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EFFECTIVE POTENTIALS IN GAUGE FIELD THEORIES

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ABSTRACT

An elementary and very efficient method for computing the effective potential of any theory containing scalar bosons is described. Examples include massless scalar electrodynamics and Yang-Mills theories.

1- Introduction

The effective potential is the generating function of the one-particle-irreducible (1PI) zero-momenta Green functions. It is mainly used to detect spontaneous symmetry breaking, that is, non-invariant vacua [1]. As one should expect, computing effective potentials is far from trivial. In principle, one should sum up all zero-momenta 1PI Green functions, an impossible task except for particularly simple lagrangians and low orders of approximation. A brilliant example of such a computation is Coleman- Weinberg's treatment of scalar electrodynamics.[1] An elegant, functional-theoretic method was devised by Jackiw [2] and is efficient up to one-loop approximations.

In 1975, Lee and Sciaccaluga [3] introduced a very clever way of computing the effective potential of the $\lambda\varphi^4$ theory, including two-loop contributions. Apparently they did not try their method in gauge theories, where some new problems arise and the interest is greater, due to the Higgs-Kibble mechanism [4].

In this paper we study the extension of the Lee-Sciaccaluga method to gauge theories. After solving some problems connected to the gauge-fixing terms, we have in hands a technique that allows for computation up to any order in the loop expansion. We perform, in the case of scalar electrodynamics, the computation up to two-loop contributions in a one-parameter class of gauges involving ghosts.

This constitutes already a rather general setting. Furthermore, we discuss the extension to non-abelian theories in detail, and perform a one-loop calculation.

The justification of the method is given in Section 2. In Section 3 we apply it to the Coleman-Weinberg lagrangian and compute the one-loop contribution. Section 4 contains a sample of the two-loop results. Section 5 discusses the nos-abelian case and some further extensions.

2 - Presenting the method

Consider the lagrangian

$$\mathcal{L}(\varphi, \partial_\mu \varphi) = -\frac{1}{2} \partial_\mu \varphi \, \partial_\mu \varphi - U(\varphi) \tag{1}$$

where $U(\varphi)$ is a polynomial in the scalar field φ . The effective potential is defined in the following way: let the functional generator of the IPI Green functions, $\Gamma[\varphi]$, be developed in a power expansion in the derivatives of the field,

$$-\Gamma[\varphi] = \int d^4x \left[V(\varphi) + \frac{1}{2} Z \, (\partial_\mu \varphi)^2 + \cdots \right] \tag{2}$$

where V,Z,......are ordinary functions of φ. $V(\varphi)$ is the effective potential.

The usual way to write is, of course,

$$\Gamma[\varphi] = \sum_n \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \, \Gamma^{(n)}(x_1, \cdots x_n) \, \varphi(x_1) \cdots \varphi(x_n) \tag{3}$$

where $\Gamma^{(n)}(x_1, \dots x_n)$ is the n-point IPI Green function. Fourier transforming and using the translation invariance, one gets

$$(4) \qquad \Gamma[\varphi] = \sum_n \frac{1}{n!} \int d^4x \, [\varphi(o)]^n \, \tilde{\Gamma}^{(n)}(0, \dots 0)$$

the Fourier components being calculated at zero momenta. Comparing to (2) one gets, for constant φ ,

$$(5) \qquad V(\varphi) = -\sum_n \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, \dots 0) \, \varphi^n$$

which allows the computation of the effective potential in terms of an infinite series whose coefficients are the IPI Green functions of the theory. Although this is hardly a convenient method of computation, Coleman and Weinberg used it to obtain, with great ingenuity, the one-loop approximation in some cases.

Symanzik [5] has shown that $V(\varphi)$ is the expectation value of the energy per unit volume in a state for which the field has the constant value φ. Clearly, in the tree approximation, $V(\varphi)$ coincides with $U(\varphi)$ of equation (1). For a theory in which the vacuum expectation value of the field operator does not vanish, but has the value v, one has, instead of [5],

$$(6) \qquad V(\varphi) = -\sum_n \frac{1}{n!} \tilde{\Gamma}^{(n)}(0, \dots 0) \, (\varphi - v)^n .$$

From this equation it follows that

$$(7) \qquad \left. \frac{d V(\varphi)}{d \varphi} \right|_{\varphi = v} = 0$$

which allows the computation of the vacuum expectation value as the solution of a minimum problen. This is, perhaps, the most useful property of the effective potential.

By the introduction of a new field,

$$\varphi'(x) = \varphi(x) - \vartheta$$

where ϑ is an arbitrary parameter, the lagrangian $\mathcal{L}(\varphi, \partial_\mu \varphi)$ is transformed into the new lagrangian $\mathcal{L}'(\varphi', \partial_\mu \varphi')$ which contains some new ϑ-dependent vertices.

We can rewrite eq.(4) as

$$(8) \qquad \Gamma[\varphi' + \vartheta] = \sum_n \frac{1}{n!} \int d^4 x_1 \ldots d^4 x_n \, \Gamma^{(n)}(x_1, \ldots x_n) [\varphi'(x_1) + \vartheta] \ldots [\varphi'(x_n) + \vartheta] .$$

Defining

$$\Gamma'[\varphi'] = \Gamma[\varphi' + \vartheta]$$

and re-summing eq.(8), one gets

$$(9) \qquad \Gamma'[\varphi'] = \sum_n \frac{1}{n!} \int d^4 x_1 \ldots d^4 x_n \, \bar{\Gamma}^{(n)}(x_1, \ldots x_n) \, \varphi'(x_1) \ldots \varphi'(x_n)$$

In the tree aprosimation, $\Gamma'[\varphi']$ coincides with the lagrangian $\mathcal{L}'(\varphi', \partial_\mu \varphi')$. It is then clear that $\bar{\Gamma}^{(n)}(x_1, \ldots x_n)$ are proper vertices computed with the lagrangian $\mathcal{L}'(\varphi', \partial_\mu \varphi')$. Fourier transforming eq.(9) one gets

$$(10) \qquad \Gamma'[\varphi'] = \sum_n \frac{1}{n!} \int d^4 x \, [\varphi'(0)]^n \, \tilde{\bar{\Gamma}}^{(n)}(0, \ldots 0)$$

where $\tilde{\bar{\Gamma}}^{(n)}(0, \ldots 0)$ is the n-point proper vertex computed, in momentum space, with the Feynman rules of the lagrangian $\mathcal{L}'$. For constant φ' one has

$$V'(\varphi') = V(\varphi'+v) = -\sum_n \frac{1}{n!}\,\tilde{\bar{\Gamma}}^{(n)}(0,\dots 0)\,\varphi'^{\,n} \quad . \tag{11}$$

It follows that

$$\left.\frac{dV(\varphi'+v)}{d\varphi'}\right|_{\varphi'=0} = -\,\tilde{\bar{\Gamma}}^{(1)}(0) \quad . \tag{12}$$

As

$$(13) \qquad \left.\frac{dV(\varphi'+v)}{d\varphi'}\right|_{\varphi'=0} = \left.\frac{dV(\varphi)}{d\varphi}\right|_{\varphi=v} ,$$

equation (12) becomes

$$(14) \qquad \left.\frac{dV}{d\varphi}\right|_{\varphi=v} = \frac{dV}{dv} = -\tilde{\tilde{\Gamma}}^{(1)}_{(0)} .$$

That is, one gets a simple differential equation involving the tadpole computed according to the rules of $\mathcal{L}'$: putting $v = \varphi$ at the end, one gets the effective potential $V(\varphi)$ of the theory defined by the lagrangian of Eq. (1).

3 - Electrodynamics - one loop.

We apply the method now to the Coleman-Weinberg lagrangian

$$(15) \quad \mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{2}(\partial_\mu\varphi_1 + eA_\mu\varphi_2)^2 - \frac{1}{2}(\partial_\mu\varphi_2 - eA_\mu\varphi_1)^2 - \frac{\lambda}{4!}(\varphi_1^2+\varphi_2^2)^2 .$$

We are looking for the effective potential as a function of φ_1 and φ_2 , that is, for $A_\mu = 0$ .To our avail, the O(1) global gauge symmetry will be used : it says that V(φ_1, φ_2) is a funtion only of $\varphi_1^2 + \varphi_2^2$, restricting the problem to that of determining the dependence on, say, φ_1 .

As is well-known, a definite gauge must be chosen. This, coupled to the method being presented, raises some questions which we discuss now. It is natural that we, at this stage,fix the gauge and only then replace φ_1 by $\varphi_1 + v$ everywhere. The effective potentialobtained as explained in the previous section is that of the theory described by the lagrangian of Eq.(15) in the gauge previously chosen. This is quite clear.

But, what if we <u>first</u> shift the field φ_1 and <u>then</u> choose a gauge? To which gauge of the unshifted theory does this potential correspond ? Let us examine an instructive example. After replacing, in Eq.(15), φ_1 by $\varphi_1 + v$, we are led to a different lagrangian, with several new vertices, one of which is particularly perverse : it comes from the term

$$e v (\partial_\mu \varphi_2) A_\mu$$

and may be called a mixed propagator. It is perverse because it appears infinitely many times in each order of the loop expansion. In order to avoid this problem, the most efficient way is the following : we cancel it by a convenient gauge choice. For instance, the popular R_ξ gauge-fixing term

$$\mathcal{L}_G = -\frac{1}{2}\xi\left(\partial_\mu A_\mu - \frac{ev}{\xi}\varphi_2\right)^2 \tag{16}$$

does precisely that. Suppose we use this gauge. To which gauge of the unshifted theory does it correspond ? Besides that, in Eq.(16) the parameter v does not appear in the way it should (that is, as a consequence of the shifting of φ_1)! We could think that, as the shifted lagrangian tends to (15) as v goes to zero, (16) would correspond to the gauge condition

$$\mathcal{L}_G = -\frac{1}{2}\xi\,(\partial_\mu A_\mu)^2 \tag{17}$$

of the unshifted theory, (17) being obtained from (16) by putting v=0 . But then, there are many gauge-fixing terms that go, for v=0, into (17) !This would mean that, given a lagrangian and a gauge-fixing term, many effective potentials could be found, differing nontrivially from each other. It is, then, clear that one must fix the gauge before shifting the theory.

It is, however, a pleasant surprise that a gauge-fixing term exists which, after the shifting is done, precisely cancels the mixed propagator.

It is

$$\mathcal{L}_G = -\frac{1}{2}\xi\left(\partial_\mu A_\mu - \frac{e}{\xi}\varphi_1\varphi_2\right)^2 \tag{18}$$

with the corresponding Faddeev-Popov [6] term

$$\mathcal{L}_{FP} = \bar{c}\left\{\partial^2 - \frac{e^2}{\xi}\left(\varphi_1^2 - \varphi_2^2\right)\right\}c \ . \tag{19}$$

In fact, by transforming φ_1 into $\varphi_1 + v$,one has for the gauge-fixing term of the shifted theory

$$\mathcal{L}_G = -\frac{1}{2}\xi\left[\partial_\mu A_\mu - \frac{e}{\xi}(\varphi_1 + v)\varphi_2\right]^2 \tag{20}$$

with the Faddeev-Popov term

$$\mathcal{L}_{FP} = \bar{c}\left\{\partial^2 - \frac{e^2}{\xi}\left[(\varphi_1 + v)^2 - \varphi_2^2\right]\right\}c \ .$$

We choose to compute the effective potential of the Coleman-Weinberg lagrangian in the class of gauges defined by equations (18) and (19). This could seem a hard task, as we will have to deal with ghost loops. In our method, however, this poses no particular difficulty. The Feynman rules of the shifted lagrangian corresponding to the diagrams of Fig.1 are

Fig.1

a) $\frac{1}{i}\,\frac{1}{k^2+e^2v^2}\left\{\delta_{\alpha\beta}+\frac{k_\alpha k_\beta\,(1-\xi)}{\xi k^2+e^2v^2}\right\}$

b) $\frac{1}{i}\,\frac{1}{k^2+\frac{\lambda v^2}{2}}$

c) $\frac{1}{i}\,\frac{1}{k^2+\frac{\lambda v^2}{6}+\frac{e^2v^2}{\xi}}$

d) $\frac{1}{i}\,\frac{1}{k^2+\frac{e^2v^2}{\xi}}$

e) $\frac{1}{i}\,\lambda v$

f) $\frac{1}{i}\,v\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)$

g) $\frac{1}{i}\,2e^2v\,\delta_{\alpha\beta}$

h) $\frac{1}{i}\,\frac{2e^2}{\xi}\,v$

i) $-2e\,(k_\mu+q_\mu)$

j) $\frac{1}{i}\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)$

k) $\frac{1}{i}\,\lambda$

l) $\frac{1}{i}\,\lambda$

m) $\frac{1}{i}\,2e^2\delta_{\alpha\beta}$

n) $\frac{1}{i}\,2e^2\delta_{\alpha\beta}$

o) $\frac{1}{i}\,\frac{2e^2}{\xi}$

p) $i\,\frac{2e^2}{\xi}$

The one-loop tadpole diagrams are given in Fig.2, and contribute

a) $$-\frac{\lambda v}{32\pi^2}\,\frac{\lambda v^2}{2}\,\log\frac{\lambda v^2}{2}$$

b) $$-\frac{v\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)}{32\pi^2}\left(\lambda v^2+\frac{e^2v^2}{\xi}\right)\log\left(\frac{\lambda v^2}{6}+\frac{e^2v^2}{\xi}\right)$$

c) $$\frac{e^4v^3}{16\pi^2}\,\frac{1}{\xi^2}\,\log\frac{e^2v^2}{\xi}$$

d) $$\frac{e^4v^3}{16\pi^2}\left[\frac{1}{2\xi^2}-\frac{1}{\xi^2}\log\frac{1}{\xi}-\left(3+\frac{1}{\xi^2}\right)\log e^2v^2-\frac{1}{2}\right].$$

Fig.2

The renormalization performed here is the 't Hooft-Veltman's " pole subtraction "[7]. To compare our results to some existing ones it will be found convenient to introduce below other renormalization prescriptions. The basic equation that gives the one-loop contribution to the effective potential is

$$\frac{dV}{dv} = -\frac{e^4 v^3}{16\pi^2}\left[\frac{1}{2\xi^2} - 3\log e^2 v^2 - \frac{1}{2}\right] + \frac{\lambda^2 v^3}{64\pi^2}\log\frac{\lambda v^2}{2} + \tag{20}$$
$$+ \frac{\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)\left(\frac{\lambda}{6}+\frac{e^2}{\xi}\right)}{32\pi^2}\, v^3 \log\left[v^2\left(\frac{\lambda}{6}+\frac{e^2}{\xi}\right)\right]$$

which gives

$$V(v) = -\left\{\frac{e^4}{16\pi^2}\left(\frac{1}{2\xi^2}-\frac{1}{2}\right) - \frac{\lambda^2}{64\pi^2}\log\frac{\lambda}{2} - \frac{3e^4}{16\pi^2}\log e^2 - \right.$$
$$\left. - \frac{\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)\left(\frac{\lambda}{6}+\frac{e^2}{\xi}\right)}{32\pi^2}\log\left(\frac{\lambda}{6}+\frac{e^2}{\xi}\right)\right\}\frac{v^4}{4} + \tag{21}$$
$$+\left\{\frac{3e^4}{16\pi^2}+\frac{\lambda^2}{64\pi^2}+\frac{\left(\frac{\lambda}{3}+\frac{2e^2}{\xi}\right)\left(\frac{\lambda}{6}+\frac{e^2}{\xi}\right)}{32\pi^2}\right\}\frac{v^4}{4}\left(\log v^2 - \frac{1}{2}\right).$$

Putting $v = (\varphi_1^2+\varphi_2^2)^{\frac{1}{2}}$ and adding the zero-loop term we have the effective potential up to one-loop contributions. If, following Coleman and Weinberg, we use the renormalization prescriptions

$$\left.\frac{\partial^2 V}{\partial \varphi_1^2}\right|_{\varphi_1=0} = 0 \tag{22}$$

$$\left.\frac{\partial^4 V}{\partial \varphi_1^4}\right|_{\varphi_1 = M} = \lambda_R \tag{23}$$

the result will be

$$\begin{aligned} V(\varphi_1,\varphi_2) &= \frac{\lambda_R}{4!}(\varphi_1^2+\varphi_2^2)^2 + \\ &+\left\{\frac{3e^4}{64\pi^2}+\frac{5\lambda^2}{1152\pi^2}+\frac{\lambda e^2}{192\pi^2\xi}\right\}(\varphi_1^2+\varphi_2^2)^2\left\{\log\frac{\varphi_1^2+\varphi_2^2}{M^2}-\frac{25}{6}\right\}. \end{aligned} \tag{24}$$

For $\xi \to \infty$ (Landau gauge) one gets exactly the result of ref. (1). Note that the gauge-fixing condition is compatible with

$$A_\mu = 0$$

the point at which the effective potential has been calculated.

We could, of course, have done our calculations in other gauges as well. For instance, we can use as gauge-fixing condition the term

$$\mathcal{L}_G = -\frac{\xi}{2}\,(\partial_\mu A_\mu)^2$$

for which no Faddeev-Popov terms are necessary. This expression is not modified by the shifting of φ_1, meaning that we will not be able to cancel the mixed propagator of the shifted lagrangian. This involves summing all the insertions of the vertex $e v k_\mu$, leading to modified propagators for the photon and for the meson φ_2. The contribution of a new tadpole must also be considered. The result is

$$V(\varphi_1,\varphi_2) = \frac{\lambda}{4!}\,(\varphi_1^2+\varphi_2^2)^2 + \\ + \left(\frac{5\lambda^2}{1152\,\pi^2} + \frac{3e^4}{64\,\pi^2} - \frac{\lambda e^2}{192\,\pi^2 \xi}\right)(\varphi_1^2+\varphi_2^2)^2\left[\log\left[\frac{\varphi_1^2+\varphi_2^2}{M^2}\right] - \frac{25}{6}\right] .$$

We would like to remark that, despite the difficulties that appear when we associate this method with the R_ξ - gauge, there is at least one instance where we can take profit of the simplicity of the diagrammatic recipes of this gauge : it is the case of $\xi \to \infty$ .This corresponds to the Landau gauge, and the results of the application of our method are reliable due to the fact that the terms containing the problematic v (see Eq.(16)) vanish in the limit. Accordingly, we will make use of an analogous gauge condition to compute, in the same limit, the effective potential of a non-abelian gauge theory.

Finally, one could be surprised by the fact that, apparently, one has in hand a method to compute effective potentials in any gauge whatsoever, in contrast to claims that the effective potential does not exist in some gauges. The fact is that for some gauge-fixing terms, the condition $A_\mu = 0$ is not compatible with nonvanishing values of $\varphi_1^2+\varphi_2^2$,as, for instance, the gauge-fixing term

$$\mathcal{L}_G = -\frac{1}{2}\left(\partial_\mu A_\mu - \varphi_1^2 - \varphi_2^2\right)^2 .$$

4- Electrodynamics - two loops.

In our method, the n-loop contribution to the effective potential is obtained from the totality of tadpoles composed of n loops. The number of diagrams to compute is, therefore, finite. The contribution of each diagram is regularized by the dimensional procedure [7] and, afterwards, a simple differential equation must be integrated. One of the advantages of the method is that the renormalization is relatively simple : we just renormalize Green functions. In fact, the key to the simplicity of the method is that it leads to the computation not of the effective potential itself, but of its derivative, which is directly connected to Green functions.

For the two-loop contribution the tadpole diagrams are those given in Fig.3.

Fig. 3

The whole two-loop contribution is, of course, too long and, accordingly, not much transparent. We choose to exhibit, instead, the contribution of a potentially complicated diagram, the one numbered 9, which has both overlapping divergences and ghosts.
The calculation, including renormalization in the sense of ref. (7) is, however, simple. The result is

$$V_9 = -\frac{e^6 |\varphi|^4}{8(4\pi)^4 \xi^3}\Big\{ \frac{1}{4} \log^2 |\varphi|^2 +$$

$$+\left[I_1\left(\frac{\lambda}{2}, \frac{e^2}{\xi}\right) - 2\gamma\right] \log|\varphi|^2 + I_2\left(\frac{\lambda}{2}, \frac{e^2}{\xi}\right) - \gamma^2 - \frac{1}{4}\Big\} \tag{25}$$

where

$$\gamma = \log \frac{e^2}{\xi} - 1$$

$$I_1(a_1, a_2) = \alpha\, G_1(a_1, a_2) - G_3(a_1, a_2)$$

$$I_2(a_1, a_2) = G_2(a_1, a_2) - G_4(a_1, a_2)$$

and

$$G_1(a_1, a_2) = \int_0^1 dz \int_0^1 dy \, \frac{z(1-z)(1-y)}{\left[zy(1-z)+1-y\right]^3} \left\{ \log\left[(a_1-a_2)zy + a_2\right] - \frac{1}{2} \right\}$$

$$G_2(a_1, a_2) = \int_0^1 dz \int_0^1 dy \, \frac{z(1-z)(1-y)}{\left[zy(1-z)+1-y\right]^3} \left\{ \left[\log\left((a_1-a_2)zy + a_2\right) - \frac{1}{2} \right]^2 + \frac{1}{4} \right\}$$

$$G_3(a_1, a_2) = \int_0^1 dz \int_0^1 dy \, \frac{1-y}{\left[zy(1-z)+1-y\right]^2} \, \frac{(a_1-a_2)z + a_2}{(a_1-a_2)zy + a_2}$$

$$G_4(a_1, a_2) = \int_0^1 dz \int_0^1 dy \, \frac{1-y}{\left[zy(1-z)+1-y\right]^2} \, \frac{(a_1-a_2)z + a_2}{(a_1-a_2)zy + a_2} \left\{ \log\left[(a_1-a_2)zy + a_2\right] - \frac{1}{2} \right\}.$$

For the complete result at the two-loop level we refer the reader to ref. (8).

5 - Non-abelian gauge theories

The computation of the effective potential of a non-abelian gauge theory is not much more difficult than that of an abelian theory. Consider the following lagrangian :

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{\alpha} F_{\mu\nu}^{\alpha} - \frac{1}{2}(D_\mu \phi, D_\mu \phi) - P(\phi) \tag{26}$$

where

$$(D_\mu \phi)_a = \partial_\mu \phi_a - \Theta_{ab}^{\alpha} \phi_b A_{\mu\alpha} \tag{27}$$

$$P(\phi) = \frac{\mu^2}{2} \phi^2 + h\phi^4 \tag{28}$$

$$F_{\mu\nu}^{\alpha} = \partial_\mu A_\nu^{\alpha} - \partial_\nu A_\mu^{\alpha} - C^{\alpha\beta\gamma} A_\mu^{\beta} A_\nu^{\gamma} \tag{29}$$

$C^{\alpha\beta\gamma}$ being the structure constants of the gauge group. We use here the notation and conventions of Weinberg (9). For a more detailed explanation see the excellent review of Costa and Tonin (10).

Introducing a new set of scalar fields

$$\phi'_a = \phi_a - \varepsilon_a^{0} \tag{30}$$

we obtain a new lagrangian written as

$$\mathcal{L}(x) = \mathcal{L}_{free} + \mathcal{L}_{int} \tag{31}$$

with

$$(32)\quad \mathcal{L}_{free} = -\frac{1}{4}\left(\partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha\right)\left(\partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha\right) -$$

$$-\frac{1}{2}\left(\mathcal{M}^2\right)_{\alpha\beta} A_\mu^\alpha A_\mu^\beta - \frac{1}{2}\left(\partial_\mu \phi', \partial_\mu \phi'\right) - \frac{1}{2}\left(M^2\right)_{ab} \phi_a' \phi_b' +$$

$$+\left(\partial_\mu \phi', \Theta^\alpha \varepsilon^\circ\right) A_\mu^\alpha$$

and

$$\mathcal{L}_{int} = \frac{1}{2} C^{\alpha\beta\gamma}\left(\partial_\mu A_\nu^\gamma - \partial_\nu A_\mu^\gamma\right) A_\mu^\alpha A_\nu^\beta - \frac{1}{4} C^{\alpha\beta\gamma} C^{\sigma\delta\gamma} A_\mu^\alpha A_\nu^\beta A_\mu^\sigma A_\nu^\delta$$

$$(33)\quad +\left(\partial_\mu \phi', \Theta^\alpha \phi'\right) A_\mu^\alpha - \left(\Theta^\alpha \phi', \Theta^\beta \varepsilon^\circ\right) A_\mu^\alpha A_\mu^\beta - \frac{1}{2}\left(\Theta^\alpha \phi', \Theta^\beta \phi'\right) A_\mu^\alpha A_\mu^\beta$$

$$-\frac{1}{6} f_{abc}\, \phi_a' \phi_b' \phi_c' - \frac{1}{24} f_{abcd}\, \phi_a' \phi_b' \phi_c' \phi_d' .$$

The new symbols introduced are

$$(34)\quad \left(M^2\right)_{ab} = \left.\frac{\partial^2 P(\phi)}{\partial \phi_a \partial \phi_b}\right|_{\phi = \varepsilon^\circ}$$

$$(35)\quad \left(\mathcal{M}^2\right)^{\alpha\beta} = \left(\Theta^\alpha \varepsilon^\circ, \Theta^\beta \varepsilon^\circ\right)$$

$$(36)\quad f_{abc} = \left.\frac{\partial^3 P(\phi')}{\partial \phi_a' \partial \phi_b' \partial \phi_c'}\right|_{\phi' = 0}$$

$$(37)\quad f_{abcd} = \left.\frac{\partial^4 P(\phi')}{\partial \phi_a' \partial \phi_b' \partial \phi_c' \partial \phi_d'}\right|_{\phi' = 0}$$

The computations with this lagrangian are complicated by the presence, in $\mathcal{L}_{free}$ of a mixed propagator, that is, a quadratic term involving two different fields. The best way out consists in using the gauge freedom to eliminate this term. This was accomplished by 't Hooft[11] with the introduction of the gauge-fixing term

$$(38) \qquad C^{\alpha}(x) = \xi^{\frac{1}{2}} \partial_{\mu} A_{\mu}^{\alpha} - \xi^{-\frac{1}{2}} (\phi', \Theta^{\alpha} \varepsilon^{\circ})$$

meaning that the langrangian to be considered is

$$(39) \qquad \mathcal{L}_{C}(x) = \mathcal{L}(x) - \frac{1}{2} \left[C^{\alpha}(x) \right]^{2} .$$

Besides that, quantization of the theory requires the addition of the Faddeev-Popov term

$$(40) \quad \mathcal{L}_{FP} = \bar{u}_{\alpha} \left\{ \xi^{\frac{1}{2}} \delta_{\alpha\beta} \Box + \xi^{-\frac{1}{2}} (\mathcal{M}^{2})_{\alpha\beta} - \xi^{-\frac{1}{2}} (\Theta^{\alpha} \varepsilon^{\circ}, \Theta^{\beta} \phi') \right\} u_{\beta}(x) .$$

It is not convenient, though possible, to proceed in so abstract a level. Let us examine the case in which the gauge group is SU (2) [12] and the scalar bosons transform like a 4 - dimensional (reducible) representation. We have, for instance,

$$\Theta_1 = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \qquad \Theta_2 = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

$$\Theta_3 = \begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 0 \end{pmatrix}$$

and, of course,

$$\left[\Theta^\alpha , \Theta^\beta\right] = - \epsilon^{\alpha\beta\gamma} \Theta^\gamma$$

For our purpose it is enough to take $(\varepsilon^0)_a = v\,\delta_{a1}$.
In this case the matrix $\mathcal{M}^2$ is diagonal, that is

(41) $$\mathcal{M}^2 = \begin{pmatrix} \frac{v^2}{4} & 0 & 0 \\ 0 & \frac{v^2}{4} & 0 \\ 0 & 0 & \frac{v^2}{4} \end{pmatrix}$$

The bosonic mass matrix is

(42) $$(M^2)_{ab} = (\mu^2 + h v^2)\,\delta_{ab} + 2 h v^2\,\delta_{a1}\,\delta_{b1}$$

and the bosonic propagator is

(43) $$\Delta^{\phi}_{ab}(k) = e_a^{(1)} \frac{1}{k^2+\mu^2+3hv^2} e_b^{(1)} + \frac{1}{k^2+\mu^2+hv^2+\frac{v^2}{4\xi}} \sum_{p=2}^{4} e_a^{(p)} e_b^{(p)}$$

with $$\left(e^{(m)}\right)_a = \delta_{ma} \, .$$

The "photon" propagator is

$$(44) \quad \overset{A}{\Delta}(k)_{\mu\alpha,\nu\beta} = \frac{1}{k^2 + \frac{v^2}{4}} \left(\delta_{\mu\nu} + \frac{(1-\xi) k_\mu k_\nu}{\xi k^2 + \frac{v^2}{4}} \right) \delta_{\alpha\beta}$$

and that of the Faddeev - Popov ghost particles is

$$(45) \quad \overset{u}{\Delta}_{\alpha\beta}(k) = \frac{\delta_{\alpha\beta}}{\xi k^2 + \frac{v^2}{4}} \, .$$

The relevant vertices are given in Fig.4 and give

1) $$\frac{(2\pi)^4}{i} \, 2h \left(\varepsilon^0_c \, \delta_{ab} + \varepsilon^0_b \, \delta_{ac} + \varepsilon^0_a \, \delta_{bc} \right)$$

2) $$\frac{(2\pi)^4}{2\xi i} \left(\epsilon^{\alpha\beta\gamma} \, \Theta^{\gamma}_{b\ell} \, \varepsilon^0_\ell + \frac{1}{2} \, \delta_{\alpha\beta} \, \varepsilon^0_\ell \right)$$

3) $$\frac{(2\pi)^4}{2i} \, \varepsilon^0_b \, \delta_{\alpha\beta} \, \delta_{\mu\nu}$$

Fig.4

The tadpoles contributing to the one-loop approximation to the effective potential are given in Fig.5. Their values are :

1) $$-\frac{3hv}{16\pi^2}\left\{(\mu^2+3hv^2)\log(\mu^2+3hv^2)+\left(\mu^2+hv^2+\frac{v^2}{4\xi}\right)\log\left(\mu^2+hv^2+\frac{v^2}{4\xi}\right)\right\}$$

2) $$-\frac{v^3}{128\pi^2\xi^2}\log\frac{v^2}{4\xi}$$

3) $$-\frac{3v}{2(2\pi)^4 i}\int dk\,\frac{k^2\left(n+\frac{1-\xi}{\xi}\right)+\frac{nv^2}{4\xi}}{\left(k^2+\frac{v^2}{4}\right)\left(k^2+\frac{v^2}{4\xi}\right)}\ .$$

Fig.5

Taking, as explained at the end of Section 3, the limit $\xi \to \infty$, one gets for the tadpoles :

1) $-\frac{3h v}{16\pi^2}\left\{(\mu^2+3hv^2)\log(\mu^2+3hv^2)+(\mu^2+hv^2)\log(\mu^2+hv^2)\right\}$

2) zero .

3) $-\frac{9v^3}{128\pi^2}\log\frac{v^2}{4}$,

that is,

$$\frac{dV}{dv}=\frac{9v^3}{128\pi^2}\log\frac{v^2}{4}+ \tag{46}$$

$$+\frac{3hv}{16\pi^2}\left\{(\mu^2+3hv^2)\log(\mu^2+3hv^2)+(\mu^2+hv^2)\log(\mu^2+hv^2)\right\}.$$

The (trivial) integration of this equation gives the effective potential as a function of v . Substituting everywhere $(\phi,\phi)^{\frac{1}{2}}$ for v one has the effective potential of the theory characterized by the lagrangian of Eq. (26) in the gauge obtained by putting, in (38), $\varepsilon^0=0$. The computation of higher order approximations is straightforward.

Conclusions

The method described here allows the explicit computation of the effective potential of any theory which contains scalar bosons to any order of the loop expansion. Higher orders introduce no new complications except the normal ones of pertubation theory. The method, because of its expliciteness, is particularly convenient for numerical computation.

Simple extensions exist that permit the calculation of the effective potential when both fermion and scalar bosons are present. The case of just fermions and vector mesons is under study.

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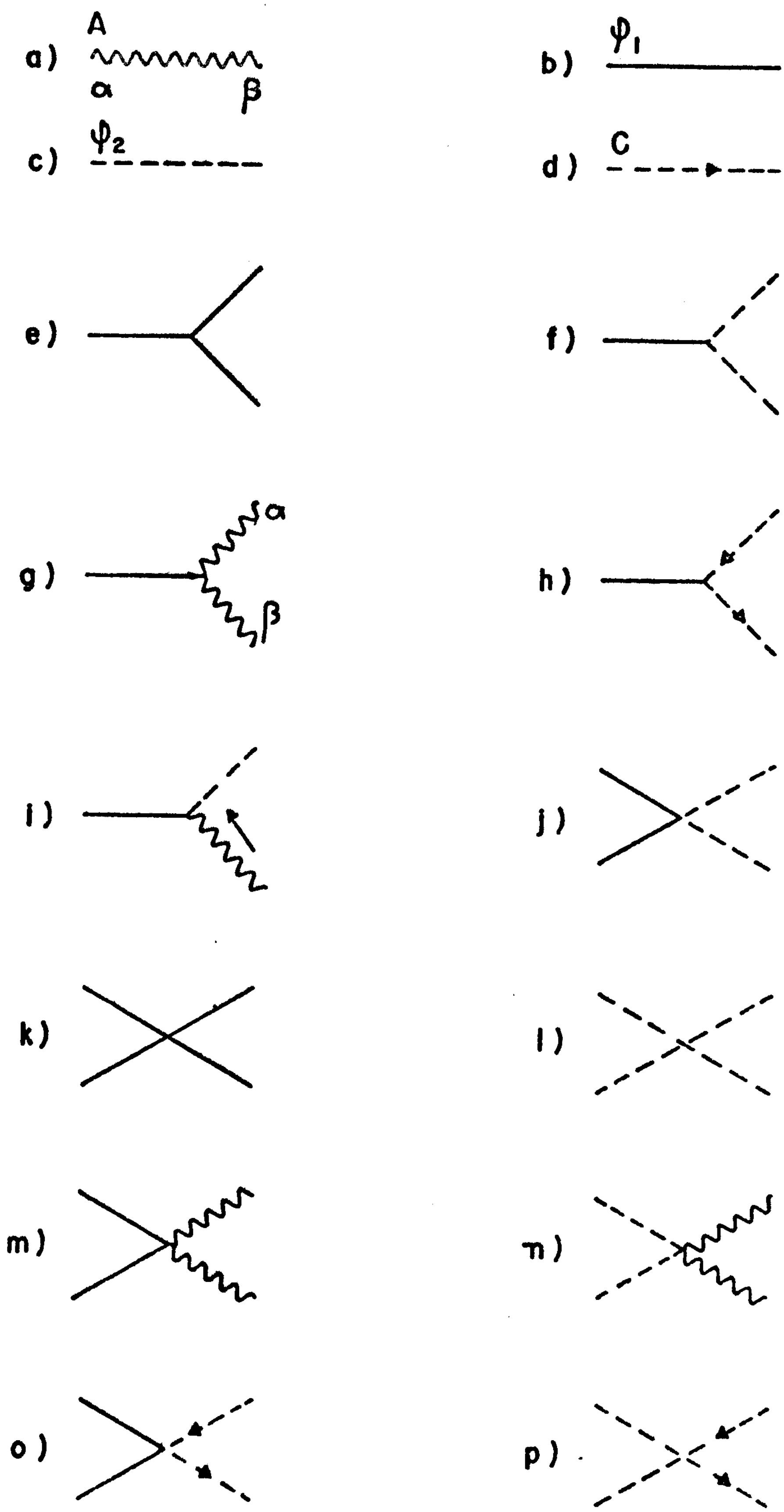

fig. 1

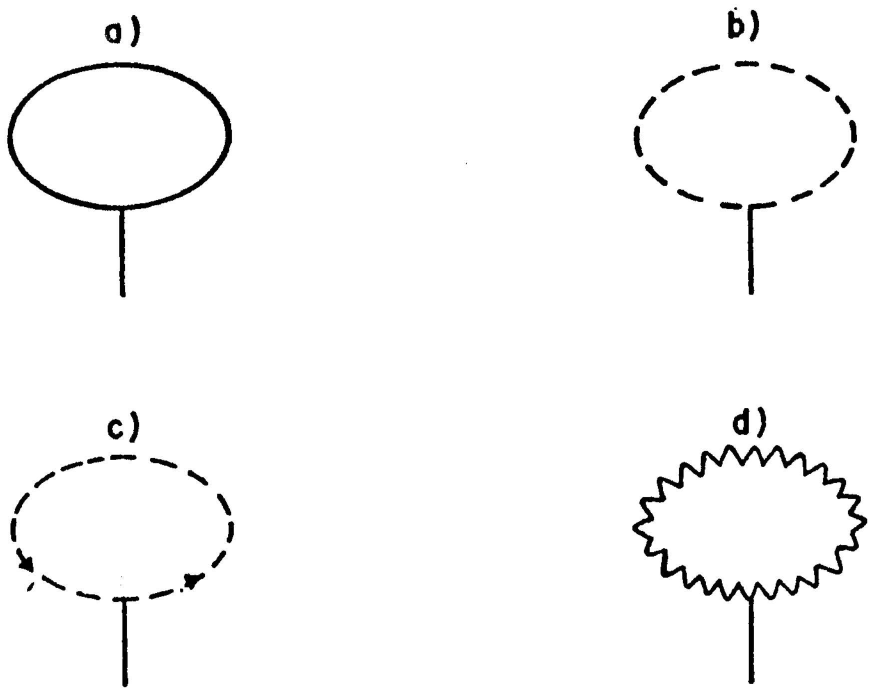

fig. 2

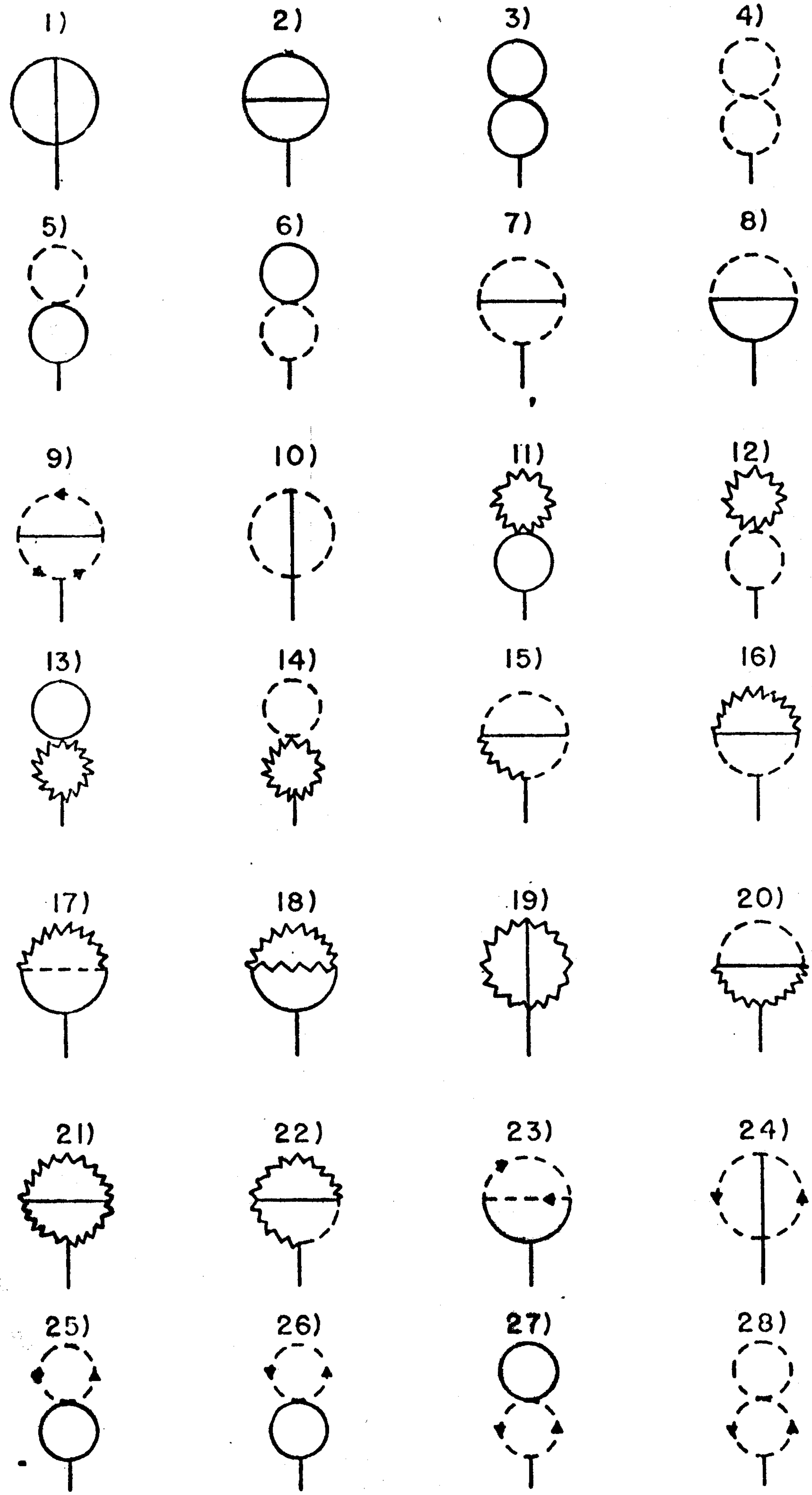

fig. 3

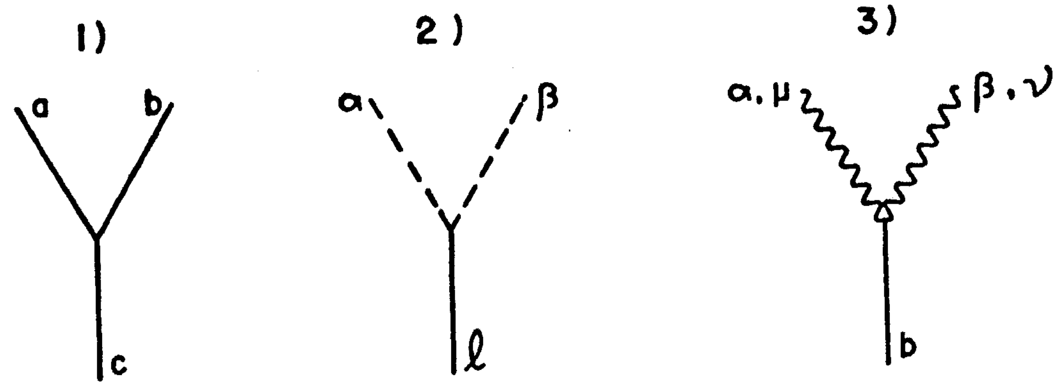

fig. 4

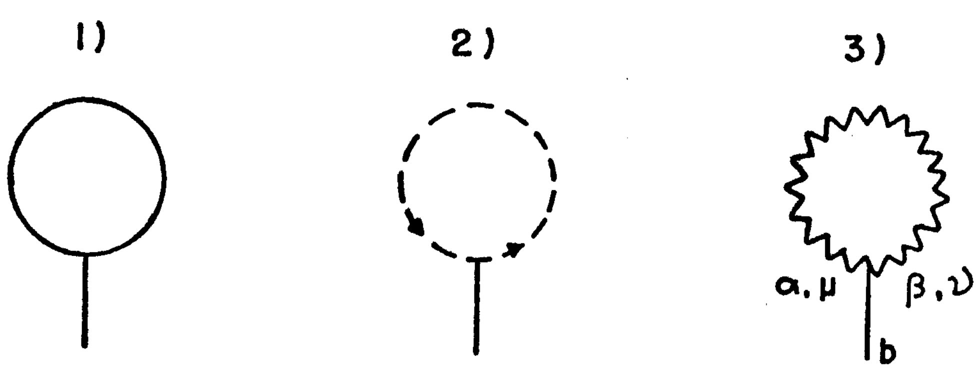

fig. 5