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PHOTO-TWO PROTON PRODUCTION: A STUDY IN  
NUCLEAR SEQUENTIAL DECAY\*

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30500046716

IFUSP/P-78

PHOTO-TWO PROTON PRODUCTION: A STUDY IN NUCLEAR  
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ABSTRACT:

A description of ( $\alpha, 2p$ ) reaction is presented. The same theory is applied to discuss certain features of the hole propagator in ( $p, 2p$ ) reactions. It is demonstrated that resonant states connected to each other through their presence in a particular sequential reaction show certain degree of correlation in their widths as well as continuum shifts which reflects the importance of memory. The fluctuation cross section relevant for the energy spectrum of the first emitted proton in ( $\alpha, 2p$ ) or the second in ( $p, 2p$ ) is derived.

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\* Research supported in part by BNDE-FUNTEC 108.  
Submitted for publication in "Annals of Physics".

## I. INTRODUCTION

The problem of describing nuclear reactions leading to a sequential decay of the compound nucleus is a difficult one because of the intrinsically many-body-in-the-continuum nature of the process. The simplest of these and yet still plagued with the same difficulty is a two-step sequential decay which makes it a three-body problem<sup>1)</sup>. Attempting to construct a general and practical theory, e.g. Feshbach's projection operator theory, one is confronted with the dilemma of defining the proper projection operators as well as the continuum-continuum coupling which is much more important in this case than in the ordinary one-step decay reactions. Nevertheless it would be of interest to attempt to use certain approximations that could shed some light on the structure of the cross section yet retain the sequential decay aspect of the process. In this paper we try to do exactly this by using the projection operator method. In order to simplify the discussion we use a prototype reaction that emphasizes the decay part of the system. The production part is considered very simple such that one could use perturbation theory to get the target nucleus to the desired excited state that could trigger the nuclear sequential decay. Thus our projectile is electromagnetic radiation. In this, one has a photon (first-order perturbation theory suffices) impinging on a target nucleus. The photon is energetic enough to excite the nucleus well into the continuum where proton emission is possible. The photon in question could be a real photon or a virtual one as in electron scattering. Experimentally, however, monochromatic photons are hard to get whereas electron scattering is a more manageable experiment. However, we will merely formulate a photo-emission reaction since

electron scattering can be described with basically the same t-matrix attached to it a photon propagator (again we assume lowest-order perturbation theory to be adequate in electron scattering just as it is in photo exciting the nucleus).

A theory of photo-excitation of the giant resonance in nuclei has already been formulated in reference<sup>2)</sup> for a conventional one-step decay (OSD) of the resonance. This present work can be considered as an extension (non-trivial!) of the work described in<sup>2)</sup>. Whereas in many-body problems one always tries to render them into a many one-body problem without too much ambiguity, in the case of a sequential-decay problem the reduction of the problem into a multi one-step decay one is not so unambiguous. We shall, in this paper, attempt to do that. Surprising results are obtained as will be shown in the last section, where an interesting role of the continuum shift in the decay process of the nuclear system is found which somehow contrasts with the role played in the more conventional OSD processes.

## II. THE TRANSITION MATRIX

In deriving an expression for the transition amplitude of a particular type of reaction one has to try to separate formation from decay. If the reaction proceeds via compound nuclear formation then the above separation is very easy to visualize. Thus in our prototype reaction of  $(\gamma, 2p)$  the formation mechanism is merely photoexciting the target nucleus  $(N, Z)$  into a highly excited state; a compound nucleus with a rather small width. The decay mechanism is, according to the nature of our prototype reaction, the decay of the compound nucleus into

a proton plus an  $(N, Z-1)^*$  nucleus which is also a resonance. This second resonance then decays into a second proton plus an  $(N, Z-2)$ . One should realize that the separation of the reaction into a formation and decay mechanism is an artificial one since there is always interference between the two mechanisms.

Thus we view  $(\gamma, 2p)$  reaction as in figure 1 where a proton with momentum  $q$  impinges on a target nucleus in its ground state (which we shall denote by  $(N, Z)$ ) resulting in two protons plus the final residual nucleus which is also in the ground state  $(N, Z-2)$ .

The transition amplitude describing the above reaction can be written down easily if one considers the electromagnetic interaction as acting only to first order<sup>2)</sup>. First order perturbation in  $V_Y$  yields the following expression for the T-matrix

$$T_{fi} = \langle \Psi_f^{(-)} | V_Y | (N, Z) \rangle \quad (1)$$

where  $(N, Z)$  is the ground state of the target nucleus and  $\langle \Psi_f^{(-)} |$  is the scattering state (full) of the Schrodinger equation

$$(E - H) | \Psi_f^{(-)} \rangle = 0 \quad (2)$$

H is the Hamiltonian of the nuclear system.

Thus all the nuclear interaction is contained in  $\langle \Psi_f^{(-)} |$

Since our aim is to construct a theory for the two-proton ejection reaction (photo-two-proton production), hereafter we shall remove the subscript f from  $\Psi_f^{(-)}$ . Separating the total nuclear Hilbert space into the

usual P and Q subspaces (P is so defined as to project out of the Hilbert space the part describing the state of a proton and a "decaying" residual nucleus  $(N, Z-1)^*$ .  $Q = 1-P$ ).

Therefore

$$\begin{aligned} (E - H_{PP}) P | \Psi^{(-)} \rangle &= H_{PQ} Q | \Psi^{(-)} \rangle \\ (E - H_{QQ}) Q | \Psi^{(-)} \rangle &= H_{QP} P | \Psi^{(-)} \rangle \end{aligned} \quad (3)$$

$$H_{PP} = P H P \quad \text{etc}$$

Since  $Q | \Psi^{(-)} \rangle$  includes only irrelevant open channels as well as closed channels one may write a formal solution for the second equation in (3) in the following fashion:

$$Q | \Psi^{(-)} \rangle = \frac{1}{E - H_{QQ} - i\eta} H_{QP} P | \Psi^{(-)} \rangle \quad (4)$$

where  $-i\eta$  is to guarantee that the Green's function  $(E - H_{QQ} - i\eta)^{-1}$  does not blow-up for the irrelevant open-channels. Naturally for the closed channels one does not have to be so careful since  $(E - H_{QQ})^{-1}$  is not singular in this latter case.

$$\text{Since } \langle \Psi^{(-)} | = \langle \Psi^{(-)} | P + \langle \Psi^{(-)} | Q$$

Thus

$$\langle \Psi^{(-)} | = \langle \Psi^{(-)} | P \left[ H \frac{Q}{E - H_{QQ} - i\eta} + 1 \right] \quad (5)$$

$\langle \Psi^{(-)} | P$  can be "evaluated" from the two coupled equations (3) by eliminating the Q components of the wave function resulting in:

$$\langle \psi^{(-)} | P = \langle \phi_p^{(-)} | + \langle \phi_p^{(-)} | H_{p\alpha} \frac{1}{E - H_{\alpha\alpha}^{(-)} - i\eta} H_{\alpha p} G_{pp}^{(-)} \quad (6)$$

where  $(E - H_{pp}) | \phi_p^{(-)} \rangle = 0$

and  $H_{\alpha\alpha} = H_{\alpha\alpha} + H_{\alpha p} \frac{1}{E - H_{pp} + i\eta} H_{p\alpha}$

$$G_{pp} = (E - H_{pp} + i\eta)^{-1}$$

The T-matrix T can then be written in the following simple form:

$$T_{fi} = \langle \psi^{(-)} | P \tilde{V}_y | (N, Z) \rangle \quad (7)$$

where  $\tilde{V}_y \equiv V_y + H \frac{Q}{E - H_{\alpha\alpha} + i\eta} V$

or  $T_{fi} = \langle \phi_p^{(-)} | \tilde{V}_y | (N, Z) \rangle + \langle \phi_p^{(-)} | H_{p\alpha} \frac{1}{E - H_{\alpha\alpha}^{(-)} + i\eta} H_{\alpha p} \times G_{pp}^{(-)} \tilde{V}_y | (N, Z) \rangle$

This is the expression used by <sup>2)</sup> to calculate the giant resonance scattering on <sup>16</sup>O. Note that because the reaction is not symmetrical in its incoming and outgoing channels, ( $\gamma, 2p$ ), the above expression looks different from the usual expression for  $T_{fi}$  in which instead of (N, Z) one has a state similar in structure to  $|\phi_p^{(-)}\rangle$ . This is due to the fact that electromagnetic scattering is treated only to first order.

Before going any further we have to define the projection operators P and Q. Since the final resonating nuclear state (after the first proton ejection) is rather long lived one may make the assumption that the first proton does not interact with, or influence the decay mechanism of, this nucleus. Thus the projection operator P is defined to be spanned by, among others, states of the form:

$$|\lambda, r\rangle = \sum_{\alpha, \beta} C_{\alpha\beta}^{\lambda} a_{\alpha}^{\dagger}(r) |\beta\rangle \quad (8)$$

where  $|\beta\rangle$  is the state describing a decaying residual nucleus (N, Z-1)<sup>+</sup> and  $a^{\dagger}(r)$  is an operator defined to create a proton at a distance r from the "target" i.e. the residual nucleus,  $\alpha$  and  $\beta$  are appropriate quantum numbers and  $C_{\alpha\beta}^{\lambda}$  is the coupling coefficient. Therefore P is defined

$$P = \sum_{\lambda} \int r^2 dr |\lambda, r\rangle \langle \lambda, r| \quad (9)$$

$$= \sum_{\lambda} \sum_{\alpha, \beta, \alpha', \beta'} C_{\alpha\beta}^{\lambda} C_{\alpha'\beta'}^{\lambda} a_{\alpha}^{\dagger}(r) |\beta\rangle \langle \beta'| a_{\alpha'}(r) r^2 dr$$

We have written P in terms of  $a_{\alpha}^{\dagger}$  and  $a_{\alpha'}$  explicitly due to the nature of the problem. We follow the prescription presented in <sup>3)</sup> for rendering the states (8) orthonormal utilizing the appropriate density matrix of the (N, Z-1)<sup>+</sup> nuclear system. In order to describe the decay of the residual nucleus (after the first proton ejection) we shall identify the states  $|\beta\rangle$  as those spanning a new subspace namely that one describing the (N, Z-1)<sup>+</sup>  $\rightarrow$  (N, Z-2) + p<sub>1</sub> process which is governed by a Hamiltonian related to  $H_{pp}$  in a simple way. Instead of modifying  $H_{pp}$  one can modify the energy by extracting a plane wave (with complex energy) for the first ejected proton from  $|\phi_p^{(-)}\rangle$  <sup>4)</sup>. Then:

$$PHP |\phi_p^{(-)}\rangle = (\widetilde{PHP} + \frac{k_p^2}{2m}) |\phi_p^{(-)}\rangle \quad (10)$$

thus

$$(E - \frac{k_p^2}{2m} - \widetilde{PHP}) |\phi_p^{(-)}\rangle = 0$$

$\widetilde{PHP}$  is now the Hamiltonian of the decaying residual nucleus (N, Z-1)<sup>+</sup>.

The second assumption that we are going to make is the following: since decaying (or resonant) nuclear states are mostly described as compound nuclear states formed by the capture of a projectile by the

target nucleus such that it remains in an orbit inside the compound nucleus until it decays, what we can do is discuss the second decay (second proton ejection) in a time-reversed way, i.e., assume that one has a proton "impinging" on a "target" nucleus  $(N, Z-2)$ . The result of the reactions is the formation of the compound nucleus  $(N, Z-1)^*$ . We do not go any further since what comes next in this time-reversed description is another reaction with the first ejected proton to form the first compound nucleus  $(N, Z)^*$ . This is an asymmetrical reaction (a sequential capture of two protons) which is just the kind of thing we are trying to simplify. Thus we are identifying the states  $|\alpha\rangle$  as those spanning a smaller subspace  $q$  of the full subspace spanned by  $\{|P\rangle\}$ . Thus we write  $P = p+q$ .

We shall call  $\widetilde{H}_P$ ,  $h_{pp}$  for convenience. Thus equation (10) can be written as:

$$\begin{aligned} (\mathcal{E} - h_{pp}) p |\Phi_p^{(+)}\rangle &= h_{pq} q |\Phi_p^{(+)}\rangle \\ (\mathcal{E} - h_{qq}) q |\Phi_p^{(+)}\rangle &= h_{qp} p |\Phi_p^{(+)}\rangle \end{aligned} \quad (11)$$

Thus we can solve for  $q |\Phi_p^{(+)}\rangle$

$$q |\Phi_p^{(+)}\rangle = \left[ \mathcal{E} - h_{qq} - h_{qp} \frac{1}{\mathcal{E} - h_{pp} + i\eta} h_{pq} + i\eta \right]^{-1} h_{qp} p |\Phi_p^{(+)}\rangle \quad (12)$$

$$(\mathcal{E} - h_{pp}) p |\Phi_p^{(+)}\rangle = 0$$

The trick is to identify the above solution  $q |\Phi_p^{(+)}\rangle$  with the states  $|\beta\rangle$  that enter in our definitions of  $P$ . The state  $p |\Phi_p^{(+)}\rangle$  describes the second, potential-scattered proton and an  $(N, Z-2)_q$  nucleus.

What (11) tells us is how the states  $|\beta\rangle$  are coupled to the

continuum, i.e., it defines the escape width of its decay, which may be written as

$$\gamma_q = 2 \text{Im} \left[ h_{qq} + h_{qp} \frac{1}{\mathcal{E} - h_{pp} + i\eta} h_{pq} \right] \quad (13)$$

The continuum shift is just

$$\delta_q = \text{Re} \left[ h_{qq} + h_{qp} \frac{1}{\mathcal{E} - h_{pp} + i\eta} h_{pq} \right] \quad (14)$$

All of these quantities naturally appear in  $P$  and thus in the width and shift of the compound nucleus  $(N, Z)^*$ . This is interesting as it tells us, in principle, what the total width,  $\Gamma_{(N, Z)^*}$ , and continuum shift,  $\Delta_{(N, Z)^*}$ , of the compound system  $(N, Z)^*$  are, given the quantities  $\gamma_q \equiv \Gamma_{(N, Z-1)^*}$  and  $\delta_q \equiv \Delta_{(N, Z-1)^*}$  of the decaying residual nucleus  $(N, Z-1)^*$ . In order to see this dependence let us make certain approximations which serve to illustrate the ideas above. One of these is to write the  $Q$ -Space in the following form  $Q = D + Q'$  where  $D$  projects the primary doorway space. These  $D$ -States are strongly coupled to the continuum ( $P$ -space) so that they deserve a separate treatment. The  $D$ -states are also coupled to the other states in  $Q$  which gives the resonance associated with  $D$  two widths; an escape and damping one<sup>5)</sup>. Table 1 contains the details of our decomposition of the Hilbert space.

Before we doorway-decompose equation (7) let us look at the first term namely:

$$\langle \Phi_p^{(-)} | \widetilde{V}_r | (N, Z) \rangle \quad (15)$$

Thus from (12) one gets:

$$\begin{aligned} \langle \varphi_p^{(-)} | &= \langle \varphi_p^{(-)} | p + \langle \varphi_p^{(-)} | q \\ &= \langle \varphi_p^{(-)} | q (1 + \Lambda(\epsilon)) + \langle \xi_p^{(-)} | \end{aligned} \quad (16)$$

$$\Lambda(\epsilon) \equiv h_{qp} (\epsilon - h_{pp} + i\eta)^{-1}$$

Equation (15) becomes:

$$\begin{aligned} \langle \varphi_p^{(-)} | \tilde{V}_Y | (N, Z) \rangle &= \langle \xi_p^{(-)} | \tilde{V}_Y | (N, Z) \rangle \\ &+ \langle \xi_p^{(-)} | h_{pq} (\epsilon - h_{qq} - h_{qp} \frac{1}{\epsilon - h_{pp} + i\eta} h_{pq} + i\eta)^{-1} \tilde{V}_Y | (N, Z) \rangle \end{aligned} \quad (17)$$

where we have defined an effective electromagnetic interaction

$$\tilde{V}_Y \equiv (1 + \Lambda(\epsilon)) \tilde{V}_Y$$

The effective electromagnetic potential is of principal importance to our subsequent discussion. We write it explicitly for reference using  $pq=0$  and  $p+q=P$  we have:

$$\begin{aligned} \tilde{V}_Y &= V_Y + h_{qp} \frac{p}{\epsilon - h_{pp} + i\eta} V_Y + H_{pQ} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y \\ &+ H_{qQ} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y + h_{qp} \frac{p}{\epsilon - h_{pp} + i\eta} H_{pQ} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y \end{aligned} \quad (18)$$

$\tilde{V}_Y$  is the effective interaction introduced by<sup>2)</sup> and its structure reflects the usual effect of the Q-space states on V i.e. an energy dependent renormalization.

The first term in (17) describes the direct photodisintegration. The state  $\langle \xi_p^{(-)} |$  as said before is a solution of (12) and since it depends on the energy the effect of the first ejected proton is described by this term's explicit dependence on  $\epsilon$  i.e. on  $\frac{k^2}{2m}$  only. This term is the one used to calculate, naively, the process envisaged in fig. (1), without considering the compound systems  $(N, Z)^*$  and  $(N, Z-1)^*$ .

The second term in (16) is more interesting as one starts getting more information about the  $(N, Z-1)^*$  nuclear system, although by-passing the first compound nuclear resonance, which is felt here only through the effective electromagnetic potential  $\tilde{V}_Y$ .

$$\langle \xi_p^{(-)} | h_{pq} [\epsilon - h_{qq} - h_{qp} \frac{1}{\epsilon - h_{pp} + i\eta} h_{pq} + i\eta]^{-1} \tilde{V}_Y | (N, Z) \rangle \quad (19)$$

It is clear that in the above expression one in dealing with the matrix  $\langle \tilde{q} | \tilde{V} | (N, Z) \rangle$  where  $\langle \tilde{q} |$  is a state in the q-subspace. In order to understand the structure of this "form factor" we use (18). There are four non-vanishing terms, namely:

$$\begin{aligned} \langle q | V_Y | (N, Z) \rangle, \quad \langle q | h_{qp} \frac{p}{\epsilon - h_{pp} + i\eta} V_Y | (N, Z) \rangle \\ \langle q | h_{q0} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y | (N, Z) \rangle, \\ \langle q | h_{qp} \frac{p}{\epsilon - h_{pp} + i\eta} H_{pQ} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y | (N, Z) \rangle \end{aligned} \quad (20)$$

If the coupling of the Q-states to the p-states is small then the fourth term may be neglected on account of  $E_{pQ} \sim 0$ . The second term corresponds to scattering of  $p+(N, Z-2)$  via  $h_{qp}$  so it cannot be neglected. The same is true for the third term. However, this last term may also be dropped if one assumes that V couples  $(N, Z)$  to  $(N, Z-2)$  stronger than to  $(N, Z)^*$ . Thus the interaction relevant for the process under study may be written as

$$\tilde{V}_Y \approx V_Y + h_{qp} \frac{p}{\epsilon - h_{pp} + i\eta} V_Y + h_{qQ} \frac{Q}{\epsilon - H_{QQ} + i\eta} V_Y \quad (20')$$

The term that describes the transition via the first compound nucleus is the second term in (7) namely

$$T_{fi}^c = \langle \Phi_P^{(-)} | H_{PQ} \frac{Q}{E - H_{QQ} + i\eta} H_{QP} G_{PP}(\epsilon^{(+)}) \tilde{V}_Y | (N, Z) \rangle \quad (21)$$

In order to analyze this expression such as to be adapted to the sequential decay problem at hand one notices first the usual decomposition

$$P G_{PP} = G_{PP} + G_{PP} H_{PQ} Q G_{PP} \quad (22)$$

and

$$Q G_{PP} = G_{QQ} + G_{QQ} H_{QP} P G_{PP}$$

From (22) one obtains the following

$$\begin{aligned} P G_{PP} P &= P (E - H_{PP} - H_{PQ} G_{QQ} H_{QP} + i\eta)^{-1} P \\ Q G_{PP} Q &= Q (E - H_{QQ} - H_{QP} G_{PP} H_{PQ} + i\eta)^{-1} Q \\ P G_{PP} Q &= P (E - H_{PP} - H_{PQ} G_{QQ} H_{QP} + i\eta)^{-1} H_{PQ} G_{QQ} Q \\ Q G_{PP} P &= Q (E - H_{QQ} - H_{QP} G_{PP} H_{PQ} + i\eta)^{-1} H_{QP} G_{PP} P \end{aligned} \quad (23)$$

where  $G_{PP} = P G_{PP} P + Q G_{PP} Q + P G_{PP} Q + Q G_{PP} P$ . With the aid of (22) and (23) one can write for (21)

$$\begin{aligned} T_{fi}^c &= \langle \Phi_P^{(-)} | \left[ P Q \frac{1}{E - H_{QQ} + i\eta} (1 + \Lambda(\epsilon)) + 1 \right] \\ &\quad \times (H_{PQ} + H_{QP}) (E - H_{QQ}(\epsilon^{(+)}) + i\eta)^{-1} (H_{QP} + H_{PQ}) \\ &\quad \times (P G_{PP} P + Q G_{PP} Q + P G_{PP} Q + Q G_{PP} P) \tilde{V}_Y | (N, Z) \rangle \end{aligned} \quad (24)$$

With the further approximation  $H_{QP} \ll H_{PQ}$  one can write:

$$(E - H_{QQ} + i\eta)^{-1} \approx (E - H_{QQ} - H_{QP} Q G_{PP} H_{PQ} + i\eta)^{-1} \quad (25)$$

through this form for the compound nucleus propagator one recognizes the

connection to the secondary compound nuclear states  $q$ . It is interesting to notice that here too one has two widths associated with a doorway state in the  $Q$ -subspace. To see this one has to extract this doorway state from the rest of the  $Q$ -subspace and then average out the  $Q'$ -subspace (since there are many resonances in  $Q'$ -subspace). Then one writes  $Q = D + Q'$  and by using simple operator reshuffling techniques one gets the following expression for  $D(E - H_{QQ} + i\eta)^{-1} D^{-1}$ :

$$D (E - H_{QQ} + i\eta)^{-1} D = D (E - H_{DD} - W_{DD} - \hat{W}_{DD})^{-1} D \quad (26)$$

where we have defined

$$W_{DD} \equiv H_{DP} G_{PP} H_{PD}$$

$$\hat{W}_{DD} \equiv H_{DQ'} G_{Q'Q'} H_{Q'D}$$

Now  $W_{DD}$  can be analyzed by decomposing  $P$  resulting in the expression

$$W_{DD} = H_{DQ} Q G_{PP} Q H_{QD} = H_{DQ} Q \frac{1}{E - H_{QQ} - \omega_{QQ} + i\eta} Q H_{QD} \quad (27)$$

If one again picks a  $q$  state to be a secondary doorway then by similar consideration as the above one arrives at

$$W_{DD} = H_{Dd} d \frac{1}{E - H_{dd} - \omega_{dd} + i\eta} d H_{dD} \quad (27')$$

where

$$\omega_{dd} \equiv H_{dq'} G_{q'q'} H_{q'd}$$

Then  $T_{fi}^c$  may be expressed as:

$$T_{fi}^c = \langle \Phi_P^{(-)} | H_{PQ} \frac{D}{E - H_{DD} - W_{DD} - \hat{W}_{DD}} H_{DP} G_{PP}(\epsilon^{(+)}) \tilde{V}_Y | (N, Z) \rangle \quad (28)$$

Using the doorway approximation again for the  $q$  subspace one arrives at

$$T_{fi}^c = \langle \xi_p^{(-)} | h_{pd} (\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd} + i\eta)^{-1} (1 + \Lambda(\epsilon)) + 1 ] (H_{dD} + H_{qD}) (\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD} + i\eta)^{-1} (H_{Dq} + H_{Dd}) (q G_{pp} q + q G_{pp} p) \tilde{V}_Y | (N, Z) \rangle \quad (29)$$

Now

$$q G_{pp} q = d G_{pp} d + q' G_{pp} q' + q' G_{pp} d + d G_{pp} q'$$

$$d G_{pp} d = d \frac{1}{\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd}} d$$

$$q' G_{pp} q' = q' (\epsilon - h_{q'q'} - \omega_{q'q'} - \hat{\omega}_{q'q'})^{-1} q' \quad \text{etc...}$$

We shall make a further approximation  $H_{q'D} = 0$ . This corresponds to the observation that in actual reactions e.g. (p,2p), the secondary compound states,  $q'$ , corresponds to n hole-(n-1) particle states which are, due to their complexity, coupled weakly to the primary doorway, D, namely in our case the state in the compound nucleus that is formed by the direct process. In the case of ( $\gamma$ ,2p) the primary doorway is the highly excited target nucleus whereas the secondary compound is again n hole-(n-1) particle excitation. The secondary doorway would be the hole state which decays by emitting the second proton.

Since  $\tilde{V}_Y$  involves the operator  $(1 + H_{pQ} \frac{Q}{\epsilon - H_{Qa} + i\eta})$  on its left then using the fact that  $V_Y$  connects the target state only weakly to the P states, we are led to conclude that  $\tilde{V}_Y \approx H_{pQ} G_{Qa} V_Y \approx H_{DQ} \frac{1}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD} + i\eta} V_Y$ . Thus the final form of the transition amplitude  $T_{fi}^c$  is

$$T_{fi}^c = \langle \xi_p^{(-)} | h_{pd} \frac{d}{\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd} + i\eta} H_{dD} \frac{D}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD} + i\eta} H_{Dd} \frac{d}{\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd}} H_{DQ} \frac{D}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD}} V_Y | (N, Z) \rangle \quad (30)$$

where we have used the fact that, by construction,  $\langle \xi_p^{(-)} | H_{dD} = 0$  and also  $\Lambda(\epsilon) H_{dD} = 0$  by definition.

One wants to cast the amplitude  $T_{fi}^c$  such that the primary doorway propagator contains information about the secondary doorway state i.e. the hole state and not necessarily the opposite since the reaction envisaged is irreversible, i.e. one loses a proton and thus the secondary doorway forgets totally its mother state the primary doorway.

It is easy to show that by using the doorway approximation for the second term in (16) the result is such as to simplify the above expression for  $T_{fi}^c$  into:

$$T_{fi}^c \approx \langle \xi_p^{(-)} | h_{pd} \frac{d}{\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd}} (1 + \Lambda(\epsilon)) H_{dD} \frac{D}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD}} V_Y | (N, Z) \rangle \quad (31)$$

where we have made use of the identity

$$\frac{D}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD}} \frac{W_{DD}}{E - H_{DD} - W_{DD}} = \frac{D}{\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD}} - \frac{D}{\epsilon - H_{DD} - \hat{W}_{DD}}$$

In the above expression for  $T_{fi}^c$  the hole state propagator  $(\epsilon - h_{dd} - \omega_{dd} - \hat{\omega}_{dd})^{-1}$  is only an approximate one as the explicit effect of the  $q'$  space on the escape width of the secondary doorway i.e.:  $|\langle \xi_p^{(-)} | h_{pd} | d \rangle|^2$  is not included. We shall consider this fluctuation contribution to the escape width in the next section.

Now we concentrate on the primary doorway propagator  $(\epsilon - H_{DD} - W_{DD} - \hat{W}_{DD})^{-1}$ . Since by definition, all operators appearing in the propagator are related to the P states one is then led to consider how any intermediate structure namely energy variation in P is going to affect the measurable quantities in the primary doorway propagator in particular its total width and continuum shift which are defined in a similar way to  $\chi_q$  and  $\xi_q$  (equations (13) and (14)).



$$\Gamma_D \equiv 2 \text{Im}(W_D + \hat{W}_D) \quad (32)$$

$$\Delta_D \equiv \text{Re}(W_D + \hat{W}_D)$$

It is obvious that  $\Gamma_D$  is composed of two terms, the total escape width  $\Gamma_D^\uparrow \equiv 2 \text{Im} W_D$  and the total damping width  $\Gamma_D^\downarrow \equiv 2 \text{Im} \hat{W}_D$ . The same decomposition is true for  $\Delta_D$ . The fact that in P there are states that are decaying gives rise to a strong energy dependence in  $\Gamma_D$  and  $\Delta_D$ . This is a dependence on the energy. It is easy to write down, explicitly, the form of the quantities in (32) in terms of those related to the secondary doorway namely  $\gamma_d$  and  $\delta_d$  as:

$$\Delta_D(\epsilon) \equiv \langle D | H_{Dd} | d \rangle \frac{\epsilon - \epsilon_d - \delta_d(\epsilon)}{(\epsilon - \epsilon_d - \delta_d(\epsilon))^2 + \frac{\gamma_d^2(\epsilon)}{4}} \langle d | H_{dD} | D \rangle \quad (33)$$

and

$$\Gamma_D(\epsilon) \equiv \langle D | H_{Dd} | d \rangle \frac{\gamma_d(\epsilon)}{(\epsilon - \epsilon_d - \delta_d(\epsilon))^2 + \frac{\gamma_d^2(\epsilon)}{4}} \langle d | H_{dD} | D \rangle$$

Where we have made use of the decomposition of the P-space Green's function  $(E - H_{pp} + i\eta)^{-1}$  as in equation (23) and neglected  $\langle D | H_{Dd} \frac{1}{E - H_{pp} + i\eta} H_{dD} | D \rangle$  in the coupling of D to d.

Equation (33) constitutes the main result of this section. It relates the resonance escape width and energy continuum shift of the primary doorway state, which in our example of  $(\gamma, 2p)$  is just an excited state of the target nucleus  $(N, Z)^*$ , to that of the secondary or subdoorway state which is an excited state of the nucleus  $(N, Z-1)^*$ . In a  $(p, 2p)$  reaction the primary doorway is again the excited target nucleus and the secondary is the hole state formed by emitting (or knocking-out if the process is very fast) a proton. If the primary doorway is very short-lived meaning that  $\Gamma_D$  is very large then one may speak of a direct reaction as responsible for the proton knock-out. In general  $\Gamma_D \geq \gamma_d$  and thus one gets a general inequality namely:

$$\frac{|\langle D | H_{Dd} | d \rangle|^2}{(\epsilon - \epsilon_d - \delta_d(\epsilon))^2 + \frac{\gamma_d^2(\epsilon)}{4}} \geq 1 \quad (34)$$

When the above ratio is very large one speaks of a usual knock-out reaction that populates the decaying hole states. On the other hand if the ratio is small, e.g. 0.3 then one speaks of a sequential decay process. Notice that both  $\delta_d(\epsilon)$  and  $\gamma_d(\epsilon)$  are assumed strongly dependent on  $\epsilon$  since the above condition should be valid throughout the spectrum of the first emitted proton (in  $(\gamma, 2p)$ ) or the "knocked out" proton (in  $(p, 2p)$ ).

The importance of the continuum shift  $\Delta_D$  becomes apparent at  $\epsilon = \epsilon_d$  then one has from (33)

$$\Delta_D(\epsilon_d) \approx - \frac{|\langle D | H_{Dd} | d \rangle|^2}{\delta_d^2(\epsilon_d) + \frac{\gamma_d^2(\epsilon_d)}{4}} \delta_d(\epsilon_d) \quad (35)$$

A positive continuum shift  $\delta_d(\epsilon_d)$  implies that  $\Delta_D(\epsilon_d)$  is negative! and vice-versa.

One can easily invert the above equation so that given the parameters  $\Delta_D(\epsilon)$  and  $\Gamma_D(\epsilon)$  at  $\epsilon = \epsilon_d$  one gets  $\delta_d(\epsilon_d)$  and  $\gamma_d(\epsilon_d)$  namely:

$$\delta_d(\epsilon_d) \approx \frac{-|\langle D | H_{Dd} | d \rangle|^2}{\Delta_D^2(\epsilon_d) + \frac{\Gamma_D^2(\epsilon_d)}{4}} \Delta_D(\epsilon_d) \quad (36)$$

$$\gamma_d(\epsilon_d) \approx \frac{|\langle D | H_{Dd} | d \rangle|^2}{\Delta_D^2(\epsilon_d) + \frac{\Gamma_D^2(\epsilon_d)}{4}} \Gamma_D(\epsilon_d)$$

This form is convenient for the discussion of sequential decay i.e. everything on the right of (36) refers to an event preceding in time those events described (or labelled) by the quantities on the left of (36). Note that one has the following suggestive formula

$$\frac{\Delta_D(\epsilon_d)}{\delta_d(\epsilon_d)} \approx - \frac{\Gamma_D(\epsilon_d)}{\gamma_d(\epsilon_d)} \quad (37)$$

Thus in principle  $\delta_d(\xi_d)$  could become large and negative depending on the value of the other parameters  $\Delta_D(\xi)$ ,  $\Gamma_d(\xi)$  and  $\gamma_d(\xi)$  if their respective signs are such that as to give an over all positive sign. Thus a right combination of the value of these parameters could render the secondary doorway resonance into a bound state. Naturally the above is only an approximate treatment as the inclusion in the above formulae for  $\delta_d$  and  $\gamma_d$  i.e. equation (36), of the coupling to the compound  $q'$ -state as well as the  $Q'$ -states will modify the conclusions.

It is interesting to observe how changing the behaviour of one resonance affects the other in such a way as to reflect the strong historical connection between the two. In the case of a direct process in the entrance channel,  $\Gamma_D \rightarrow \infty$  and  $\Delta_D \rightarrow 0$  and thus one practically gets a zero-width resonance in the final channel namely the hole-state channel. The fact that the hole state does have a non-zero width suggests that at least part of the excitation process in the entrance channel goes via a short-lived resonant primary state.

### III. THE CROSS SECTION

We have already mentioned the way to obtain the fluctuating components of the cross section without interference from the smooth ones by using the idea of the optical background representation (OBR)<sup>6</sup>. Since the problem of averaging is closely connected to the energy resolution of the detecting apparatus it should be clear that if one is looking for the energy spectrum of the first outgoing proton namely the cross section as a function of  $\xi \equiv E - \frac{k_p^2}{2m}$  the averaging procedure will affect only those quantities that depend on  $\xi$ . Thus it is the wave

function  $\langle \Psi_p^{(-)} | p \rangle$  that suffers from the limitation imposed by the energy resolution of the detector.

The averaged cross section associated with (31) can be cast in a form that represents the incoherent contribution of two parts, a rather smooth  $\xi$ -dependent part and a fluctuating part. One can "evaluate" the fluctuation part by employing the (OBR) treatment on the  $p$ -part of wave function i.e.  $\langle \Phi_p^{(-)} | p \rangle$ . The details of the derivation that leads to equation (38) below are presented in appendix (I). The final form assumed by the  $\xi$ -averaged cross section ( $\xi$ -averaged spectrum of first emitted proton) is:

$$\left\langle \frac{d\sigma_{cc'}(\xi)}{d\Omega d(\xi)} \right\rangle_I = \frac{\overline{d\sigma_{cc'}}}{d\Omega d(\xi)}(\xi) + \frac{d\sigma_{cc'}^{fl}}{d\Omega d(\xi)}(\xi) \quad (38a)$$

where  $I$  is much larger than the averaged  $q'$ -resonance width and smaller than  $\gamma_d$ : the hole state (doorway) widths, and

$$\frac{d\overline{\sigma_{cc'}}}{d\Omega d(\xi)}(\xi) = \left| X_{cd}(\xi) \right|^2 \frac{\gamma_{d,c'}^\uparrow}{(\xi - \xi_d - \delta_d)^2 + \frac{\gamma_d^2}{4}} \quad (38b)$$

where  $|X_{cd}(\xi)|^2$  is a formation form factor which, in most cases, is a slowly varying function of  $\xi$  as it should be regardless to whether one reaches  $d$  via a primary doorway or not since a primary doorway corresponds to a complex pole in  $|X_{cd}(\xi)|^2$  as a function of the total energy  $E$  and this will not be reflected in the  $\xi$ -averaged spectrum unless one is at the extreme end of the spectrum namely near  $\xi \sim \frac{k_p^2}{2m}$  then one should observe some kind of threshold phenomenon in which the first doorway is partially "observed" in the spectrum if the total energy happens to be near the first doorway resonance.  $\gamma_{d,c'}^\uparrow(\xi)$  is the hole state escape width to channel  $c'$ ,  $\xi_d + \delta_d$  is the position of its resonance and  $\gamma_d$  is its total width namely  $\gamma_d^\uparrow + \gamma_d^\downarrow$  where  $\gamma_d^\downarrow$  is the damping width

to the  $q'$ -states.

The second part of (38a),  $\frac{d\sigma_{cc'}^{fl}(\epsilon)}{d\Omega d(\epsilon)}$  is the built-in fluctuation cross section that arises from averaging out all the  $q'$ -states. This fluctuation cross-section still contains real energy-dependence due to the coupling of the hole state both to the continuum as well as the  $q'$ -states.

Explicitly written,  $\frac{d\sigma_{cc'}^{fl}(\epsilon)}{d\Omega d(\epsilon)}$  has the following form (see appendix I)

$$\frac{d\sigma_{cc'}^{fl}(\epsilon)}{d\Omega d(\epsilon)} = |X_{cd}(\epsilon)|^2 \frac{1}{(\epsilon - \epsilon_d - \delta_d)^2 + \gamma_d^2/4} \frac{2\pi}{\gamma D} \times \left\{ \langle |R_{dq'}(\epsilon)|^2 \rangle_{q'} \langle |P_{q'p'}(\epsilon)|^2 \rangle_{q'} + \langle |R_{dq'}(\epsilon) P_{q'p'}^\dagger(\epsilon)|^2 \rangle_{q'} \right\} \quad (38c)$$

where  $\gamma$  and  $D$  are the average width and spacing of the  $q'$ -resonances.  $R_{dq'}(\epsilon)$  and  $P_{q'p'}(\epsilon)$  are complicated "widths" that are not completely independent as there is a strong correlation among the  $q'$ -states. This level-level correlation gives rise to the second term in the curly brackets. The first term can be further reduced to a Hauser-Feshbach form involving intermediate-structure-modulated transmission coefficients if one neglects the second term completely as was done by Hufner et al<sup>7)</sup>. This evaporation of the  $q'$ -resonances never materializes as long as the second term is present which is apparently the case. We could not reduce the second term further into a simpler one, nevertheless we cannot escape the conclusion that the result of reference<sup>7)</sup> is only partly true namely that the  $q'$ -states do not evaporate protons but there is some kind of non-equilibrium processes that would induce the emission of the protons namely a pre-equilibrium emission from the  $q'$ -states, that accompanies, and thus modifies, the decay of the hole states. This

conclusion is further substantiated by the observation that evaporation would result if one can really separate very-delayed processes namely those involving  $q'$ -states from those not so delayed namely corresponding to a time  $\sim (\gamma_d^2)^{-1}$ . This separation is hard to visualize in ( $\gamma, 2p$ ) or ( $p, 2p$ ) as observed by<sup>8)</sup>.

#### IV. DISCUSSION

Among the most obvious consequences of our theory is the effect of one decay on the other. This is seen from the expression in (36) relating the width and continuum shift of the secondary doorway resonance to that of the primary doorway where the right-hand side is evaluated at a particular energy  $\epsilon_d$ . The case  $\Gamma_D(\epsilon_d) > \gamma_d(\epsilon_d)$  has already been discussed as it is relevant for ( $p, 2p$ ).

In cases where  $\Gamma_D(\epsilon_d) \ll \gamma_d(\epsilon_d)$ , one has

$$\frac{|\langle D | H_{Dd} | d \rangle|^2}{\Delta_D^2(\epsilon_d) + \frac{\Gamma_D^2(\epsilon_d)}{4}} \gg 1 \quad (39)$$

Obviously the above will be true only if  $\Gamma_D^2/4 + \Delta_D^2 \ll |\langle D | H_{Dd} | d \rangle|^2$ . If  $\Delta_D(\epsilon_d) \gg 0$  and large, such that  $\delta_d(\epsilon_d)$  is negative and very large, then as one changes  $\Delta_D(\epsilon_d)$  a point is reached where  $\delta_d(\epsilon_d)$  is so large and negative that it renders the secondary doorway resonance into a bound state since  $\epsilon_d + \delta_d(\epsilon_d)$  becomes negative. The case  $\Gamma_D(\epsilon_d) < \gamma_d(\epsilon_d)$  is quite interesting indeed because it says that even though the secondary doorway state is a bound state, i.e. its decay is stopped, the doorway state is still a normal resonance. Thus the important quantity that specifies the nature of resonance in the case of sequential decays of unstable system seems to

be the continuum shift and not the width. This fact seems to result from the important role played by the historical connection of resonances with each other.

This is to be contrasted with the conventional relation between widths and shift, through dispersion relations, in the case of a one-step decay. Approximations that render many-step decay of an unstable system into many one-step decays should take into account this new feature of the sequential nature of the decay. It is obvious that the important ingredient that induces the above effect is the matrix element  $\langle D | H_D | H \rangle$ . An estimate of this coupling may be made by measuring the total cross-section at first doorway resonance; then at that "incident" energy one measures the energy spectrum at second doorway resonance, and by comparing the two numbers one gets a simple relation that determines the matrix element  $\langle D | H_D | H \rangle$ , since one assumes that one knows  $\Delta_D$  as well as  $\gamma_D$ .

One way of checking the above ideas is to detect the two outgoing protons in coincidence, one proton comes from the decay of the primary doorway, the other comes from the decay of the hole state. One anticipates that the cross section for such events is quite small compared with other competing direct as well as direct-compound processes. However, it is certainly worthwhile looking at a genuine sequential decay process in order to check the validity of approximating the hole-state propagator by a Breit-Wigner expression as most seem to be doing.

The second finding in this work is the form of the cross-section as a function of the energy of the first emitted proton i.e. the energy

spectrum. As was shown, the presence of strong level-level correlation among the  $q'$ -resonances results in a non-evaporation expression for the fluctuation cross-section. Only in the limit of no level-level correlation would one then get a Hauser-Feshbach formula. This finding is contrasted with that of ref. 7) where a Hauser-Feshbach expression for the fluctuation cross-section was obtained. This should then serve as a warning for those who are extracting information about hole states and assuming the simplest form for the fluctuation cross section associated with these states. Not only  $(p, 2p)$  and  $(\sigma, 2p)$  reactions are affected by our conclusions, but also the decay of muonic atoms assuming that they are prepared secondary-doorway states as discussed in 7).

APPENDIX I

We shall derive equation (38) for the average cross-section. We decompose the projection operator P into the following

$$P = p + q' + d \quad A.1$$

where p, q and d are also projection operators. p projects out open channels, d projects out the secondary doorway (hole state) subspace, and q' the compound components of the hole state. The Schrodinger equation for  $|\phi_p^{(-)}\rangle$  is then:

$$(\epsilon - \bar{h}_{pp}) p |\phi_p^{(-)}\rangle = \bar{h}_{pd} d |\phi_p^{(-)}\rangle \quad A.2$$

$$(\epsilon - \bar{h}_{dd}) d |\phi_p^{(-)}\rangle = \bar{h}_{dp} p |\phi_p^{(-)}\rangle$$

where the q' part has been eliminated as usual.

Following the procedure of reference (6), we take the energy average of A.2

$$(\epsilon - \bar{h}_{pp}) \overline{p |\phi_p^{(-)}\rangle} = \bar{h}_{pd} \overline{d |\phi_p^{(-)}\rangle} \quad A.3$$

$$(\epsilon - \bar{h}_{dd}) \overline{d |\phi_p^{(-)}\rangle} = \bar{h}_{dp} \overline{p |\phi_p^{(-)}\rangle}$$

The energy average is taken in an interval  $\Delta\epsilon$  such that  $\bar{\gamma}_q \ll \Delta\epsilon < \bar{\gamma}_d$ . Thus  $\overline{p |\phi_p^{(-)}\rangle}$  will show intermediate structure due to d. To derive the fluctuation part of the wave functions  $p |\phi_p^{(-)}\rangle$  and  $d |\phi_p^{(-)}\rangle$  we add and subtract  $\bar{h}$  to the

effective Hamiltonians in A.2 arriving at:

$$\begin{pmatrix} \epsilon - \bar{h}_{pp} & -\bar{h}_{pd} \\ -\bar{h}_{dp} & \epsilon - \bar{h}_{dd} \end{pmatrix} \begin{pmatrix} p |\phi_p^{(-)}\rangle \\ d |\phi_p^{(-)}\rangle \end{pmatrix} = \begin{pmatrix} \mathcal{V}_{pp}^f & \mathcal{V}_{pd}^f \\ \mathcal{V}_{dp}^f & \mathcal{V}_{dd}^f \end{pmatrix} \begin{pmatrix} p |\phi_p^{(-)}\rangle \\ d |\phi_p^{(-)}\rangle \end{pmatrix} \quad A.4$$

where the  $\mathcal{V}_{f,s}^f$  are responsible for the fluctuation due to the q-states and are just  $(\bar{h}_s - \bar{h})$ .

The equations A.4 may be written

$$M_{opt} \Phi = N_{fluc} \Phi \quad A.5$$

$M_{opt}$  is the matrix on the left-hand side of A.4 and refers to optical potentials with intermediate structure.

A.4 permits a formal solution

$$\Phi = \Phi_{opt} + (M_{opt})^{-1} N_{fluc} \Phi$$

Since

$$\langle \Phi \rangle_{\Delta\epsilon} = \Phi_{opt} \quad A.6$$

Thus

$$\langle (M_{opt})^{-1} N_{fluc} \Phi \rangle_{\Delta\epsilon} = 0$$

The quantity of interest is  $d |\phi_p^{(-)}\rangle$ . From A.6 and by writing the separable form of the  $\mathcal{V}_{f,s}^f$ ,  $d |\phi_p^{(-)}\rangle$  is found to be

$$d |\phi_p^{(-)}\rangle = G_{dd}^{(-)} \bar{h}_{dp} |\bar{\phi}_p^{(-)}\rangle + B [V_{qt} + V_{qd} G_{dd}^{(-)} \bar{h}_{dp} + V_{qt} \int_{opt} \bar{h}_{pd} G_{dd}^{(-)} \bar{h}_{dp}] |\bar{\phi}_p^{(-)}\rangle \quad A.7$$

where  $G_{dd}(\epsilon) = (\epsilon - \bar{h}_{dd} - \bar{h}_{dp} \mathcal{G}_{opt} \bar{h}_{pd})^{-1}$

$B = C V_{pq}(\text{Det})_{ii} \mathcal{G}_{ii} + D V_{dq}(\text{Det})_{ii} \mathcal{G}_{ii}$

where C + D are determined from

$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \equiv (M_{opt})^{-1}$

and  $(\text{Det})_{ii} \equiv (1 - V_{qp} A V_{pq} - V_{qp} B V_{dq} - V_{qd} C V_{pq} - V_{qd} D V_{dq})^{-1}$  is the propagator:

$(\epsilon - \bar{h}_{pp} + i\gamma)^{-1}$

i.e. the usual (no intermediate-structured) propagator.  $|F_p^{(-)}\rangle$  is also the optical potential wave function with no intermediate structure namely  $|F_p^{(-)}\rangle$  is just a potential scattering wave function describing the second proton and the final residual nucleus.

Eq. A.7 exhibits all wanted features of the secondary doorway wave function, namely  $d|\Phi_p^{(-)}\rangle = \overline{d|\Phi_p^{(-)}\rangle} + d|\Phi_p^{(-)}\rangle$  fluctuation. Since the energy-averaged cross section is

$\langle \frac{d\sigma}{d\Omega} \rangle_{\Delta\epsilon} \propto |\langle \Phi_p^{(-)} | d A(\epsilon) | (N, z) \rangle|^2$

thus

$\langle \frac{d\sigma}{d\Omega} \rangle_{\Delta\epsilon} \propto |\langle \Phi_p^{(-)} | d A(\epsilon) | (N, z) \rangle|^2 + \langle \langle \Phi_p^{(-)} | d^{fluc} A(\epsilon) | (N, z) \rangle \rangle_{\Delta\epsilon}^2$

where we have used the fact that :

$\langle \langle \Phi_p^{(-)} | d^{fluc} A(\epsilon) | (N, z) \rangle \rangle_{\Delta\epsilon} = 0$  by construction. By using the explicit form of  $d|\Phi_p^{(-)}\rangle^{fluc}$  we then arrive at the form in equation (38) with  $R$  and  $P$  given by:

$R_{dq'}(\epsilon) \equiv \langle d | V_{dq'}(\epsilon) (\text{Det})_{ii} \mathcal{G}_{ii}^{-1} | q' \rangle$

A.8

$P_{qp'}(\epsilon) \equiv \langle q' | [V_{qp'} + V_{qd} G_{dd} \bar{h}_{dp} + V_{qp'} \mathcal{G}_{opt} \bar{h}_{pd} G_{dd} \bar{h}_{dp}] | F_p \rangle$

where

$V_{dq'} = G_{dd}^{-1}(\epsilon) (C_{dp} V_{pq'} + D_{dd} V_{dq'})$

Throughout, similar statistical assumptions were used as in <sup>6)</sup>.

#### ACKNOWLEDGEMENTS

Part of this work was done while the author was visiting the Technischen Universität München. He would like to thank Professor Dr. Klaus Dietrich for his hospitality. He also thanks A.P.R. de Toledo Piza and G. Jacob for useful discussions. Thanks are also due to Dr. P.E. Hodgson for hospitality at the Nuclear Physics Laboratory at Oxford. A travel grant from FAPESP is also acknowledged.

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TABLE CAPTION

Table 1: The decomposition of the nuclear Hilbert space into different orthogonal subspaces through the relevant projection operators used in the text.

FIGURE CAPTION

Fig.1: The  $(\gamma, 2p)$  process, the two shaded circles represent the two compound systems. Double arrows 1 and 2 refer to the target and the final nucleus respectively.



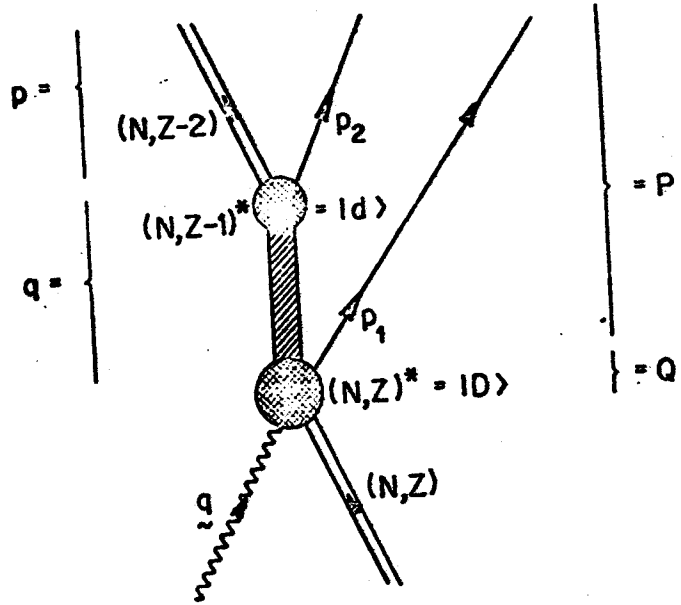


Fig 1.

Projection Operators	Hilbert Space Spanned	Doorway Decomposition	Doorways	Resonance Parameters
$Q$	$\{ (N,Z)^* \rangle\}$	$Q = D + Q'$	D=Primary doorway in $(N,Z)^*$	$\Gamma_D$ & $\Delta_D$
$P$	$I - Q = P + q$	_____	_____	_____
$P$	$P_1 + P_2 + (N,Z-2)^* q$	_____	_____	_____
$q$	$\{ (N,Z-1)^* \rangle\}$	$q = d + q'$	d=secondary doorway: in $(N,Z-1)^*$ i.e. the hole state	$\Gamma_d$ & $\Delta_d$

TABLE 1