 2.60$ MeV. Since we have considered the 6.2 MeV peak as a state in the second well, our barrier heights are greater than those estimated recently by Back et al.⁽²⁸⁾. Our barriers are also narrower than ref.(28) in order to reproduce the correct halflives. Recent results of angular distribution measurements^{23,29)} provide evidence that 6.2 MeV peak is a photofission resonance. The peak is also narrower than what would be expected for a resonance above the barrier. These facts provided the rationale for including this resonance as a state in the second well, in the present calculation. The isomer excitation energy obtained in our calculation is also in reasonable agreement with that obtained by Russo et al.⁽³⁰⁾.

CONCLUSIONS

The purpose of the present investigation was to examine the process of photofission near and below threshold and to try to explain the observed resonance structure in subbarrier photofission cross sections of ^{238}U in terms of the quasi-stable states in the second well of a double humped barrier in fission. We have also attempted to determine whether a single set of parameters for the double humped barrier are consistent with a number of experimental data taken together: resonances in the photofission cross sections, isomeric half-life, and the ground state spontaneous fission half-life of ^{238}U .

Our results as presented above indicate that fission half-lives and cross section peaks are satisfactorily reproduced. Our calculation also predicts several low energy resonances in photofission cross sections of ^{238}U , the resulting cross sections however are extremely small. In a parabolic potential parametrization as used in our calculation, the spacing between sub-barrier fission resonances provides direct information about the curvature of the second well. It is also necessary to have some direct measurements of photoabsorption cross sections at low gamma energies to determine if any gross resonance structure is also present in photoabsorption cross section.

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FIGURE CAPTIONS

- Fig. 1 : Measurements of photofission cross sections for ^{238}U . The solid line represents the results of Rabotnov et al.³⁾; the dashed curve is the result of Khan and Knowles⁵⁾ and dash-dot curve is from Anderl et al.⁶⁾. The crosses represent the results of Dickey and Axel⁷⁾. Open circles are points from Mafra et al.⁴⁾ using gamma rays from neutron capture on various elements as labeled in the diagram.
- Fig. 2 : An illustration of a double humped potential barrier showing classical turning points.
- Fig. 3 : Dashed curve is modulus-squared wavefunction integrated over primary well (eq.25). The solid curve is the transmission coefficient. The peaks in both curves are shown with open tops meaning that no attempt was made to identify the maxima of these narrow resonances.
- Fig. 4 - Average over the fine structure of primary well states (dashed curve) compared with transmission coefficient (solid curve).
- Fig. 5 : Photon transmission factor for an even-even nucleus. The dashed curve is from ref.(19); solid and dash-cross curves are used in the present work to reproduce photofission of ^{238}U measured in refs.(5) and (7) respectively. (see text).
- Fig. 6 : Photofission cross sections of ^{238}U near threshold. Dashed curve is the cross section measured in ref.(5); dash-dot and solid curves are calculated using T_γ shown by the dashed and solid curves in Fig.(5), respectively.

Fig. 7 : Photofission cross sections of ^{238}U near threshold. The crosses are the results of Dickey and Axel⁷⁾. The solid line is calculated using T_γ shown by the dash-cross curve in Fig.(5).

Fig. 8 : Sub-barrier photofission cross sections of ^{238}U . Dash-circle curve represents the measurements of ref.(3); dash-dot curve is the result of measurements indicating a "Shelf" in the low energy region²⁶⁾. Calculated sections are shown for prompt (solid curve) and prompt-plus-delayed fission (dashed curve).

Fig. 9 : ^{238}U transmission coefficients for prompt (solid curve) and prompt-plus-delayed fission (dashed curve).

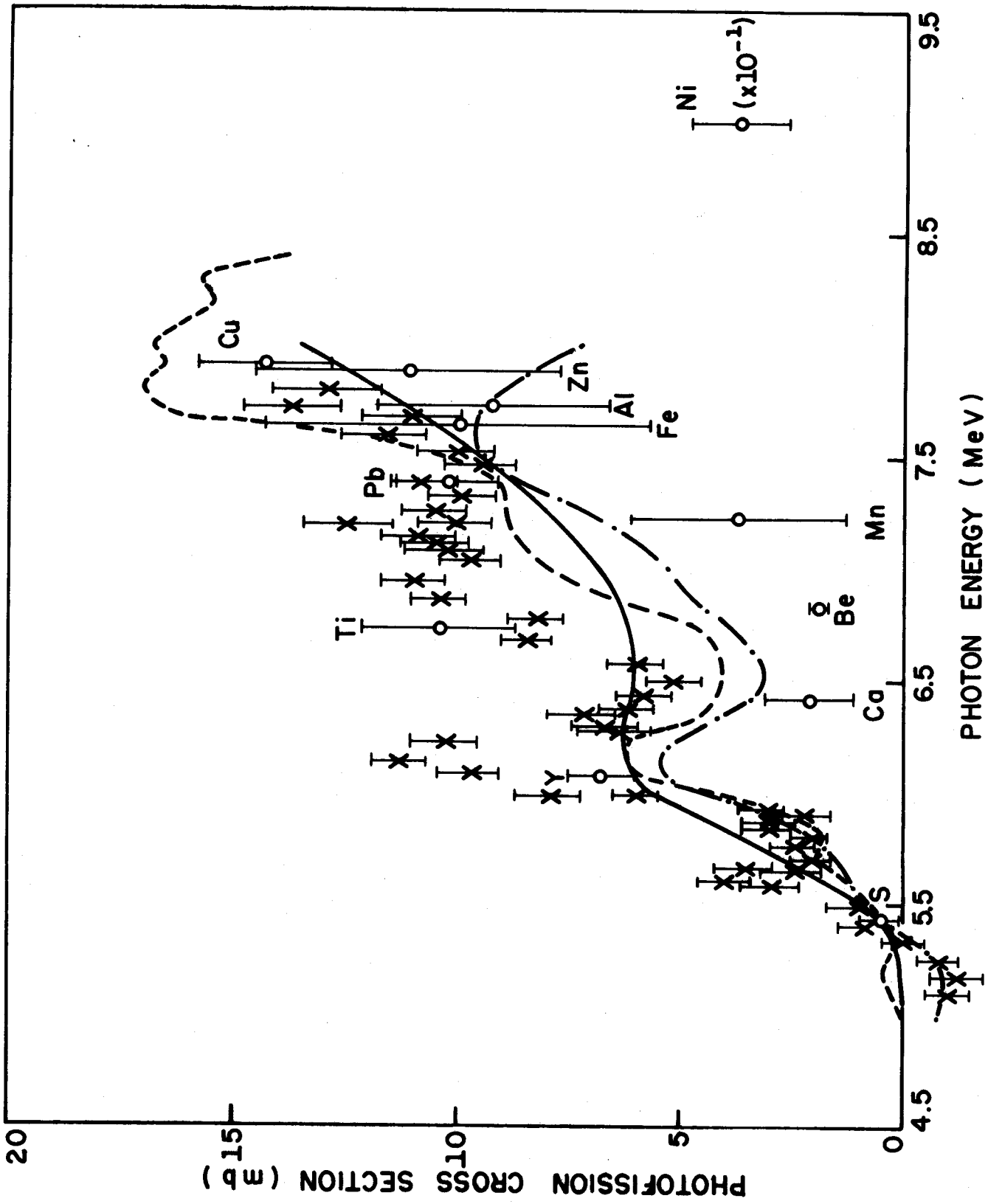


FIG. 1

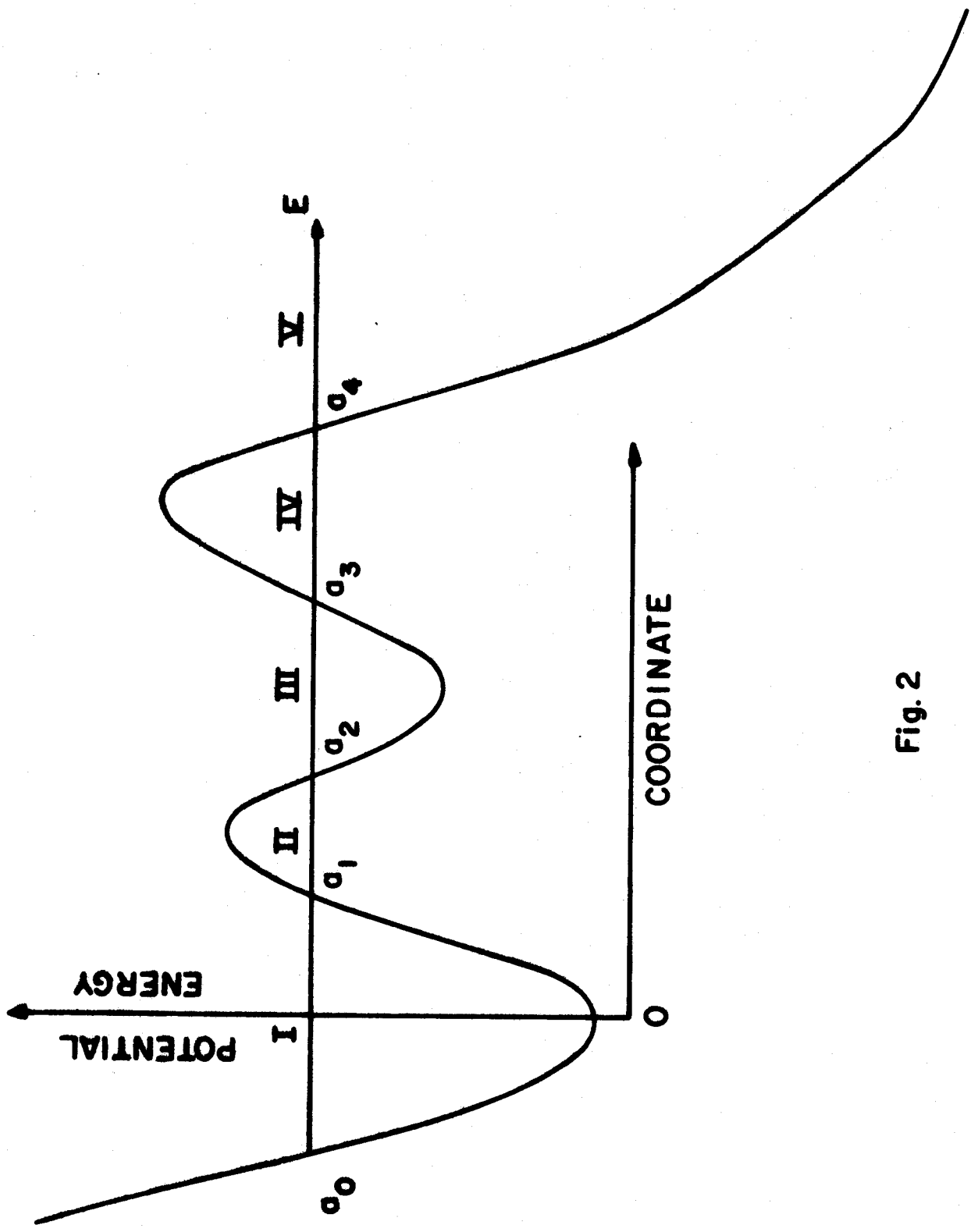


Fig. 2

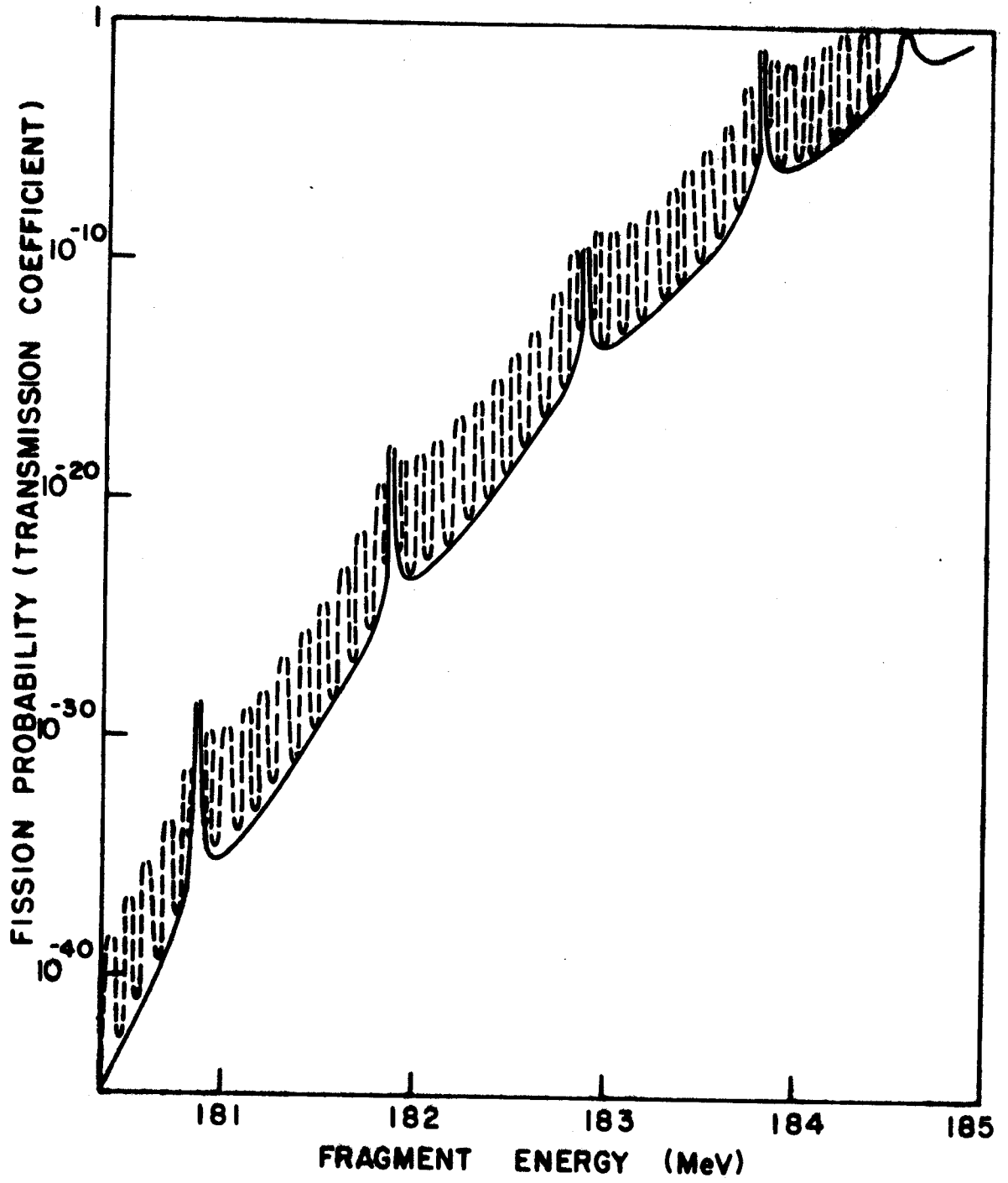


Fig. 3

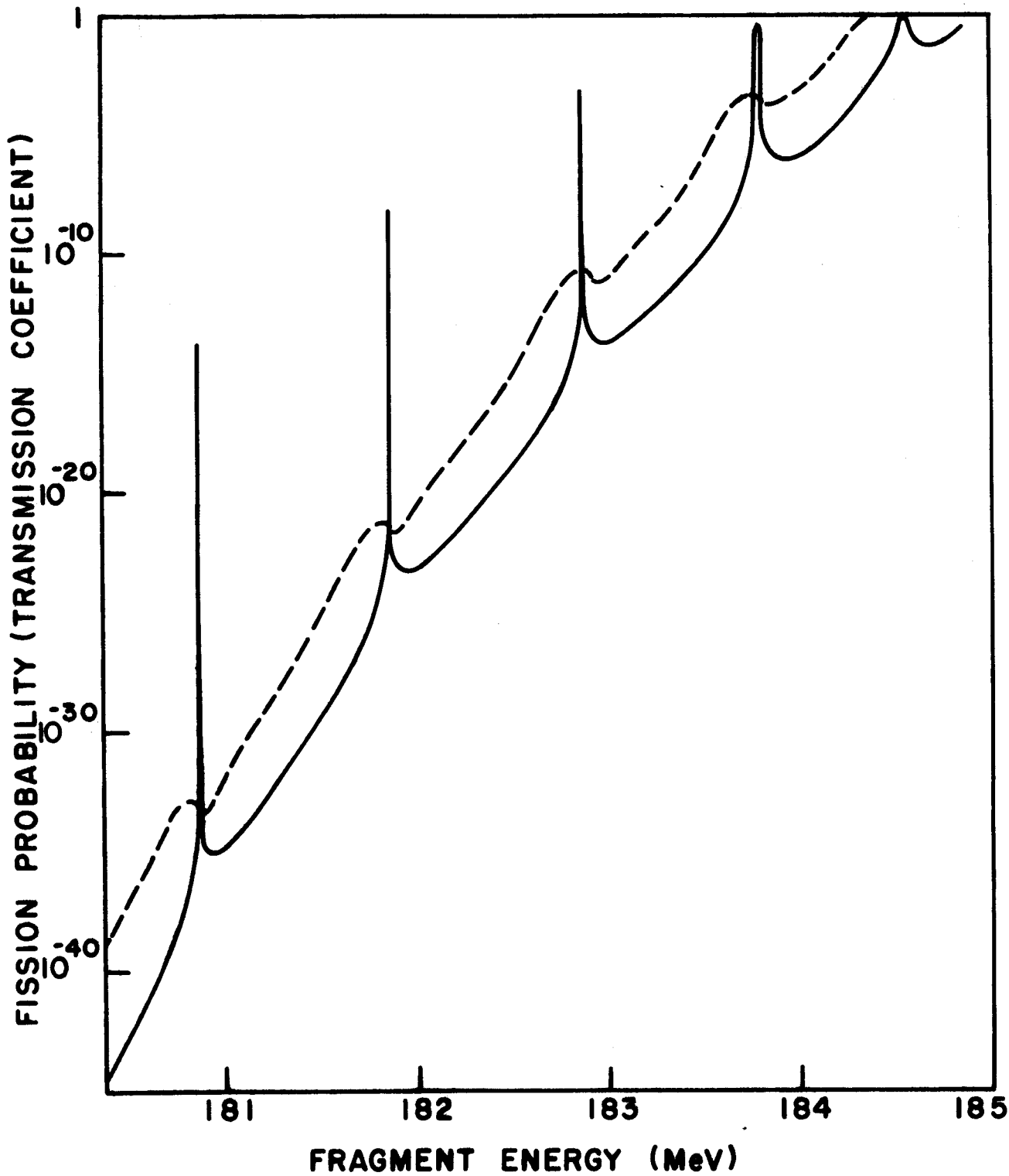


Fig. 4

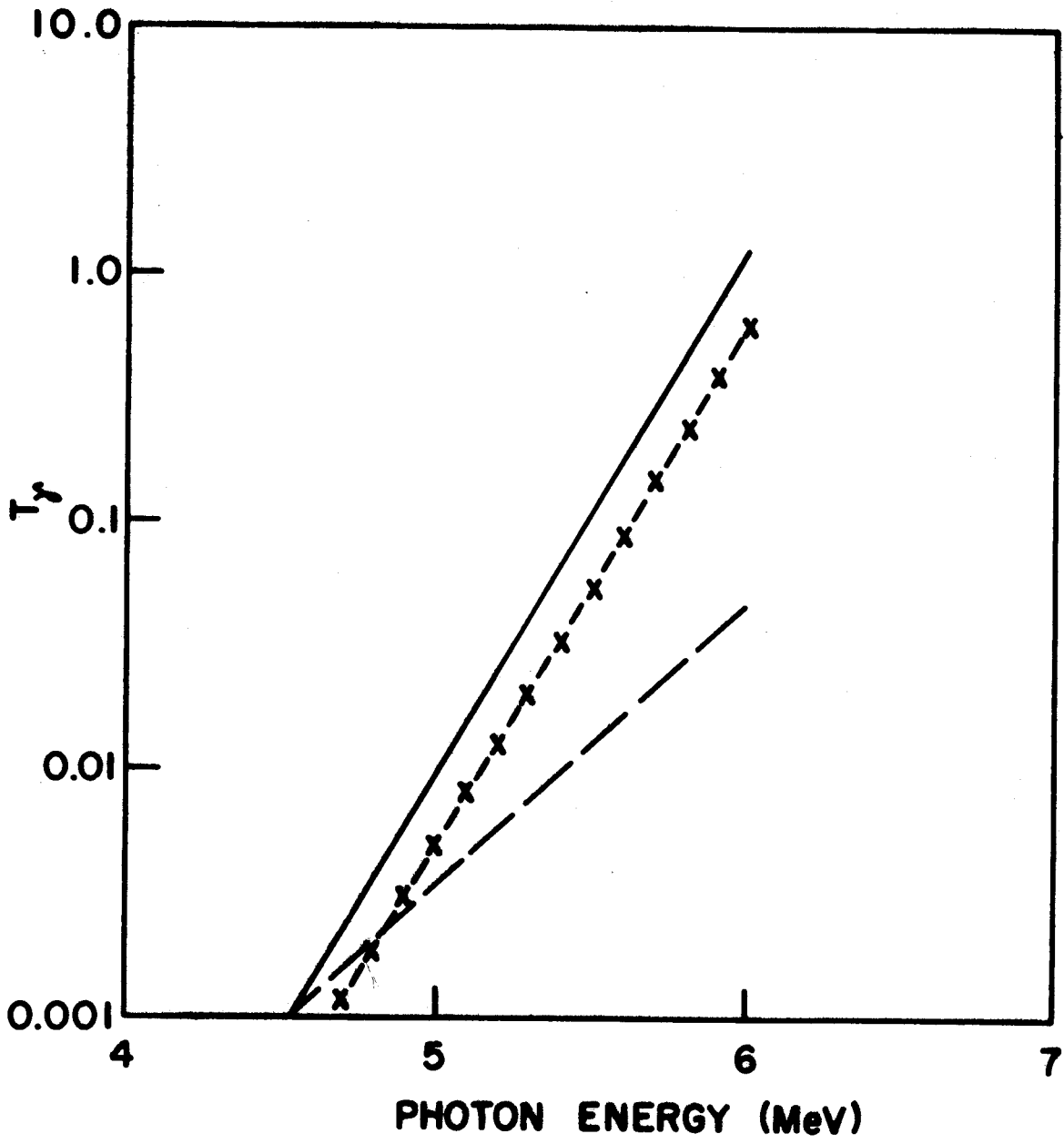


Fig. 5

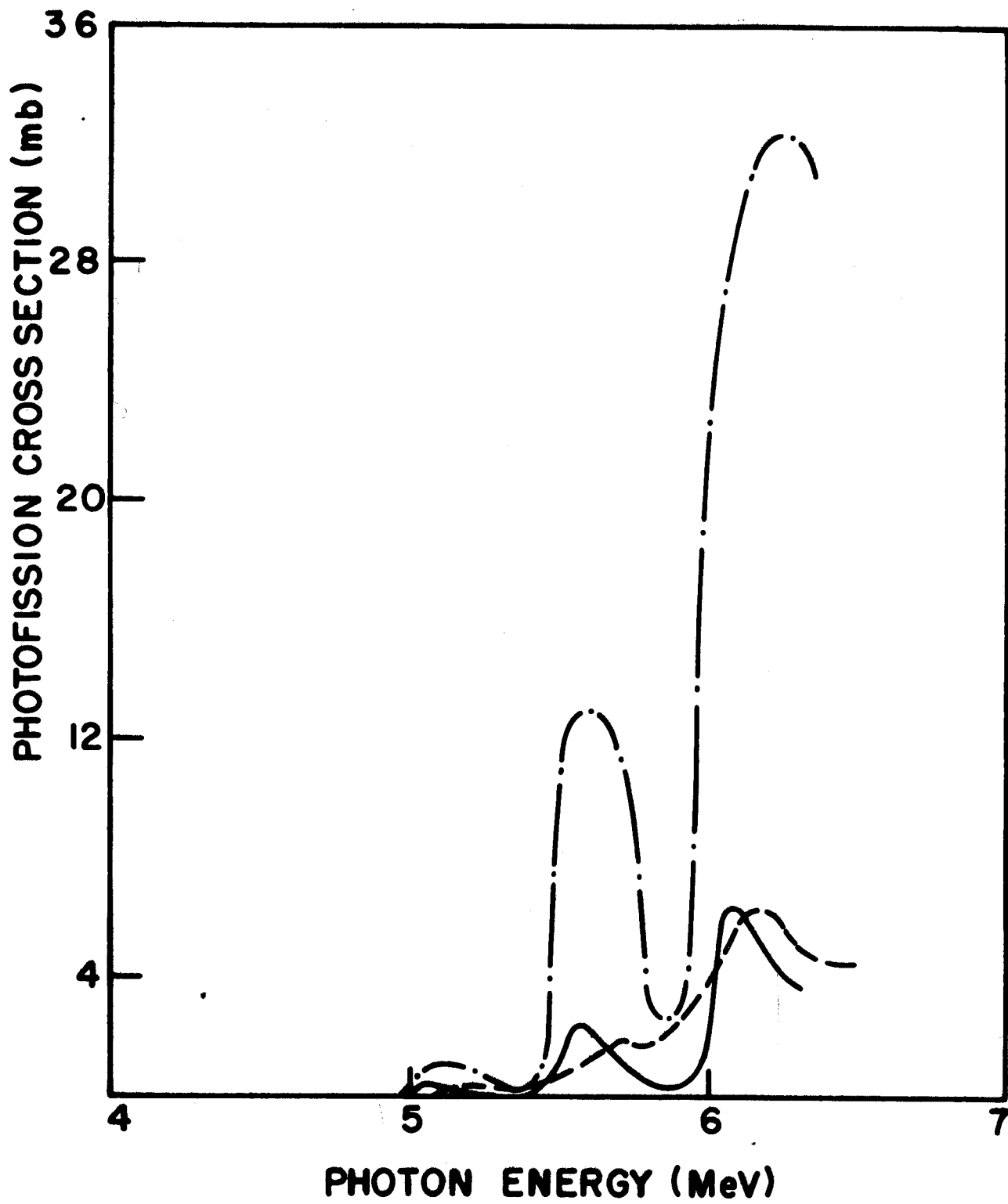


Fig. 6

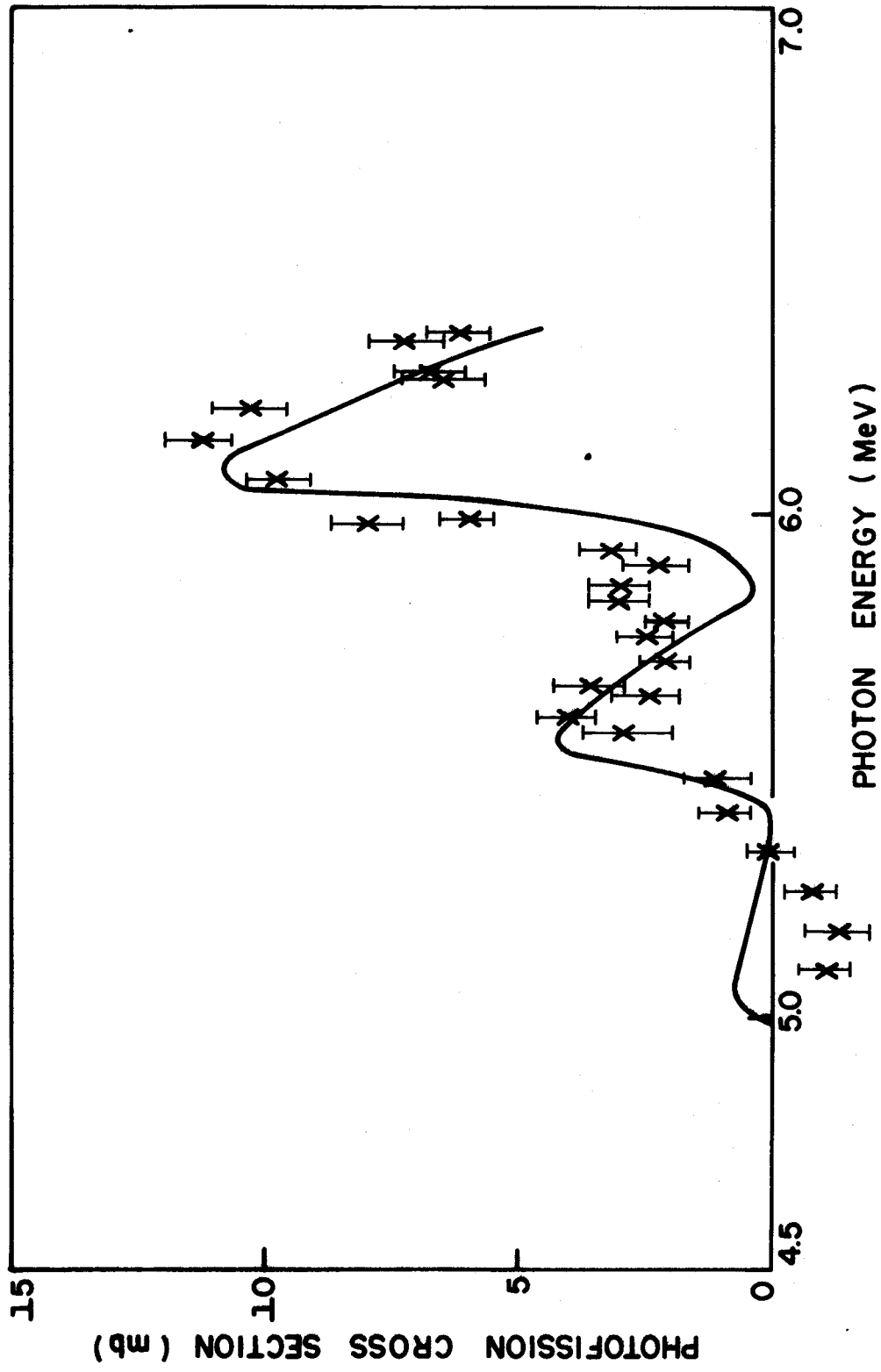


FIG. 7

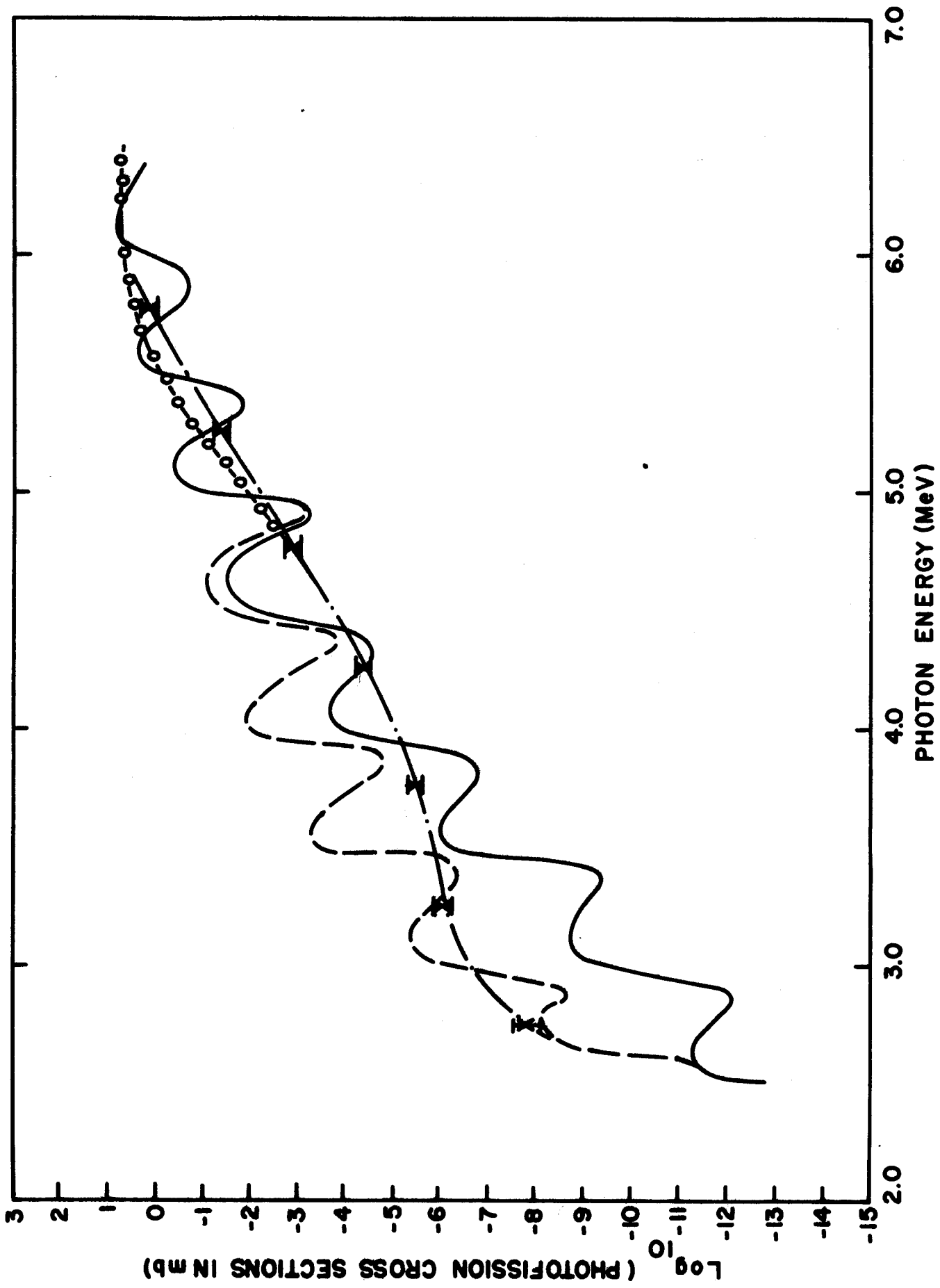


Fig. 8

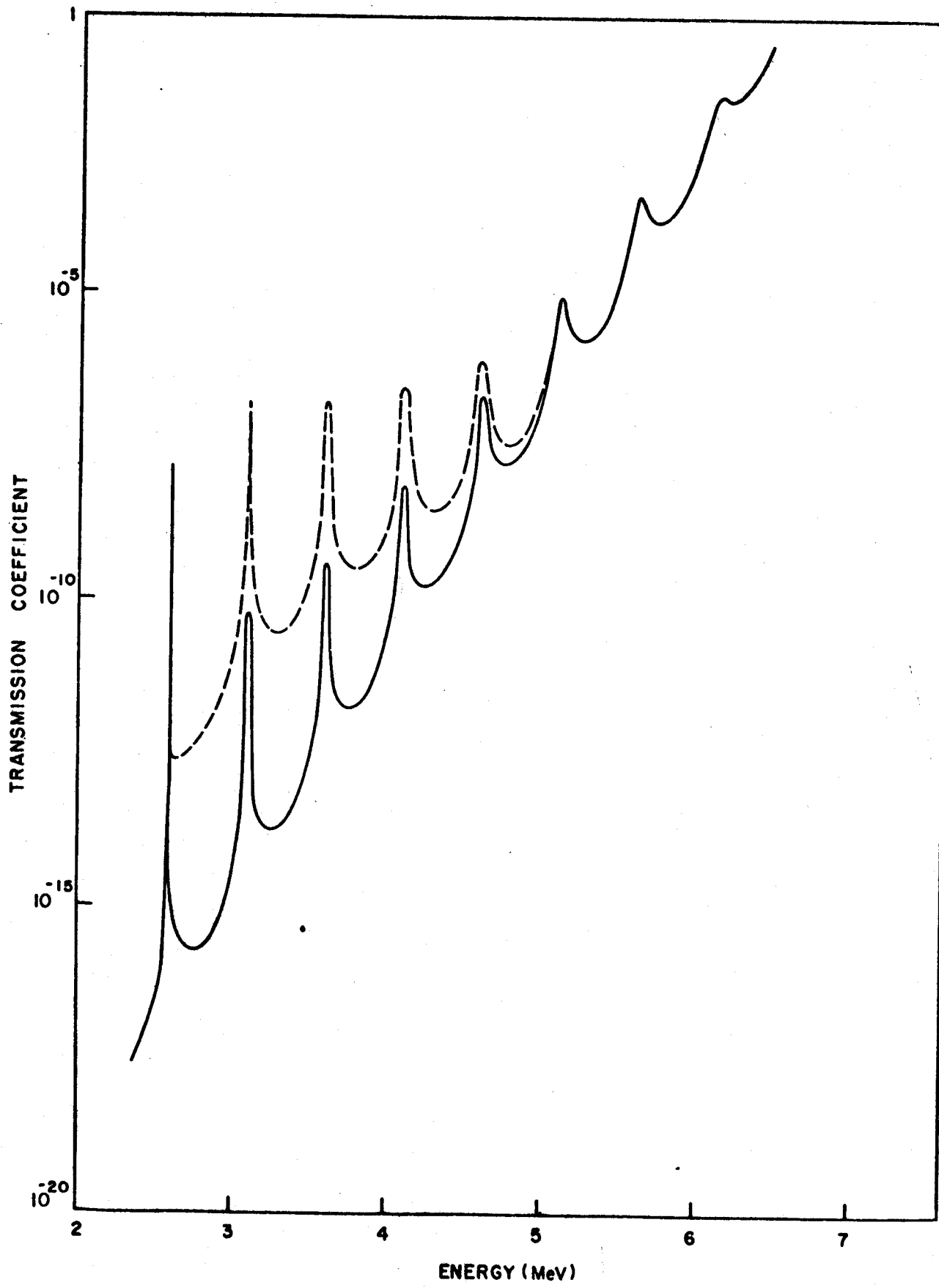


Fig. 9