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CONNECTION BETWEEN ELASTIC RELATIVISTIC FORM FACTORS AND CHARGE DISTRIBUTION

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A B S T R A C T

A scheme by means of which one can establish the connection between form factors and charge distribution (for particles of any spin) is proposed. Except for the nonrelativistic domain our results differ from previous ones. Consequences of our relations (some of them in agreement with experimental and previous theoretical results) are briefly discussed.

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On the basis of some diffraction Models[1,2], the conceptually familiar notion of spatial charge distribution should play a relevant role in the understanding of some features of high energy scattering of hadrons. Since the implementation of such models requires further hypothesis on the shape of the charge and current densities, we found worth investigating, in the relativistic context, the basis for the relationship between form factors and electric charge distribution which is taken for granted in the literature .

For spin 1/2 particles like the nucleon, the most commonly employed relation is [3-5]:

$$G_E(-\vec{q}^{\,2}) = \int d^3x \, \rho(|\vec{x}|) \, e^{i\vec{q}\cdot\vec{x}} \tag{1}$$

where $G_E(-\vec{q}^{\,2})$ is the electric form factor in the Breit system and $\rho(|\vec{x}|)$ is the charge distribution.

As has been pointed before[1,6] the above relationship is clear only in the case that the recoil velocity in elastic scattering is zero, i.e. in the limit where the nucleon mass is very large ($M\to\infty$) and the magnetic moment is held fixed.

In this paper we propose a new way of getting the desired relationship, without making any assumption about smallness of the recoil. Our result differs from equation (1) in two points; first a different dependence of G_E for large $\vec{q}^{\,2}$ and second a (spin dependent) function of $\vec{q}^{\,2}$ multiplying G_E, which is responsible for the fact that a Dirac electron ($G_E=1$) shows some properties characteristic of a particle of finite extension[7]. The method is easily extensible to particles of any spin.

Our approach is similar to that used a long time ago by Weisskopf[8] (in the case of free fields) and consists in defining a correlation operator $\hat{W}$ which would have the following physical meaning

$$\left\langle {}^{\text{nucleon}}_{\text{at rest}} \right| \hat{W} \left| {}^{\text{nucleon}}_{\text{at rest}} \right\rangle = \int d^3x \, \rho\left(\left|\vec{x}+\tfrac{\vec{\xi}}{2}\right|\right) \rho\left(\left|\vec{x}-\tfrac{\vec{\xi}}{2}\right|\right) \tag{2}$$

or, in words, its expectation value in a state with one nucleon at rest, should be interpreted as the probability of finding charge simultaneously at two points in a distance $\vec{\xi}$.

A first look at expression (2) suggests that the natural definition of $\hat{W}$ should be:

$$\hat{W} = \int d^3x \, \hat{J}_0(x_+) \, \hat{J}_0(x_-) \tag{3}$$

where $\hat{J}_0(x_\pm)$ is the formal nucleon charge density operator at the point $x_\pm = \left(x_0, \vec{x} \pm \frac{\vec{\xi}}{2}\right)$ and x_0 is an arbitrary instant.

Let us see now, how the use of equation (2) coupled with the naive identification (3) allows us to establish the connection between form factors and charge distributions within the nonrelativistic Schröedinger theory. This can be achieved due to the fact that, after the introduction of a complete set of one particle states, equation (2) can be written as:

$$\sum_{\vec{p}} \int d^3x \langle \text{rest} | \hat{J}_0(x_+) | \vec{p} \rangle \langle \vec{p} | \hat{J}_0(x_-) | \text{rest} \rangle = \int d^3x \, \rho\left(\left|\vec{x}+\tfrac{\vec{\xi}}{2}\right|\right) \rho\left(\left|\vec{x}-\tfrac{\vec{\xi}}{2}\right|\right) \tag{4}$$

By expressing the matrix elements $\langle \vec{p} | \hat{J}_0(x) | \vec{p}' \rangle$ in terms of the nonrelativistic form factor[6], equation (4) will lead after a straightforward calculation, to the desired connection (equation (1) with G_E replaced by the nonrelativistic form factor).

Let us consider now the expectation value of (3) in a state $|\vec{p}=0; s\rangle$ of one proton at rest with spin s. By inserting a complete set of states, within the relativistic context, we get:

$$\langle 0,s|\hat{W}|0,s\rangle = \sum_{n}\int d^3x \langle 0,s|\hat{J}_0(x_+)|n\rangle\langle n|\hat{J}_0(x_-)|0,s\rangle \quad (5)$$

Looking at (5), one can see that the interpretation summarized by equation (2) no longer holds if one insists on the naive identification (3). The prime reason for this is that there are many contributions (experimentally accessible innelastic channels) in (5), which wouldn't lead to any information on what is the charge distribution of the proton and, consequently, should not be included in the identification sketched by equation (2). A close look at the procedure by means of which one measures charge densities in the relativistic quantum theory will allow us to abstract from (5) those contributions which should be identified with the right hand side of (2).

The measurement of charge densities requires the control of all momenta transferred to appropriate test bodies (from an analysis of the change of the momenta one can infer about the electric field produced by the charge distribution). It is well known, however, that any experimental apparatus devised to probe the particle charge distribution, disturbs the system during the measurement (creation of pairs for instance[9]). Due to that some care should be taken - a position first advanced by Bohr and Rosenfeld[9] - in order to eliminate these effects.

From the previous remarks one has come to grips with the question of which experiments can be interpreted as measuring the charge of a system in a previously prepared state. The answer is very simple: in order to make ourselves sure that all momenta transferred to the test body, were solely due the electromagnetic field produced by the system to the probed, the experimental outcome should be such that one has the same system (which one wants to probe) at the initial and final states.

The prescription of subtracting in any experiment, all those effects which cannot be associated with the measurement of the charge density of a proton, implies that the sole part of (5) which can be given the interpretation of equation (2), is the contribution involving just one proton in the intermediate states, namely:

$$\sum_{s'}\sum_{\vec{p}}\int d^3x \langle 0,s|\hat{J}_0(x_+)|\vec{p},s'\rangle\langle\vec{p},s'|J_0(x_-)|0,s\rangle = \int d^3x\, \rho\left(\left|\vec{x}+\frac{\vec{\xi}}{2}\right|\right)\rho\left(\left|\vec{x}-\frac{\vec{\xi}}{2}\right|\right) \qquad (6)$$

Our expression (6) differs from an analogous one derived by Weisskopf[8] in the context of free field theory. One can show that the difference stems from the fact that we are subtracting those contributions associated with creation of pairs out of the vacuum.

In order to get the explicit expression for (6), in terms of the Dirac and Pauli form factors, we take the usual parametrization of the current matrix elements[10]:

$$\langle\vec{p}',s'|\hat{J}_\mu(x)|\vec{p},s\rangle \qquad (7)$$

$$= \bar{u}(p',s')\left(\gamma_\mu F_1(p'-p) + i\frac{\sigma_{\mu\nu}(p'-p)_\nu}{2M}F_2(p'-p)\right)u(p,s)\times$$

$$\times\frac{M}{V\sqrt{p_0p_0'}}\exp\left[i(p'-p)\cdot x\right]$$

Where V is the normalization volume (at the end we take the continuous limit) and $p_0 = \sqrt{M^2+\vec{p}^2}$

After substituting (7) in (6) we get, after a straightforward calculation, the following result:

$$\sqrt{\frac{1-t/4M^2}{1-t/2M^2}}\, G_E(t) = \int d^3x\, \rho(|\vec{x}|)\, e^{i\vec{q}\cdot\vec{x}} \qquad (8)$$

where $G_E(t) = F_1(t) + \frac{t}{4M^2} F_2(t)$ and $t = 2M\left(M - \sqrt{M^2+\vec{q}^2}\right)$ is the square of the four momentum transferred to the nucleon at rest.

The same calculations, as sketched above, can be done for charged spin 0 particles. In this case we use the decomposition of the current matrix elements:

$$\langle \vec{p}\,'|\, J_\mu(x)\, |\vec{p}\rangle = \frac{(p'+p)_\mu}{2V\sqrt{p_0 p_0'}}\, F(p'p)\, e^{i(p'-p)\cdot x} \qquad (9)$$

where F is the eletromagnetic form factor and $p_0 = \sqrt{m^2+\vec{p}^2}$ (here m is the mass of the scalar particle).

The resulting relativistic connection with the effective charge density will be:

$$\frac{1-t/4m^2}{\sqrt{1-t/2m^2}}\, F(t) = \int d^3x\, \rho(|\vec{x}|)\, e^{i\vec{q}\cdot\vec{x}} \qquad (10)$$

where now $t = 2m\left(m - \sqrt{m^2+\vec{q}^2}\right)$.

Our main results are summarized by equations (8) and (10). The conclusions that can be drawn are the following:

a) The usual nonrelativistic connection, like equation (1), can be obtained from ours by taking the limit of low transferred momentum $\left(-t \simeq \vec{q}^{\,2} << 4M^2\right)$ where the recoil velocity is negligible.

b) By looking at (8) and (10) one sees that our results differ from those in the literature (besides the different dependence of G_E and F on $\vec{q}^{\,2}$) by the presence of spin dependent factors. The possible existence of such contributions has been realized by Yennie et al [(3)] (see for instance expression A.23 of ref. (3)). These terms affect the expression for the mean square radius of the particle when expressed in terms of the form factors. In the case of the proton we get from (8):

$$\langle r^2 \rangle = \frac{3}{4M^2} + 6 \left. \frac{dG_E(t)}{dt} \right|_{t=0} \tag{11}$$

which differs from that obtained from (1) by the additional term $3/4M^2$. It is worth noting that expression (11) was obtained in ref. (3) by using a completely different line of reasoning.

In the case of an electron, if one takes $G_E \simeq 1$, one gets from (11) a value for the effective radius which is equal to that obtained from Darwin's term [(7)] in the hydrogen atom, being thus in perfect agreement with experiment (Weisskopf's [(8)] results predict $\langle r^2 \rangle_{electron} = \frac{3}{2m_e^2}$).

The mean square radius for the spin 0 particle can be obtained from (10), giving:

$$\langle r^2 \rangle = 6 \left. \frac{dF(t)}{dt} \right|_{t=0} \tag{12}$$

and it is easy to see that if $F(t) \simeq 1$ we get $\langle r^2 \rangle = 0$ and $\langle r^4 \rangle = \frac{15}{4m^4}$ both of which are in agreement with Darwin's nonrelativistic corrections[11] of a Klein-Gordon particle in an external static field.

c) The short distance $(r = |\vec{x}| \to 0)$ behavior of $\rho(r)$ fixes the asymptotic $(-t \simeq 2M|\vec{q}| \to \infty)$ behavior of $G_E(t)$, or explicitly[12].

$$G_E(t) \xrightarrow[-t\to\infty]{} 4\pi\sqrt{2} \sum_{\ell=1}^{\infty} \left(-\frac{4M^2}{t^2}\right)^{\ell} \left(\frac{d}{dr}\right)^{2(\ell-1)} (r\rho(r)) \Big|_{r=0} \tag{13}$$

From this it immediately follows that, if $\rho(r)$ is sufficiently smooth at the origin, then G_E falls off at least as $G_E(t) \underset{t\to-\infty}{\sim} 1/t^4$. The asymptotic behavior $G_E(t) \underset{t\to-\infty}{\sim} 1/t^2$ is compatible, within our approach, with a nonsmooth behavior at the origin $\left(\rho(r) \underset{r\to 0}{\sim} 1/r\right)$.

As a final remark we would like to point out that, for a given $G_E(t)$, the different short distance behavior of $\rho(r)$ predicted by equation (8) (in contradistinction with that obtained from (1)), might play a role in some predictions of diffrative models. The implications of this to Chou-Yong model[1] is presently being investigated.

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