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ABSTRACT:

The "Bohr-Sommerfeld quantization rule" is used in the treatment of the massive Thirring Model. A semi-Classical bound state spectrum is achieved. This spectrum is similar to that of the sine-Gordon Theory.

Recently Coleman[1] has proved that the sine-Gordon Theory (SGT) is equivalent to the massive Thirring Model (MTM). On the other hand Dashen, Hasslacher and Neveu[2] (DHN) were able to quantize the SGT by means of a semi-classical method, namely, the WKB approximation applied to Field Theory[2,3]. Then it is natural to ask ourselves about the possibility of treating the MTM by using semi-classical methods.

The purpose of this note is solely to show that the Bohr-Sommerfeld quantization rule (BSQR) when applied to the MTM leads to results that are similar to that obtained by DHN[2] in the context of the S.G.T.

Consider the quantum field $\phi(x)$ and its canonical momentum $\pi(x)$ of a given theory. If $\phi^{cla}(\vec{x},t)$ and $\pi^{cla}(\vec{x},t)$ are periodic solutions (with period τ) of the classical equations of motion, the BSQR reads[4]

$$\int_0^{\tau} dt \int d\vec{x}\; \pi^{cla}(\vec{x},t)\; \dot{\phi}^{cla}(\vec{x},t) = 2\pi n \tag{1}$$

where n is an integer.

Applying the BSQR to the breather solution of the SGT the discrete spectrum obtained is[4]

$$E_n = 2 M_c \sin\left(\frac{\mu}{2 M_c}\, n\right) \tag{2}$$

where M_c and μ are respectively the classical soliton and the elementary meson masses. We recall that if instead of the BSQR we had used the full WKB[2,3] method, the soliton mass would be corrected by renormalization effects ($M=M_c-\frac{\mu}{\pi}$; M being the corrected mass).

The massive Thirring Model is a two dimensional theory defined by the following Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{g}{2}(\bar{\psi}\gamma_\mu\psi)^2 \tag{3}$$

Its classical Hamiltonain density is

$$\mathcal{H} = \bar{\psi}(i\gamma^1\partial^1 + m)\psi + \frac{g}{2}(\bar{\psi}\gamma_\mu\psi)^2 \tag{4}$$

whereas the classical equation of motion will be

$$(i\not{\partial} - m - g\bar{\psi}\gamma^\mu\psi\gamma_\mu)\psi = 0 \tag{5}$$

Since the canonical momentum associated to $\psi(x)$ is $i\psi^+(x)$, the BSQR for this fermion field shall be

$$\int_0^\tau dt \int dx\; i\psi^{+\,cla}(x,t)\;\dot{\psi}^{cla}(x,t) = 2\pi n \tag{6}$$

where $\psi^{cla}(x,t)$ is a solution (with period τ) of eq. (5). A large class of such periodic solutions (with integrable energy) was obtained in ref. (5). They are written as follows

$$\psi_\omega(x,t) = \begin{pmatrix} \eta(x)\cos\phi(x) \\ \eta(x)\sin\phi(x) \end{pmatrix} e^{-i\omega t} \tag{7}$$

where $\omega = 2\pi/\tau$ is the frequency.

$$\eta^2(x) = \frac{2(m-\omega)}{g}\frac{1}{(\cosh^2 kx + \beta\sinh^2 kx)} \tag{8-a}$$

and

$$\phi(x) = \tan^{-1}\left(\sqrt{\beta}\, \tanh kx\right) \tag{8-b}$$

where

$$\beta = \frac{m-\omega}{m+\omega} \quad \text{and} \quad k = (m^2-\omega^2)^{1/2}$$

It is important to observe that our interpretation of the solutions of eq. (5) differs from that of ref. (5). Whereas those authors consider that ψ describes an one fermion state that must be normalized (they assume $\int dx\, \psi^{+cla}\, \psi^{cla} = 1$) we interpret ψ^{cla} as a classical field that can be quantized by using a "Bohr-Sommerfeld quantization rule". It is easy to see that the normalization condition of ref. (5) is not compatible with the BSQR.

We point that in order to have a confined Hamiltonian density it is necessary that $\omega < m$. From eq. (4) we get the energy

$$E(\omega) = (\alpha/g)\sqrt{m^2-\omega^2} \tag{9}$$

Now, putting the solution (7) into eq. (6) we obtain the allowed ω values

$$\omega_n = m \cos\left(\frac{g}{\alpha} n\right) \tag{10}$$

From eqs. (9) and (10) we are led to the discrete spectrum

$$E_n = \frac{2m}{g} \sin\left(\frac{g}{2} n\right) \tag{11}$$

Now, we can see that the two spectra (2) and (11) are equivalent if[6]

$$g = \mu / M_c \tag{12-a}$$

$$\mu = m \tag{12-b}$$

The current feeling is that the soliton (of the SGT) is the fermion (of the MTM). If we insist on that, we conclude that the fermion mass will be $M_f = m/g$; i.e., the mass of the physical fermion is not the one that appears on the Lagrangian, even on classical grounds.

After the introduction of the rescaled field $\varphi' = (8\mu/M_c)^{1/2} \varphi$ in the SGT (φ is the bosonic field), Coleman[1] concluded that semi-classical approximations for the quantum SGT are reliable if

$$8\mu / M_c \ll 1 \tag{13}$$

Similarly, rescaling our ψ field by $\psi' = \sqrt{g}\, \psi$ we conclude that our semi-classical approach is expected to be reliable whenever

$$g \ll 1 \tag{14}$$

Here, we point out the consistency of relations (12-a), (13) and (14).

The similarity observed between our spectrum and that of DHN is by all means wellcome since Coleman's[1] investigation antecipated a strong equivalence between the SGT and the MTM. However there exists a point in our treatment that deserves a more careful study: expression (11) seems to imply that there exists fermion-fermion bound states ($g < 0$ case). This question must be clarified in a more refined treatment. We conjecture that when $g < 0$ the periodic classical solutions (with $\omega \neq 0$) are unstable - they may generate complex stability angles with negative imaginary part. Such instability of the classical solutions will forbid the existence of the undesirable bound states.

It is known that renormalization effects will change the parameters of expression (11). We hope to improve our spectrum by using a treatment like that of DHN[2,3]. A program for quantizing the Massive Thirring Model by using a WKB[2,3] technique is in progress. The present authors are trying to use the DHN method in order to treat renormalizable fermionic theories. At present this program seems to be a hard task because the linearized stability equations[2,3] are difficult to deal with.

FOOTNOTES

(1) - S.Coleman; Phys.Rev. D11, 2088 (1975).

(2) - R.Dashen, B.Hasslacher, A.Neveu; Phys.Rev. D11, 3424 (1975).

(3) - R.Dashen, B.Hasslacher, A.Neveu; Phys.Rev. D10, 4114 (1974); Ibid D10, 4130 (1974).

(4) - R.Jackiw - Lecture given at the XV Cracow School of Theor. Phys.; to be published in Acta Physica Polonica B.

(5) - S.J.Chang, S.D.Ellis, B.W.Lee; Phys.Rev. D11, 3572 (1975).

(6) - We point out that our g is not the same as that of Coleman. Actually in the quantum theory g is a free parameter which can be defined by imposing arbitrary renormalization conditions on the four point vertex function.