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On renormalization-invariant masses

by

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ABSTRACT

It is shown that spontaneous generation of renormalization invariant mass is possible in infra-red stable theories with more than one coupling constant. If relations among the coupling constants are permitted the effect can be made compatible with perturbation theory.

1 - Introduction

Spontaneous mass generation is the appearance, through spontaneous breakdown of symmetry , induced by renormalization , of massive particles, in a theory described by a massless lagrangean.

In this work we study the problem of spontaneous mass generation in theories with more than one coupling constant. The attention of physicists has, up to now, been directed mainly to theories with a single coupling constant: in this case some rather strong results have been proved [(1),(2)],showing that the generated masses are, first, nonperturbative effects, and, second, only possible for asymptotically free theories. The extension of these results to theories with several coupling constants was attempted in ref.(1),but the argument is unreliable. In fact, when more than one coupling constant exist , spontaneous mass generation can appear even at a perturbative level, a result contained in a celebrated paper by S.Coleman and E.Weinberg[(3)]. What we do here is to show that theories with two coupling constants, as, for instance ref. (3), escape the criticism of Gross and Neveu[(1)], based on the requirement of renormalization invariance of the alleged spontaneously generated mass, and to give a simple criterion to decide whether such an effect may be understood at the level of perturbative calculations.

The paper is organized as follows: in section 2, we review the main results on mass generation in theories with one coupling constant and discuss the attempt, by Gross and Neveu[(1)], of extending these results to the general case. Section 3 considers a theory with two coupling constants. To be concrete, we discuss the Coleman-Weinberg lagrangean, solve the pertinent renormalization--group equations and explicitly exhibit a (nonperturbative) invariant mass. Section 4 shows how a perturbative invariant mass can be obtained.

This turns out to be the very mass computed by Coleman and

Weinberg, following a different route. A general prescription for this kind of effect is abstracted.

2 - Theories with one coupling constant.

Suppose λ is the coupling constant and m is the spontaneously generated mass. To renormalize the massless theory, a parameter μ, with the dimension of mass must be introduced and we have, on dimensional grounds,

$$m = \mu \, f(\lambda) \tag{1}$$

f being an arbitrary function. If m is the mass of a particle, its value should not depend on the renormalization, that is, on μ. We must, therefore, have $\frac{dm}{d\mu} = 0$,or , equivalently,

$$f(\lambda) + \beta(\lambda) \frac{df}{d\lambda} = 0 \tag{2}$$

where

$$\beta(\lambda) = \mu \frac{\partial \lambda}{\partial \mu} \, .$$

The general solution of (2) is

$$m = C \exp\left(- \int^{\lambda} \frac{d\lambda'}{\beta(\lambda')} \right) \tag{3}$$

C being an arbitrary constant. It must be required that m vanish as λ goes to zero. It is easily seen from (3) that this is the case only if the theory is asymptotically free[4]. If, for instance,

$$\beta(\lambda) = -a \lambda^{r} \tag{4}$$

we will have

$$m(\mu,\lambda) \sim \mu \exp\left[\frac{-1}{a(r-1)\lambda^{r-1}}\right] . \qquad (5)$$

Because of the λ^{-1} behavior, this is a nonperturbative result. Accordingly, the known examples are some rather unrealistic two-dimensional models studied in the $N \to \infty$ approximation, as in refs.(1) and (2) .

Gross and Neveu attempted to extend these results to theories with more coupling constants in the following way:

For an infrared-stable ($\beta > 0$) theory, in the case of many coupling constants, the physical masses must satisfy

$$\begin{aligned} m(\mu,\lambda_1,\dots\lambda_n) &= \mu f(\lambda_1,\dots,\lambda_n) \\ &= \mu' f\left[\bar\lambda_1\left(\ln\frac{\mu'}{\mu}\right),\dots,\bar\lambda_n\left(\ln\frac{\mu'}{\mu}\right)\right] \end{aligned} \qquad (6)$$

where

$$\frac{d\bar\lambda_i^2}{dt} = \beta_i(\bar\lambda_1,\dots\bar\lambda_n)$$

If the theory is infrared-stable, then $\bar\lambda_i^2(t) \approx -\frac{1}{t}$ when $t \to -\infty$ Therefore, as $\mu' \to 0$, $f(\bar\lambda_i)$ must diverge as $\exp\left(\frac{1}{\bar\lambda_i^2}\right)$ when all the coupling constants vanish at the same rate.

Against this argument we will explicitly solve the renormalization-group equation for m in the next section, and show that it is, instead, possible to obtain a solution which vanishes as the coupling constants go to zero. The limit $\mu' \to 0$ in eq.(6) is a singular one , as the basic property of the parameter μ is being different from zero. Besides that, what really matters is the limit $\lambda_i \to 0$ with fixed μ , which is the limit discussed in the case of one coupling constant.

3 - Theories with several coupling constants.

To be specific, let us consider the Coleman-Weinberg lagrangean

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{2}(\partial_\mu\phi_1 - eA_\mu\phi_2)^2 - \frac{1}{2}(\partial_\mu\phi_2 + eA_\mu\phi_1)^2 - \frac{\lambda}{4!}(\phi_1^2+\phi_2^2)^2 \qquad (7)$$

containing massless hermitian scalar fields ϕ_1 and ϕ_2 . We work in the Landau gauge and renormalize the Green functions at an auxiliary mass μ . We say that a field ϕ acquires mass m if the effective potential[5] $V(\phi)$ has an absolute minimum at $\phi = v$ and if, in terms of the new field $\phi' = \phi - v$, the 2-point vertex obeys

$$i \Gamma^{(2)}(p^2) \Big|_{p^2=0} > 0 \tag{8}$$

and

$$\Gamma^{(2)}(p^2) \Big|_{p^2=-m^2} = 0 \quad . \tag{9}$$

From (9) and from the renormalization-group equation for $\Gamma^{(2)}$ it follows that

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_e \frac{\partial}{\partial e} + \beta_\lambda \frac{\partial}{\partial \lambda} \right) m(\mu, \lambda, e) = 0 \tag{10}$$

This is the renormalization-group equation for the mass, and its content is that the mass of a physical particle must be independent of renormalization.

To repeat the analysis of the one-coupling-constant case let us write

$$m = \mu f(e, \lambda) \tag{11}$$

and then

$$\beta_e \frac{\partial f}{\partial e} + \beta_\lambda \frac{\partial f}{\partial \lambda} = -f \; ; \tag{12}$$

here ,

$$\beta_e = \mu \frac{\partial e}{\partial \mu} = \frac{e^3}{48\pi^2} \tag{13}$$

and

$$\beta_\lambda = \mu \frac{\partial \lambda}{\partial \mu} = \frac{1}{4\pi^2}\left(\frac{5}{6}\lambda^2 - 3e^2\lambda + 9e^4\right) \tag{14}$$

in the one-loop approximation. The equation (12) becomes

$$\frac{e^3}{48\pi^2}\frac{\partial f}{\partial e} + \frac{1}{4\pi^2}\left(\frac{5}{6}\lambda^2 - 3e^2\lambda + 9e^4\right)\frac{\partial f}{\partial \lambda} = -f \tag{15}$$

and its general solution can be easily found by the method of the characteristics[(6)]. The invariant mass is

$$m = \mu f_0\left(\arctan \frac{1}{\sqrt{719}}\left[\frac{10\lambda}{e^2} - 19\right] - \sqrt{719}\,\ln e\right)\exp\left(\frac{24\pi^2}{e^2}\right) \tag{16}$$

where f_0 is an arbitrary function. The last factor of eq.(16) explodes as $e \to 0$ .To get a reasonable invariant mass we must then exhibit some f_0 that cancels this singularity. An example is

$$m = \mu \exp\left(\frac{24\pi^2}{e^2}\right)\exp\left[-\frac{24\pi^2}{e^2}\exp\left\{\frac{2}{\sqrt{719}}\arctan\left[\frac{1}{\sqrt{719}}\left(\frac{10\lambda}{e^2} - 19\right)\right]\right\}\right] . \tag{17}$$

Taking

$$\lambda = \frac{19}{5}e^2$$

we have

$$a \equiv \exp\left(\frac{2}{\sqrt{719}}\arctan\frac{19}{\sqrt{719}}\right) > 1$$

and

$$m = \mu \exp\left[-\frac{24\pi^2}{e^2}(a-1)\right]$$

which vanishes as $e \to 0$. As it is, the dependence on e is clearly non perturbative, so that, unfortunately, we have no way to verify whether the theory actually takes profit of this possibility.

4 - Perturbative solutions.

A solution of eq.(10) which is compatible with perturbation theory may be obtained with a convenient relation between λ and e . Coleman and Weinberg themselves needed such a relation. We think we are able to strengthen its necessity by arguing for it in a different, more easily generalizable way. Our treatment is inspired in the beatiful paper of Iliopoulos and Papanicolau[7] on the gauge invariance of Coleman-Weinberg's results.

Assume that λ is of the same order as e^4 .Explicitly, let us take $\lambda = \rho e^4$, ρ being a new parameter of order 1 .In terms of e and ρ , equation (15) reads

$$\frac{e^3}{48\pi^2}\frac{\partial f}{\partial e} + \frac{1}{4\pi^2}\left(\frac{5}{6}\rho^2 e^4 - 3\rho e^2 + 9\right)\frac{\partial f}{\partial \rho} = - f \ . \tag{18}$$

We look now for solutions which not only vanish, but are analytic for $e=0$ .In this case $\frac{\partial f}{\partial e}$ is also analytic, and the first term of the left-hand side of eq.(18) vanishes with e . For small e , then ,

$$\frac{9}{4\pi^2}\frac{\partial f}{\partial \rho} = - f . \tag{19}$$

The general solution gives tha mass

$$m = \mu f_0(e) \exp\left(-\frac{4\pi^2}{9}\rho\right) \tag{20}$$

where f_0 is an arbitrary analytic function of e that vanishes for $e=0$. This is an important result, as it gives, for $f_0 = e$, the vector meson mass computed in ref.(3). It is therefore inequivocally shown that the results of Coleman and Weinberg are renormalization-invariant, and that the masses they compute can be masses of physical particles.

Technically, this is due to the fact that the coefficient of

$\frac{\partial f}{\partial \rho}$ in eq.(18) has a constant term. This simulates a negative β in equation (10) , giving the exponent the correct sign. To find the convenient combination between the coupling constants, we must therefore look for those which originate constant terms in the coefficients of the renormalization group equation for the mass. In this way not only does one get mass generation in an infra-red stable theory, but this is done in a way consistent with perturbation theory.

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R E F E R E N C E S

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