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In the course of the last few years we tried to derive consequences of the analyticity and unitarity of the scattering amplitudes on rising Regge trajectories in an essentially model-independent frame[1]. The main result obtained was that the widths of the resonances interpolated by the trajectories grew linearly with the mass of the resonance. A crucial test for our result was thought to be the eventual discovery of a resonance at the same time massive and narrow.

Would the recent finding of the psion family[2] point, in this sense, to the breakdown of the standard analyticity[3]? In the following analysis we show that the answer is "not necessarily".

According to an investigation of Chang and Nelson[4], the psions are consistent with a Regge classification under the O(4) group, in the sense of Freedman and Wang[5]. The details are not relevant to our discussion, but the result that the narrow resonances sit on Regge trajectories that are linear in s is very important. As the proposed mechanisms to explain the inhibition of strong decays are not much dependent on the growth of the mass of the resonance we will consider, for simplicity, the case of narrow resonances of very high mass. Is the existence of these particles compatible with the usual assumptions of analyticity and unitarity?

We start by assuming that the mesonic trajectory $\alpha(s)$ has the following properties:

a) is analytic in the complex s plane cut along the real axis above the physical threshold and continuous in the real axis;

b) is real analytic, i.e., $\alpha^*(s)=\alpha(s^*)$;

c) grows slower than an exponential in s for $|s|\to\infty$ on the upper half-plane of the physical sheet;

d) $$\lim_{s \to \pm\infty} \frac{\alpha(s)}{(-s)^{\varepsilon}|\ln(-s)|^{\beta}} = -C_{\pm} \qquad (1)$$

ε and β being real numbers.

Under these conditions the Phragmēn-Lindelöf[6] theorem can be applied to the function $\alpha(s)/(-s)^{\varepsilon}|\ln(-s)|^{\beta}$, with the consequence that $C_+ = C_- = C$, a positive constant, if the trajectory is to rise with s.

For large values of s it follows that

$$\mathrm{Re}\,\alpha(s) = -C\cos(\pi\varepsilon)s^{\varepsilon}(\ln s)^{\beta} + \beta\pi C\sin(\pi\varepsilon)s^{\varepsilon}(\ln s)^{\beta-1} \qquad (2)$$

$$\mathrm{Im}\,\alpha(s) = C\, s^{\varepsilon}(\ln s)^{\beta-1}\,(\sin(\pi\varepsilon)\ln s + \pi\beta\cos(\pi\varepsilon)) \qquad (3)$$

The widths of the resonances interpolated by $\alpha(s)$ are

$$\Gamma(s) = \frac{\mathrm{Im}\,\alpha(s)}{\sqrt{s}\ \mathrm{Re}\,\alpha'(s)} \qquad (4)$$

the prime denoting differentiation with respect to s. Using (2) and (3) one gets

$$\Gamma(s) = \sqrt{s}\,\ln s\left[\sin(\pi\varepsilon)\ln s + \beta\pi\cos(\pi\varepsilon)\right] \cdot$$

$$\cdot\left[-\varepsilon\cos(\pi\varepsilon)(\ln s)^2 + \beta\ln s(\pi\varepsilon\sin(\pi\varepsilon) - \cos(\pi\varepsilon)) + \beta(\beta-1)\pi\sin(\pi\varepsilon)\right]^{-1} \qquad (5)$$

The unitarity requirement (from potential theory) of positivity of the imaginary part gives origin to the restrictions

$$\frac{1}{2} \leq \varepsilon \leq 1 \qquad (6)$$

where the extreme values are included only if $\beta \neq 0$.

Assume, according to Chang and Nelson[4], that the trajectory is very close to a straight-line, taking ε to be very close to 1. Trajectories of this kind have been extensively studied and are, in our opinion, good candidates to be the "real life" trajectories that interpolate the resonances known before the psions. Their prominent feature is that, as follows from Eqts. (2-5),

$$\Gamma(s) = -\frac{\tan(\pi\varepsilon)}{\varepsilon}\sqrt{s} \qquad (7)$$

that is, a width of a very general form which grows linearly with the mass and does not depend on any trajectory parameter other than ε. The problem then is: putting the psions on trajectories of this type that are almost linear means giving them widths of the same order of the widths of "ordinary" resonances. Is this an evidence that the psions lie on trajectories of different analyticity properties?

Not so. Observe, in fact, that we can get very near a linear trajectory in a different way. Take, in equation (1), $\varepsilon=1$ and $\beta \neq 0$. The trajectory is then, for large s, linear except for a logarithmic factor. From equation (5) we have for the width now

$$\Gamma(s) = -\beta\pi\frac{\sqrt{s}}{\ell n s+\beta} \qquad (8)$$

and for the trajectory,

$$\mathrm{Im}\,\alpha(s) = -\beta C\pi s(\ell n s)^{\beta-1} \qquad (9)$$

$$\mathrm{Re}\,\alpha(s) = Cs(\ell n s)^{\beta} \qquad (10)$$

Positivity demands that β be negative. By choosing it large enough in modulus one can, as follows from Eq.(8), obtain widths that are orders of magnitude smaller than the widths predicted by Eq.(7).

So, if the psions lie on a trajectory of this kind, the analysis of Chang and Nelson is compatible with the asymptotic consequences of the standard analyticity of mesonic Regge trajectories.

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