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GAUGE DEPENDENCE OF RENORMALIZATION GROUP PARAMETERS IN GHOST-FREE NON-ABELIAN GAUGE THEORIES

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ABSTRACT

We discuss the gauge dependence of the renormalization group parameters in a class of ghost-free non-abelian gauge theories. We show, using the n-dimensional regularization with the "minimal" renormalization procedure, that these parameters are gauge independent.

The renormalization group equations [1] play an important role in the study of the asymptotic behaviour of renormalized field theories, especially in view of the fact that non-abelian gauge theories are asymptotically free [2]. An interesting problem is the gauge independence of the renormalization group parameters, or lack thereof. This was already discussed by a number of authors [3] for the class of covariant Lorentz gauges.

Here, using the n-dimensional regularization scheme [4], we would like to discuss this question in the context of non-covariant "axial" gauges, which do not lead to the appearence of Faddeev-Popov ghosts [5]. For simplicity, we will restrict ourselves to a pure Yang-Mills theory [6], since all essential features are already present here. This theory is described by the (unrenormalized) Lagrangian :

$$L^0 = -\frac{1}{4}\left(\partial_\mu A_\nu^{0a} - \partial_\nu A_\mu^{0a} + g^0 f^{abc} A_\mu^{0b} A_\nu^{0c}\right)^2 - \frac{1}{2\beta}\left(n_\mu^0 A_\mu^{0a}\right)^2 \qquad (1)$$

where f^{abc} are the totally antisymmetric structure constants of the gauge group G and n_μ^0 is any fixed four-vector ($n_\mu^0 n_\mu^0 \neq 0$). In order to obtain finite Green functions we rescale the fields A_μ^{0a}

$$A_\mu^{0a} = Z_3^{1/2} A_\mu^a \qquad (2)$$

In the dimensional regularization scheme it is convenient to rescale g^0 such that the renormalized coupling constant is dimensionless. As a consequence of the Slavnov-Taylor identities [7], which hold in the absence of ghosts, one finds $Z_1 = Z_3$, so that we can write :

$$g^0 = \mu^{\varepsilon/2} Z_3^{-1/2} g \qquad (3)$$

where $\varepsilon = 4 - n$ and μ is an arbitrary mass, which sets the scale of the theory. Furthermore, it is necessary to rescale the gauge parameter n_μ^0 such that the gauge fixing term in (1) remains invariant. This requires :

$$n_\mu^0 = Z_3^{-1/2} n_\mu \qquad (4)$$

We will determine the (dimensionless) counterterm Z_3 using the "minimal" regularization scheme [8], which in this case contains only inverse powers of ε in an expansion about $\varepsilon = 0$.

The propagator for the vector-boson fields, for $\beta = 0$:

$$D^{ab}_{\mu\nu}(p,n) = \delta^{ab} \frac{1}{p^2} \left[\delta_{\mu\nu} - \frac{n_\nu p_\mu + n_\mu p_\nu}{n.p} + n^2 \frac{p_\mu p_\nu}{(n.p)^2} \right] \tag{5}$$

leads, by power counting arguments, to a renormalized theory. Remark that, since $D^{ab}_{\mu\nu}$ is a function of zeroth degree in n, it has the important property of being invariant under the scaling $n \to sn$, where s is an arbitrary ($s \neq 0$) parameter :

$$D^{ab}_{\mu\nu}(p,n) = D^{ab}_{\mu\nu}(p, sn) \tag{6}$$

Using this relation, we will show that Z_3 is independent of the gauge parameter n, which implies, as we will see, the gauge independence of the renormalization group parameters. The proof is inductive.

Consider a Feynman diagram, which is made finite, order by order in a loop expansion, up to l loops, by counterterms which are independent of n. This is trivially true in the tree-approximation (l=0), where $Z_3 = 1$. To order (l+1) the diagram will exhibit poles (up to degree l+1), with the property that the residues are polynomial in the external momenta [4,9]. Furthermore, using (6) and by the induction hypothesis, all terms, including the residues, must be invariant under the scaling $n \to sn$. Note that this property must be satisfied independently by rational and logarithmic functions. We now observe (from (3)), that μ only appears as a power of $\mu^{\varepsilon/2}$, multiplying coupling constants. When the Laurent expansion of each Feynman integral is made, only logarithms of μ will appear. In view of the first property, the residues could depend on μ, via these logarithmic functions, only through dimensionless ratios of n^2/μ^2. However, these terms cannot satisfy the second requirement, mentioned above, so that we conclude that the residues are independent of μ (for a discussion of this point see also reference 10). We now remark that Z_3 is a dimensionless quantity, which can depend on n only via dimensionless ratios of n^2/μ^2. As its poles are defined to be precisely those needed to substract the poles associated with the above residues, which are independent of μ, Z_3 must also be μ-independent. It follows then, that also to order (l+1), we can remove the divergences with counterterms independent of n, which concludes the proof.

We can reach the same conclusions by observing that, since the theory is renormalizable, the residues must have the same form as the counterterms generated in the Lagrangian (1) by the rescalings (2), (3) and (4). Separating out explicitly the dependence on the external momenta, we see that the residues could depend on n only through functions of n^2/μ^2. For the reasons discussed above, such functions cannot be present, whence we conclude the gauge independence of Z_3.

As a consequence of this property, we can easily show with the help of (5) that any N-particle (one-particle irreducible) Green function $\Gamma^N(p_i, g, n, \mu)$ satisfies the relation :

$$n_\alpha \frac{\partial}{\partial n_\alpha} \Gamma^N(p_i, g, n, \mu) = 0 \tag{7}$$

On the other hand, Γ^N satisfies the renormalization group equation [1] :

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \delta_\nu \frac{\partial}{\partial n_\nu} - N\gamma\right) \Gamma^N(p_i, g, n, \mu) = 0 \tag{8}$$

where β, δ_ν and γ are given by :

$$\beta = \mu \frac{\partial g}{\partial \mu}\bigg|_{g_0, n_0} ; \quad \delta_\nu = \mu \frac{\partial n_\nu}{\partial \mu}\bigg|_{g_0, n_0} ; \gamma = \mu \frac{\partial}{\partial \mu} \log Z_3^{1/2}\bigg|_{g_0, n_0} \tag{9}$$

Although the dimensionless parameters β, $\delta = \frac{\delta_\nu n_\nu}{n^2}$, γ could a priori depend on n via functions of n^2/μ^2, we will now show that they are, in fact, independent of the gauge parameter n. To see this, let us apply the operator $n_\alpha \partial/\partial n_\alpha$ on equation (8). Using relation (7) we arrive at :

$$\left[\left(n_\alpha \frac{\partial \beta}{\partial n_\alpha}\right) \frac{\partial}{\partial g} + \left(n_\alpha \frac{\partial \delta_\nu}{\partial n_\alpha} - \delta_\nu\right) \frac{\partial}{\partial n_\nu} - N\left(n_\alpha \frac{\partial \gamma}{\partial n_\alpha}\right)\right] \Gamma^N = 0 \tag{10}$$

We have imposed no new conditions on Γ^N, so that this equation must be satisfied identically. Thus, the coefficients vanish and we obtain the relations :

$$n_\alpha \frac{\partial \beta}{\partial n_\alpha} = 0 \tag{11a}$$

$$n_\alpha \frac{\partial \delta}{\partial n_\alpha} = 0 \tag{11b}$$

$$n_\alpha \frac{\partial \gamma}{\partial n_\alpha} = 0 \tag{11c}$$

which imply the gauge independence of the parameters β, δ and γ since they are momentum independent. From (9) we see that (11c) is trivially satisfied as the counterterm Z_3 is gauge independent. Equations (11a) and (11b) can also be derived by noting that, from equations (3), (4) and (9), we have (in the limit $\varepsilon \to 0$) :

$$\beta = \gamma g \qquad \delta = \gamma \tag{12}$$

Using these relations we see that (11a) and (11b) follow from (11c).

As an illustration, we obtain, without making any assumption about n_μ, in the lowest order of perturbation theory :

$$Z_3 = 1 - \frac{11C}{24\pi^2} g^2 \frac{1}{n-4} \tag{13}$$

where C is the value of the quadratic Casimir operator in the regular representation of G and is given by $\delta_{ab} C = f_{acd} f_{bcd}$. Relation (13) shows that Z_3 is independent of the gauge parameter n and yields, using (9) and (12), the same gauge invariant result for the β-function, as the one obtained in covariant Lorentz gauges [2].

Finally, we would like to remark that, despite the gauge invariance of the result (13), it is not possible to choose the gauge $n^2 = 0$, which would greatly simplify the calculations (see equation (5). In fact, an explicit calculation performed in this gauge yields for Z_3 a different (and, hence, incorrect) result. The reason for this difference is due to the singularities n^{-2} which appear when computing the Feynman integrals [11]. These singularities, when multiplied by the factor n^2, which appears in the propagators, yield well defined results which, in general, do not vanish. Due to this fact, in order to obtain consistent results, it is (unfortunately) necessary to use the full expression (5) for the vector boson propagator, with $n^2 \neq 0$.

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