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MODEL"

A.J.da Silva, M.Gomes

Instituto de Física, Universidade de São Paulo

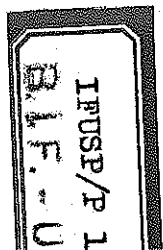
and

R.Köberle

Instituto de Física e Química de São Carlos -  
Universidade de São Paulo

**B.I.F. - USP**

UNIVERSIDADE DE SÃO PAULO  
INSTITUTO DE FÍSICA  
Caixa Postal - 20.516  
Cidade Universitária  
São Paulo - BRASIL



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SPONTANEOUS CHIRAL SYMMETRY BREAKING AND THE 1/N EXPANSION IN TWO DIMENSIONS IN AN EXACTLY SOLUBLE MODEL

A.J. da Silva, M.Gomes

Instituto de Física, Universidade de São Paulo, C.Postal 20516, São Paulo, SP, Brasil

and

R.Köberle

Instituto de Física e Química de São Carlos, Universidade de São Paulo, C.Postal 369 - São Carlos, SP, Brasil

ABSTRACT

In an exactly soluble two dimensional model we show how mass generation for a fermion field can be reconciled with the absence of chiral symmetry breaking.

SPONTANEOUS CHIRAL SYMMETRY BREAKING AND THE 1/N EXPANSION IN TWO DIMENSIONS IN AN EXACTLY SOLUBLE MODEL

Due to the absence of particle production a large number of two-dimensional models<sup>(1)</sup> have been solved exactly for their S-matrices.

Recently an S-matrix has been proposed for the chiral Gross-Neveu model<sup>(2)</sup>.

$$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi + \frac{g^2}{2} \left( (\bar{\Psi} \Psi)^2 - (\bar{\Psi} \gamma^5 \Psi)^2 \right) \quad (1)$$

The 1/N-expansion, which one uses to check whether the proposed S-matrix<sup>(3)</sup> belongs to the Lagrangian (1) or not, shows spontaneous symmetry breaking of the U(1) chiral symmetry and an associated Goldstone boson. These features cannot exist in two space-time dimensions<sup>(4)</sup>.

In an interesting paper<sup>(5)</sup>, Witten has shown how one could reconcile spontaneous mass generation with the absence of spontaneous symmetry breaking in the chiral Gross-Neveu model. Since no explicit computations verifying Witten's proposal have been carried out yet, we feel it instructive to exhibit the problems involved, in an exactly soluble model described by the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & \bar{\Psi} i \not{\partial} \Psi + \frac{(\partial_\mu \Psi)^2}{2} + \\ & + g \bar{\Psi} \gamma^\mu \gamma^5 \Psi \partial_\mu \Psi + \\ & - M \bar{\Psi} \exp(-i 2 g \delta^5) \Psi \end{aligned} \quad (2)$$

where  $\psi$  is an  $N$ -plet of fermion fields,  $\phi$  a isoscalar field and  $g^2$  plays the role of  $1/N$ .

On the classical level we have a  $U(1) \times U(1)$  symmetry, the chiral part being:

$$\begin{aligned} \psi &\rightarrow e^{-i\beta\gamma^5} \psi \\ \phi &\rightarrow \phi + \frac{\beta}{g} \end{aligned} \quad (3)$$

with associated Noether current:

$$j_{\mu 5} = \frac{\beta}{g} \partial_{\mu} \phi \quad (4)$$

From the equations of motion:

$$\begin{aligned} i\not{\partial}\psi + g\gamma^{\mu\nu}\gamma^5\psi\partial_{\mu}\phi - M e^{-i2g\gamma^5\phi}\psi &= 0 \\ \partial^2\phi + g\partial_{\mu}(\psi\gamma^{\mu}\gamma^5\psi) + 2igM\phi\gamma^5 e^{-i2g\gamma^5\phi}\psi &= 0 \end{aligned} \quad (5)$$

it follows that the chiral current is conserved, since  $\phi$  is a free massless field

$$\partial^{\mu} j_{\mu 5} = \frac{\beta}{g} \partial^2 \phi = 0 \quad (6)$$

Notice that to zeroth order in  $g$  we have a massive fermion field  $\psi$  breaking chiral invariance, whereas a mass term  $m^2\psi^2$  for the  $\psi$  field would be compatible with that invariance. On the other hand, for  $g \neq 0$  the equations of motion (5) show that chiral symmetry is exact and requires  $\phi$  to have zero mass like a bona fide Goldstone boson.

After quantization a mild anomaly appears<sup>(6)</sup> which can be eliminated by rescaling  $\phi$ , and one easily obtains the operator solution:

$$\psi = : \exp(i \vec{g} \delta^5 \phi) : \psi_M \quad (7)$$

where  $\vec{g} = g/(1 - \frac{g^2 N}{\pi})$  and  $\psi_M$  is a free massive fermion field.

Although the zero mass field  $\phi$  does not exist in two dimensions it is well known<sup>(7)</sup> that exponentials  $: \exp i a \phi :$  can be defined by introducing a mass  $m$  for  $\phi$  and taking the zero mass limit after a multiplicative renormalization

$$: \exp i a \phi : \rightarrow \left( \frac{m e^\gamma}{2\mu} \right)^{\frac{a^2}{4\pi}} : \exp i a \phi : \quad (8)$$

where  $\gamma$  is Euler's constant and  $\mu$  plays the role of an infrared regulator. This motivates the following definition:

$$\left\langle T \prod_{j=1}^N e^{i a_j \phi(x_j)} \right\rangle \stackrel{\text{def.}}{=} e^{-\sum_{i < j} a_i a_j \Delta_F(x_i - x_j)} \delta\left(\sum_j a_j; 0\right) \quad (9)$$

where  $\Delta_F(x) = -\frac{1}{4\pi} \ln(-\mu^2 x^2 + i0)$ .

With respect to the exact solution (7) we now note several facts:

a) The Lagrangian describes a free massive fermion field  $\psi_M$  and a field  $\phi$  which can be decoupled by using equation (7). We expect this type of procedure to be useful also in less trivial models.

b) In spite of its mass  $\psi_M$  does not break the chiral symmetry generated by  $J_{5\alpha}$  because it carries no chiral charge.

$$[\psi_m, j_{05}] = [\psi_m, \frac{3}{g} \partial_0 \phi] = 0 \quad (10)$$

since  $\psi_m$  and  $\phi$  are independent fields.

c) In distinction to  $\psi_m$ ,  $\phi$  does carry chiral charge and consequently cannot develop a mass. This can easily be seen calculating its two-point function :

$$\begin{aligned} & \langle T \phi(x) \bar{\phi}(y) \rangle \\ &= \langle T : \exp i \vec{g} \delta_x^5 \phi(x) : \exp i \vec{g} \delta_y^5 \phi(y) : \langle T \psi_m(x) \bar{\psi}_m(y) \rangle \rangle \\ &= C \int \frac{d^2 p}{(2\pi)^2} e^{-i p(x-y)} \int \frac{d^2 k}{(2\pi)^2} \frac{k}{k^2 - \mu^2 + i0} \frac{1}{(p-k)^2 + i0} \left( \frac{2\pi}{\mu} \right)^2 \end{aligned} \quad (11)$$

where C depends only on  $\mu^2$  and  $g^2$ . In obtaining this result it was essential to respect the selection rule represented by the Kronecker delta in equation (9): the contribution of the mass term of  $\langle T \psi_m(x) \bar{\psi}_m(y) \rangle$  to the numerator of the propagator (11) was scaled to zero by the multiplicative renormalization (8). Notice that this propagator satisfies the correct equation of motion (5) inspite of the absence of the mass in the numerator, if one defines the normal product  $N(\exp(-i \vec{g} \delta^5 \phi) \psi)$  occurring in equation (5) as:

$$N(\exp(-i \vec{g} \delta^5 \phi(x)) \psi(x)) = : \exp(-i \vec{g} \delta^5 \phi(x)) : \psi(x) \quad (12)$$

d) Although our model shows no spontaneous generation of mass, the existence of a mass term in (2), evading chiral symmetry breaking is made possible because the two-point function of  $: \exp(-i \vec{g} \delta^5 \phi) :$  behaves for large distances as

$$\begin{aligned}
& \langle T : e^{-i2g\int\phi(x)} : e^{-i2g\int\phi(y)} : \rangle \\
& \cong (\mu^2(x-y)^2)^{-\frac{2g^2}{\pi}}
\end{aligned}
\tag{13}$$

exactly as in less trivial situations like the X-Y model<sup>(8)</sup> and in Witten's proposal for the chiral Gross-Neveu model.

\* \* \*

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