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IN AXISYMMETRIC MAGNETIC CONFINEMENT CONFIGURATIONS

by

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ABSTRACT

An expression for the second adiabatic invariant J is derived including the effects of plasma diamagnetism and displaced magnetic surfaces. It is shown that for values of $\beta \lesssim \varepsilon$, where β is the ratio of kinetic to magnetic pressure and ε is the inverse aspect ratio of the torus, J becomes a decreasing function of $\bar{\psi}$, the flux function, in the outer region of the plasma column.

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1. INTRODUCTION

Recent experimental results have shown that high density (GONDHALEKAR et al, 1978) and high temperature (EUBANK et al, 1978) plasmas can be produced and contained in tokamaks. Temperatures obtained with supplementary heating by neutral beam injection ($\sim$6 keV for ions and $\sim$3.5 keV for electrons (EUBANK et al, 1978)) are such that the plasma is in the low collisionality regime. This regime is characterized by the trapping of a fraction of the electron and ion populations in the local wells of the confining magnetic field. In this situation trapped-particle instabilities can be excited, leading to anomalous particle and energy transport across the field (TANG, 1978). These instabilities are driven by density and temperature gradients coupled with collisional dissipation mechanisms, or with unfavourable average magnetic drifts. In particular, in the limit of very high temperatures, the interchange, or collisionless, trapped-particle instability may be excited (KADOMTSEV and POGUTSE, 1967 and 1971). This is a purely growing, non-resonant flute mode. Using strong turbulence theory, WADDELL (1975) has shown that the corresponding anomalous diffusion coefficient is given by $D=\sqrt{\varepsilon}\ \gamma/2k_r^2$, where γ is the mode growth rate and k_r is the mode wavenumber in the radial direction. ROSENBLUTH and SLOAN (1971) have shown that the diamagnetic well generated by finite plasma pressure diminishes the region where the magnetic drifts are unfavourable, which has a stabilizing effect on the mode.

There are three separate mechanisms responsible for the finite-β stabilization of trapped-particle modes (β is the ratio of plasma kinetic to magnetic pressure). The first two are "geometric" effects related to the diamagnetic well, mentioned above, and to the finite-β enhancement of the non-concentric shifts of the magnetic flux surfaces. The third is the coupling between drift and shear-Alfvén waves (TANG, 1978). In the case of the interchange trapped-particle mode, the square of the linear growth rate, $\gamma^2 \simeq -\sqrt{2\varepsilon}\, \nabla p . \nabla J$ (KADOMTSEV and POGUTSE, 1971), where p is the plasma pressure and J is the second adiabatic invariant. J is defined by

$$J = \oint v_{\parallel}\ dl \quad , \tag{1}$$

where $v_{\parallel}$ is the particle velocity parallel to the magnetic field and the integral is carried out along the closed banana orbit of a trapped particle (KADOMTSEV and POGUTSE, 1970). The "geometric" finite-β effects act as follows: the diamagnetic well decreases the toroidal component, and the outward shift of the magnetic surfaces increases the poloidal component, of the magnetic field in the region where the trapped particles are located. As will be seen later, this results in an increase in J towards the axis and a decrease in J at the edge of the plasma; in some cases ∇J becomes negative over a substantial part of the plasma column, stabilizing the interchange trapped-particle mode. The reduction in anomalous diffusion in this situation has been pointed out by OHKAWA (1971).

GLASSER, FRIEMAN and YOSHIKAWA (1974) and DOBROTT and GREENE (1975) have shown that the growth rate of the interchange trapped-particle mode can be decreased by elongating the cross-section of the plasma column. For circular cross-sections, SIGMAR and FREIDBERG (1975) have examined the possibility of obtaining ∇J negative for high values of β. They found that ∇J decreased as β increased but that ∇J did not become negative for reasonable values of β. However, it appears that their calculation may be in error because they obtained finite values of J at the magnetic axis. Since the parallel velocity for a trapped-particle goes to zero at the axis, J should also go to zero.

In the present work, we re-evaluate J for circular cross-section tokamaks retaining terms of order $\varepsilon^{3/2}$. Our calculation takes into account the "geometric" effects described above and allows for various profiles of the pressure and toroidal current density. In particular, we investigate stability for the case of a pressure profile peaked off-axis. Such a profile might occur in flux-conserving tokamaks (CLARKE and SIGMAR, 1977).

2. GENERAL EXPRESSION FOR J

The integral which appears in expression (1) should be carried out along a trapped-particle path. However it is sufficiently accurate to evaluate the integral along a field line, neglecting the width of the banana orbit (KADOMTSEV and POGUTSE, 1970). In this case, expression (1) reduces to

$$J = 2 \int_{-\theta_1}^{\theta_1} \frac{dl_\chi}{B_\chi} B v_\parallel \qquad , \qquad (2)$$

where $\chi=\pm\theta_1$ are the turning points ($v_\parallel = 0$) of the trapped-particle orbit and (ψ, χ, ζ) are the usual orthogonal, curvilinear flux co-ordinates with magnetic surfaces ψ = constant and with χ and ζ in the poloidal and toroidal directions respectively (CALLEN and DORY, 1972). Since $B_\chi^2/B_\zeta^2 \sim \varepsilon^2$, we can replace B above by its toroidal component, B_ζ. We define the average over a flux surface of any quantity, X, as follows:

$$\langle X \rangle = \frac{2\pi}{V'(\psi)} \oint_\psi \frac{dl_\chi}{B_\chi} X \quad , \tag{3}$$

where $V'(\psi)$ is the derivative of the volume of the magnetic surface ψ = constant with respect to ψ. Then, defining the inverse rotational transform, $q(\psi)$, as (CALLEN and DORY, 1972)

$$q(\psi) = \oint_\psi \frac{dl_\chi}{2\pi R} \frac{B_\zeta}{B_\chi} = \frac{I(\psi)V'(\psi) \langle R^{-2} \rangle}{4\pi^2} \quad , \tag{4}$$

where $I(\psi)=RB_\zeta$ and R is the distance from the axis of symmetry, expression (2) becomes

$$J = 8R_o q(\psi) \left[\frac{\pi}{2} \frac{\langle Hv_\parallel \; h^{-1} \rangle}{\langle h^{-2} \rangle} \right] \quad , \tag{5}$$

where R_o is the distance of the geometric centre of the plasma cross-section from the axis of symmetry; $h=R/R_o$, and H is a step function defined by

$$H = \begin{cases} 0 & |\chi| > |\theta_1| \\ 1 & |\chi| \leqslant |\theta_1| \end{cases} .$$

The parallel component of the particle velocity is given by

$$v_\parallel = \sqrt{\frac{2}{m} (\mathcal{E} - \mu B)} \quad ,$$

where $\mathcal{E}$ is the total energy of the particle;

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

is the first adiabatic invariant, and $v_\perp$ is the component of particle velocity perpendicular to the magnetic field. Defining the pitch angle variable, λ , as (SIGMAR and FREIDBERG, 1975)

$$\lambda = \frac{\mu I_o}{\mathscr{E} R_o} ,$$

with I_o the vacuum value of I , we find

$$v_\parallel = v \sqrt{1 - \frac{\lambda I B}{h I_o B_\zeta}} , \tag{6}$$

where $v = \sqrt{2\mathscr{E}/m}$. In expression (6), we have retained terms of order ε^2 . Since, for trapped particles, $v_\parallel /v \sim \varepsilon^{1/2}$, terms of order ε^2 must be included in the square root to obtain the next approximation.

3. EQUILIBRIUM MODEL

We wish to evaluate J in toroidal plasma columns with circular cross-sections, for various profiles of the pressure and current density. In order to determine the equilibrium flux surfaces, we employ the YOSHIKAWA (1974) formulation of the toroidal equilibrium problem. In this treatment, the profiles for the toroidal current density, j , and the pressure, p , are specified a priori. We choose

$$\frac{p}{p_o} = 1 + \alpha_p \rho^2 - (1 + \alpha_p)\rho^4$$

$$\frac{j}{j_o} = 1 + \alpha_j \rho^2 - (1 + \alpha_j)\rho^4 , \tag{7}$$

where ρ is the distance from the geometric centre of the plasma column normalized to the minor radius, a (Fig. 1), and α_p and α_j are parameters which are chosen to give different profiles. For $\alpha=-1$, the profile is parabolic; for $\alpha=0$, quartic, and for $\alpha>0$, there is an off-axis maximum with $\rho<1/\sqrt{2}$. It should be mentioned that, in the final solution, these profiles will be distorted slightly from the simple cylindrical form above, however this distortion is not relevant to the results presented here.

YOSHIKAWA (1974) writes the poloidal flux function, ψ, as the sum of a zero-order function, ψ_o, plus a first-order toroidal correction, ψ_1 :

$$\psi = \psi_o(\rho) + \psi_1(\rho,\theta) \quad , \tag{8}$$

where $\psi_o \sim 1$, $\psi_1 \sim \varepsilon$ and θ is the angle about the geometric centre of the plasma column (Fig. 1). ψ_o is found from the solution of

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\psi_o}{d\rho}\right) = -\mu_o a^2 R_o j(\rho) \tag{9}$$

and ψ_1 is given by

$$\psi_1 = \varepsilon\cos\theta\,\frac{d\psi_o}{d\rho}\,[f(\rho) - f(1)] \quad , \tag{10}$$

where

$$f(\rho) = \int_o^\rho \frac{d\rho'}{\rho'}\,\frac{1}{\left(\frac{d\psi_o}{d\rho'}\right)^2}\int_o^{\rho'} d\rho''\left[\rho''\left(\frac{d\psi_o}{d\rho''}\right)^2 - 2\mu_o a^2 R_o^2 \rho''^2\,\frac{dp}{d\rho''}\right]. \tag{11}$$

To first order, this solution corresponds to circular cross-section flux surfaces shifted in the R-direction (Fig. 1). The normalized radius, ρ_o , of the surface $\psi=\psi'$ is found from

$$\rho_o(\psi') = \psi_o^{-1}(\psi') \quad , \tag{12}$$

ψ_o^{-1} being the inverse function of $\psi_o(\rho)$. For the normalized outward shift, Δ , we have, to first order,

$$\Delta(\psi') = \varepsilon\left[f(1) - f(\rho_o)\right] \quad . \tag{13}$$

Substituting equations (7) into equation (9) and the resulting expression for ψ_o into equation (11) allows $f(\rho)$ to be evaluated analytically for our profiles. The final expression for $f(\rho)$ is of the form

$$f(\rho) = \frac{1}{24}\left[x + \frac{A_1 x}{x_1(x_1-x)} + \frac{A_2 x}{x_2(x_2-x)} + B_1 \ln(1 - \frac{x}{x_1}) + B_2 \ln(1 - \frac{x}{x_2})\right], \tag{14}$$

where $x=\rho^2$, and x_1 , x_2 , A_1 , A_2 , B_1 and B_2 are constants (see Appendix) which depend on α_p , α_j and the poloidal beta, β_p , defined here as

$$\beta_p = \frac{2\mu_o\bar{p}}{B_\theta^2(a)} \simeq 2a^2R_o^2\mu_o\bar{p}\left(\frac{d\psi_o}{d\rho}\right)^{-2}_{\rho=1} \quad ,$$

where $\bar{p}$ is the mean pressure and $B_\theta(a)$ is the poloidal field at the edge of the plasma.

The flux quantity, $I(\psi)$, which appears in expression

(6), is found from (YOSHIKAWA, 1974)

$$\frac{I^2(\psi')}{I_o^2} = 1 + \frac{2\mu_o \dot{R}_o}{I_o^2} \int_1^{\rho_o} j(\rho) \frac{d\psi_o}{d\rho} \, d\rho - \frac{2\mu_o R_o^2}{I_o^2} \, p(\rho_o) \quad .$$

After carrying out the integration, it is convenient to express $I(\psi')$, to second order, as

$$\frac{I(\psi')}{I_o} = 1 - \frac{\varepsilon^2}{q_o^2} \, I_w \, (\rho_o) \quad , \tag{15}$$

where q_o is the value of $q(\psi)$ at the magnetic axis, and $I_w(\rho_o)$ is a polynomial in ρ_o (see Appendix) with coefficients depending on α_p, α_j and β_p. I_w determines the depth of the diamagnetic well.

4. EVALUATION OF J

To evaluate J, it is necessary to express equations (3) to (6) in terms of the equilibrium quantities of the preceding section. For the flux surface averages (definition (3)), it is useful to choose a co-ordinate system (ρ',θ') (Fig. 1) with the origin at the geometric centre of the flux surface, $\psi=\psi'$, for which the averages are to be calculated. Then the surface $\psi=\psi'$ corresponds to the circle $\rho'=\rho_o$ (ρ_o is defined in equation (12)), and definition (3) becomes

$$\langle X \rangle = \frac{2\pi a^2 R_o \rho_o}{V'(\psi')} \int_{-\pi}^{\pi} \frac{h d\theta' X}{\left(\frac{\partial\psi}{\partial\rho'}\right)_{\rho'=\rho_o}} \quad .$$

To evaluate $(\partial\psi/\partial\rho')_{\rho'=\rho_o}$ to first order, we note that

$$\rho' = \rho - \Delta(\psi') \cos\theta + \mathcal{O}(\varepsilon^2)$$

and $\cos\theta' = \cos\theta + \mathcal{O}(\varepsilon)$.

A Taylor expansion of equation (8) yields an expression for ψ in terms of the new variables ρ' and θ' :

$$\psi \simeq \psi_o(\rho') + \Delta(\psi') \cos\theta' \left(\frac{d\psi_o}{d\rho}\right)_{\rho=\rho'} + \psi_1(\rho',\theta')$$

and thus, using relations (10) and (13),

$$\langle X\rangle \simeq \frac{2\pi a^2 R_o}{V'(\psi')} \frac{\rho_o}{\left(\frac{d\psi_o}{d\rho}\right)_{\rho=\rho_o}} \int_{-\pi}^{\pi} h d\theta' \left(1-\left(\frac{df}{d\rho}\right)_{\rho=\rho_o} \varepsilon\cos\theta'\right)X. \quad (16)$$

Since, on the surface $\psi=\psi'$,

$$h = 1 + \varepsilon\rho_o \cos\theta' + \varepsilon\Delta(\psi') , \quad (17)$$

it can readily be verified that, to first order

$$\langle h^{-2}\rangle = 1 ,$$

and using expression (15), equation (4) becomes

$$q(\psi') \simeq \frac{I_o a^2}{R_o} \frac{\rho_o}{\left(\frac{d\psi_o}{d\rho}\right)_{\rho=\rho_o}}$$

Substituting relations (15) and (17) in equation (6), it is possible to evaluate $v_{\parallel}$ to terms of order $\varepsilon^{3/2}$:

$$v_{\parallel} = v\sqrt{\varepsilon\rho_o}\,(2k^2 - 1 + \cos\theta')^{\frac{1}{2}}\,(1 - \frac{\varepsilon\rho_o}{2}\cos\theta') \quad ,$$

where $$k^2 = \frac{1 - \lambda + \varepsilon\rho_o + \varepsilon^2 v_1\ (\rho_o)}{2\varepsilon\rho_o}$$

(18)

and $$v_1(\rho_o) = f(1) - f(\rho_o) + \frac{I_w(\rho_o)}{q_o^2} - \frac{\rho_o^2}{2q^2} \quad .$$

Finally, using equation (16) to evaluate $<Hv_{\parallel}\ h^{-1}>$ in equation (5), we have the required expression for J^* :

$$J(\psi') = 8R_o vq(\psi')\sqrt{2\varepsilon\rho_o}\,\Big[(k^2-1)K(k) + E(k) - \frac{\varepsilon}{3}\Big(\frac{\rho_o}{2} + \Big(\frac{df}{d\rho}\Big)_{\rho=\rho_o}\Big) \times$$

$$((1-k^2)K(k) + (2k^2-1)E(k))\Big] \quad , \qquad (19)$$

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds respectively.

If all but the lowest order terms are neglected in the expression for J we find

$$J(\psi') = 8R_o vq(\psi')\sqrt{2\varepsilon\rho_o}\,[(k^2-1)K(k) + E(k)] \quad , \qquad (19a)$$

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* To convert the integrals which appear to a form available in standard tabulations, a new variable, t , was defined such that $k \sin t = \sin(\theta'/2)$.

where $$k^2 = \frac{1 - \lambda + \varepsilon\rho_o}{2\varepsilon\rho_o} \quad .$$

To lowest order, this expression is in agreement with that derived by KADOMTSEV and POGUTSE (1970) for circular and concentric magnetic surfaces.

5. RESULTS

Using expressions (18) and (19), we have calculated J as a function of ψ/ψ_a (ψ_a is the value of ψ at the boundary) in various situations (Figs. 2 to 5). For all the figures, ε=0.3, q_o=1 , and J is normalized by $8R_o v q_a$, where q_a is the boundary value of $q(\psi)$. Figure 2 shows curves of J calculated with parabolic pressure and current profiles for several values of β_p and also for the zero-order case (equation (19a)); figure 3 shows the same curves with quartic pressure and current profiles, and in figure 4, the current profile is quartic but the pressure profile is peaked off-axis (α_p=3). In all three figures, λ is given the "average" value λ_A . We see from the definiton of k^2 (equations (18)) that the range of λ for trapped particles is

$$1 + \varepsilon^2 v_1(\rho_o) - \varepsilon\rho_o < \lambda < 1 + \varepsilon^2 v_1(\rho_o) + \varepsilon\rho_o \quad .$$

A satisfactory definition of λ_A is the value of λ such that the particle remains trapped at the magnetic axis:

$$\lambda_A = 1 + \varepsilon^2 v_1(0) \quad . \tag{20}$$

To zero-order, λ_A takes the usual "average" value of 1 .

In figure 5, curves of J are plotted for $\lambda=0.85$ (weakly-trapped particle) and $\lambda=1.15$ (strongly-trapped particle). For the weakly-trapped case, the curves stop where the turning points exceed $\theta'=\pm\pi$ and the particle becomes a free, circulating particle. For the strongly-trapped case, J goes to zero as the turning points approach $\theta'=\pm 0$. The dashed curve in figures 2 to 5 is the quantity $\sqrt{\rho_o}\ \frac{p}{p_o}$, which is approximately proportional to the trapped-particle pressure.

6. DISCUSSION

From the figures, it can be seen that ∇J falls when first order terms are taken into account, and when β_p is increased to values $\sim \varepsilon^{-1}$, ∇J becomes negative over a large part of the plasma column. The three principal mechanisms responsible for this change are:

i) Diamagnetic Well.

The decrease in toroidal field at large values of β_p is a second order effect as far as the trapped-particle path is concerned, however it does change the particle's parallel velocity and its turning points to first order. At the plasma boundary, the effect is zero; for magnetic surfaces approaching the magnetic axis, the toroidal field decreases below its vacuum value, the particles can move further along the field lines before being reflected, and J is increased.

ii) Outward Shift of Magnetic Surfaces.

The shift of magnetic surfaces in the R-direction reduces the toroidal field at a given surface. This extends the trapped-particle orbits and increases J in a similar fashion to mechanism i). This effect is also zero at the boundary and maximum at the magnetic axis. Mathematically, i) and ii) appear in the expression for $v_1(\rho_o)$ (equations (18)).

iii) Increase in the Poloidal Field.

Where the flux surfaces are packed closely together at the outside of the torus, the poloidal field increases. This reduces the length of the trapped-particle orbits because the field lines spiral inward more steeply, and J is diminished. This effect is zero at the magnetic axis and increases towards the plasma boundary, producing the marked fall in J in that region. Mathematically, the effect arises in the $\frac{df}{d\rho}$ term of equation (19).

The influence of mechanisms i) and ii) on trapped-particle turning points can be seen in figure 5: as β_p increases, strongly-trapped particles are able to penetrate closer to the magnetic axis; weakly-trapped particle become circulating particles further from the axis.

Mechanisms ii) and iii) above reduce ∇J more than mechanism i) (the diamagnetic well). Previous work by other authors (DOBROTT and GREENE, 1975; ROSENBLUTH and SLOAN, 1971; SIGMAR and FREIDBERG, 1975) based upon the diamagnetic effect have shown that large values of β_p ($\sim \varepsilon^{-1}$ in circular cross-section tokamaks) are required to reduce ∇J significantly. Our calculations show that values of $\beta_p \gtrsim 1$ are already sufficient to make the J profile more uniform over the plasma column. Since the diffusion coefficient for the interchange trapped-particle instability is proportional to $|\nabla J|^{\frac{1}{2}}$, this coefficient diminishes for values of $\beta_p \gtrsim 1$.

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APPENDIX

The expression for $f(\rho)$ (equation (14)) is obtained by integrating equation (11) for our profiles (equations (7)). For the coefficients A_1 and B_1 we have

$$A_1 = \frac{9}{(1+\alpha_j)^2(x_1-x_2)^2}\left[2-\beta_o\alpha_p+(\tfrac{4}{3}\beta_o(1+\alpha_p)+\alpha_j)x_1+(\frac{\alpha_j^2}{8}-\frac{1+\alpha_j}{3})x_1^2\right.$$

$$\left.-\frac{\alpha_j(1+\alpha_j)}{15}x_1^3\right]$$

and

$$B_1 = \left[\frac{9}{(1+\alpha_j)^2}\left(2-\beta_o\alpha_p-\frac{\alpha_j(1+\alpha_j)}{15}x_2x_1^2\right)-A_1x_2^2-A_2x_1^2\right]/x_1x_2(x_1-x_2).$$

A_2 and B_2 are found by interchanging the subscripts 1 and 2 in these expressions. x_1 and x_2 are given by

$$x_{1,2} = \frac{3\alpha_j \pm \sqrt{9\alpha_j^2+48(1+\alpha_j)}}{4(1+\alpha_j)}$$

and

$$\beta_o = \frac{(4+\alpha_j)^2}{4+\alpha_p}\beta_p .$$

The expression for $I_w(\rho)$ (equation (15)) is

$$I_w(\rho) = g(x) - g(1) + \frac{\beta_o}{12}(1+\alpha_p x-(1+\alpha_p)x^2)$$

where $x=\rho^2$ and

$$g(x) = x + \frac{3\alpha_j}{4} x^2 + \left[\frac{\alpha_j^2}{6} - \frac{4}{9}(1+\alpha_j)\right] x^3 - \frac{5\alpha_j}{24}(1+\alpha_j)x^4 + \frac{(1+\alpha_j)^2}{15} x^5.$$

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FIGURE CAPTIONS

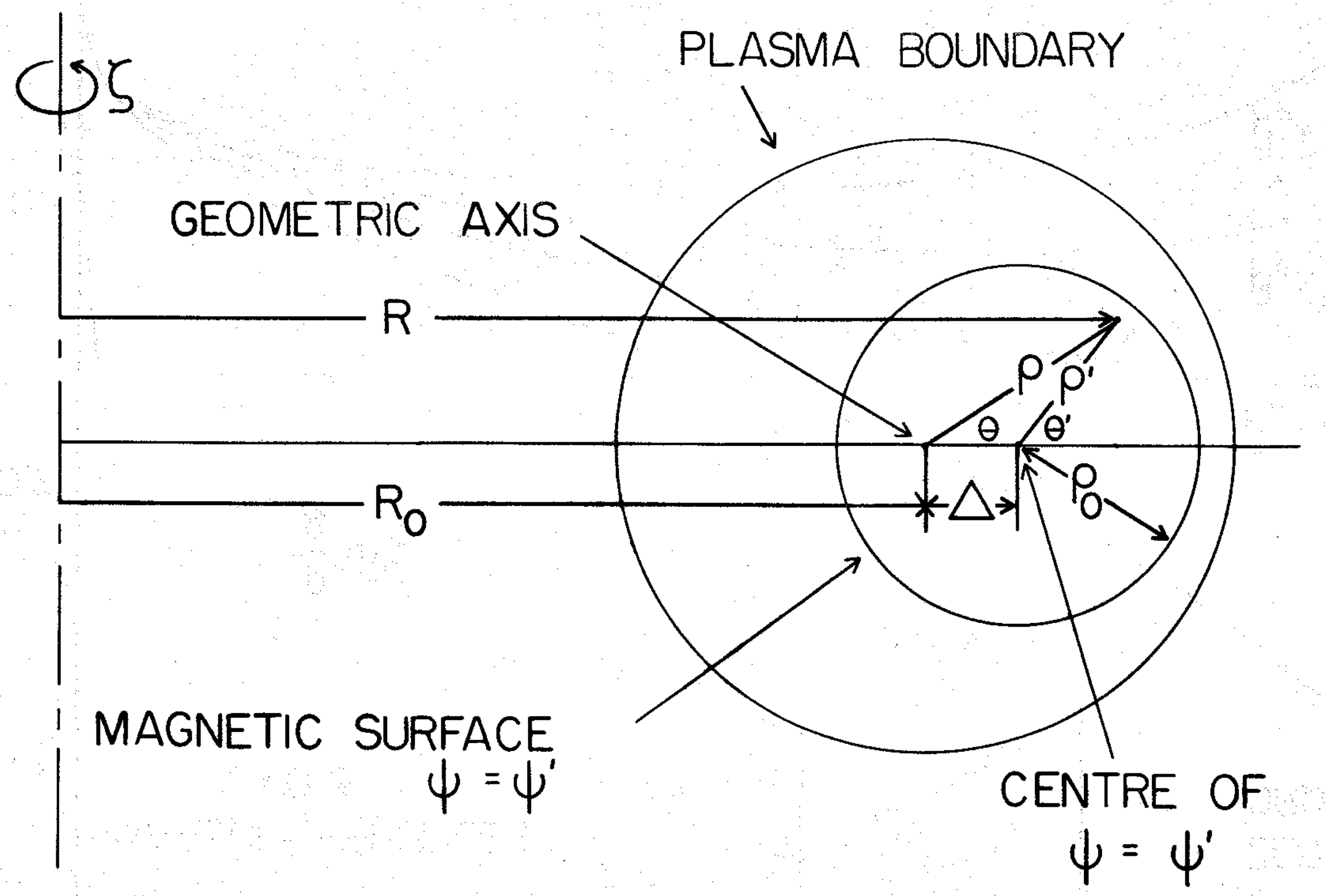
Fig. 1. Cross-section of plasma showing co-ordinate system (ρ', θ') used for the averages over magnetic surface $\psi = \psi'$.

Fig. 2. Curves of the second adiabatic invariant, J, as a function of ψ/ψ_a, the normalized flux function. The pressure and toroidal current profiles are given by expressions (7) with $\alpha_p = \alpha_j = -1$, and are shown in the inset as functions of ψ/ψ_a. λ, the pitch angle variable, has the value λ_A (equation (20)). The dashed curve, $\frac{p}{p_o}\sqrt{\rho_o}$, is approximately proportional to the trapped-particle pressure.

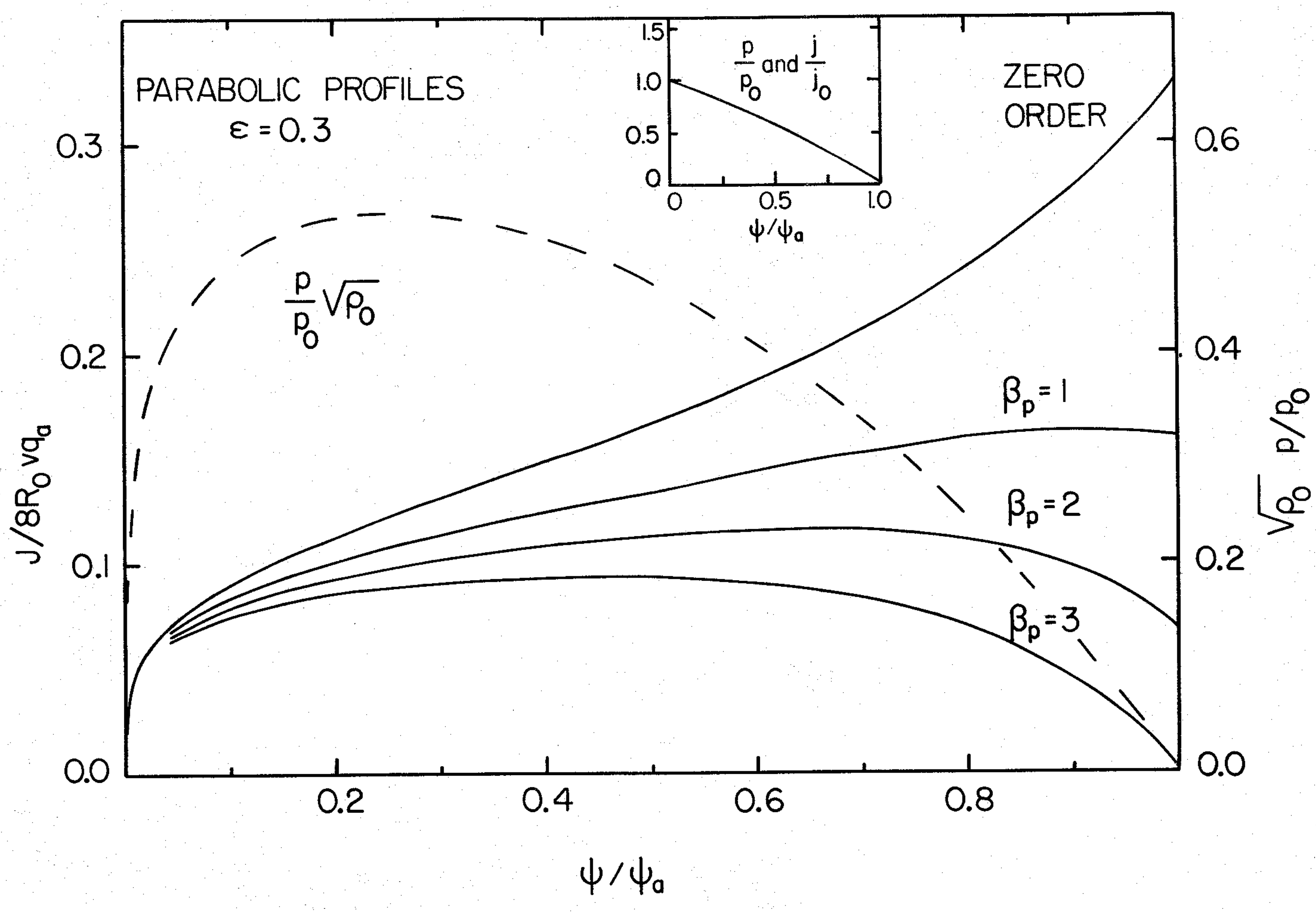
Fig. 3. Curves of the second adiabatic invariant, J, as a function of ψ/ψ_a, the normalized flux function. For this figure $\alpha_p = \alpha_j = 0$ in expressions (7) and $\lambda = \lambda_A$ (equation (20)).

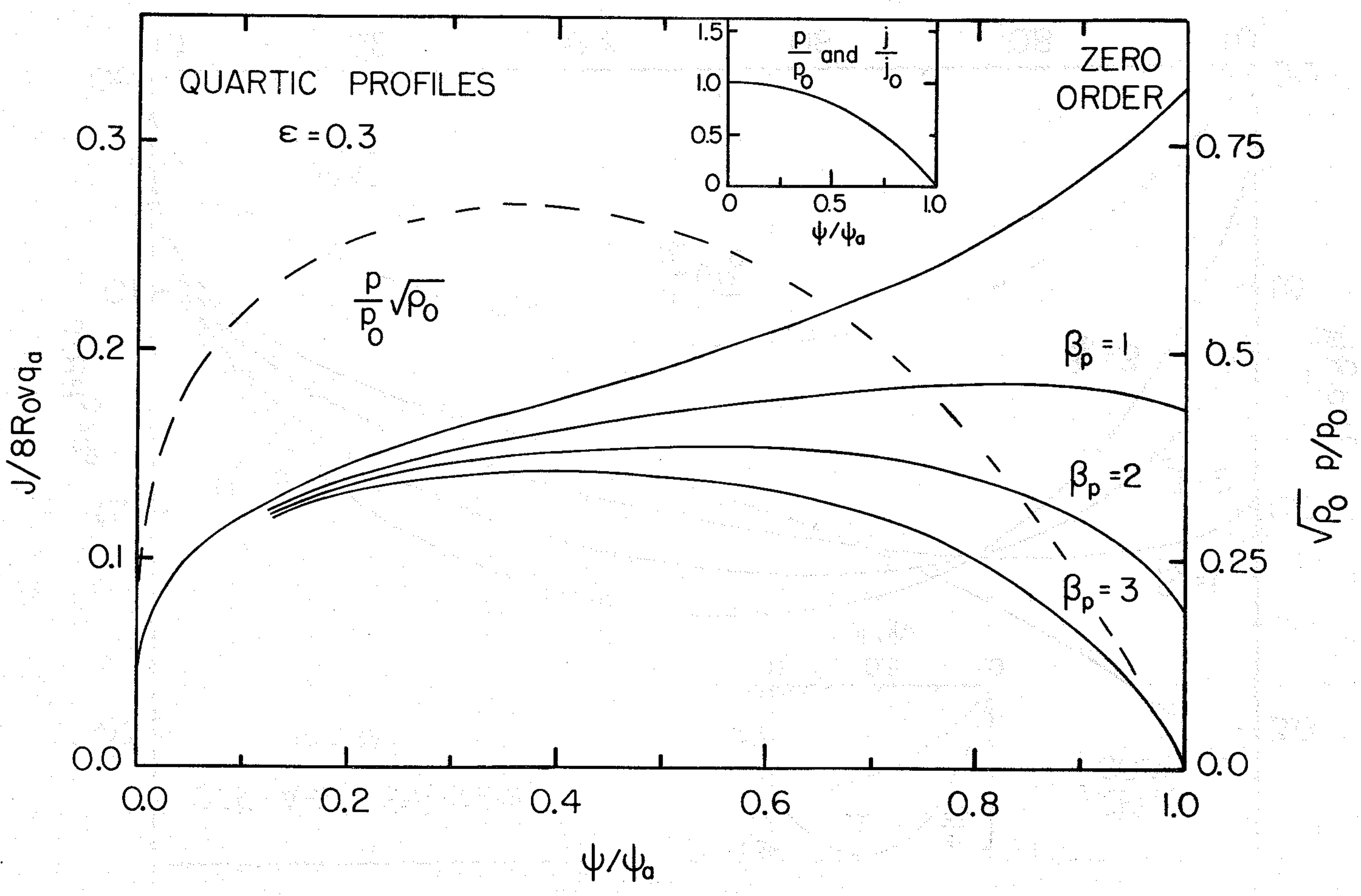
Fig. 4. Curves of the second adiabatic invariant, J, as a function of ψ/ψ_a, the normalized flux function. For this figure, $\alpha_p = 3$ and $\alpha_j = 0$ in expressions (7) and $\lambda = \lambda_A$ (equation (20)).

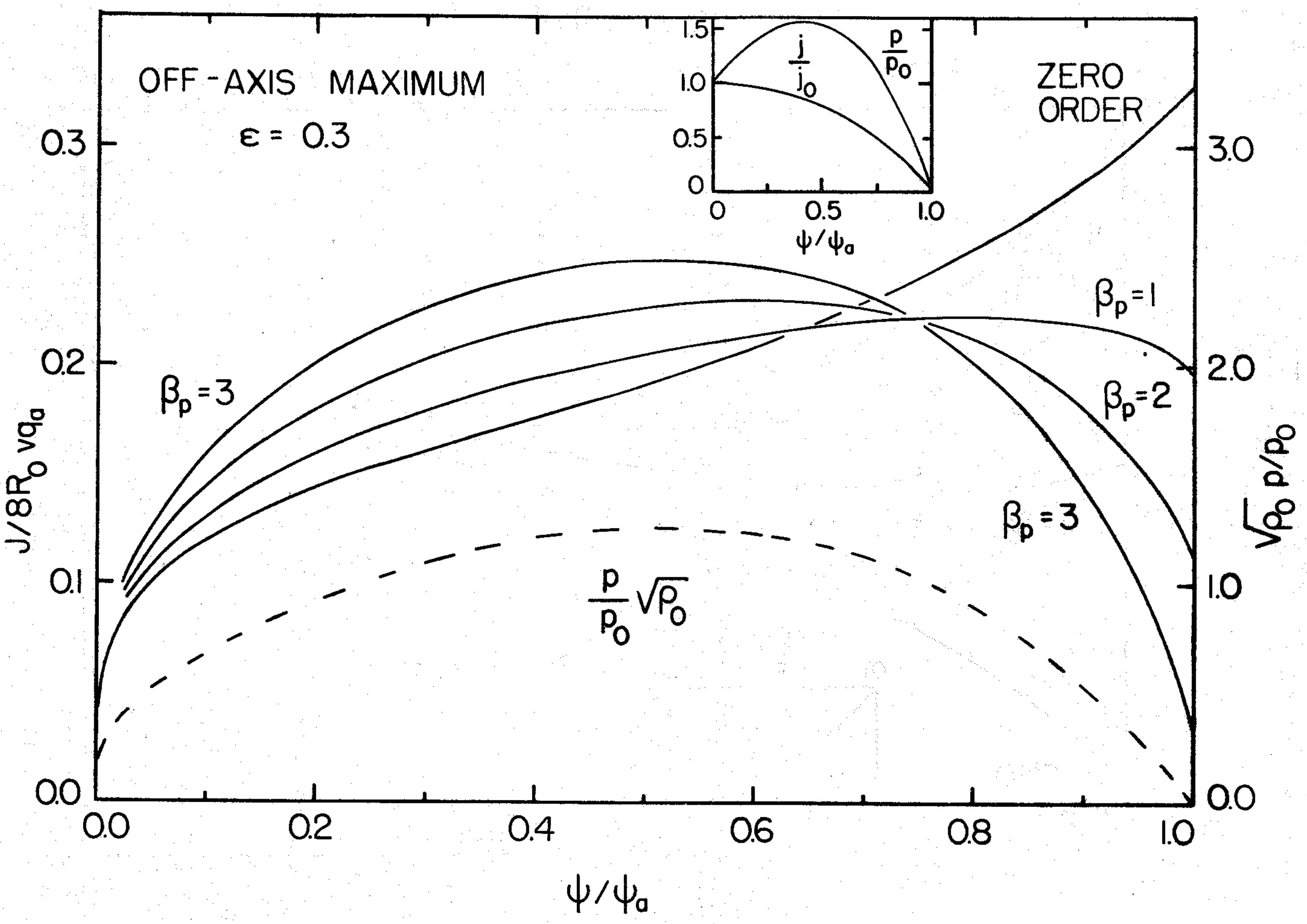
Fig. 5. Curves of the second adiabatic invariant, J, as a function of ψ/ψ_a, the normalized flux function. For this figure $\alpha_p = \alpha_j = -1$ in expressions (7) and J is shown both for $\lambda = 0.85$ (weakly-trapped particle) and $\lambda = 1.15$ (strongly-trapped particle). The variation of the curves with the parameter β_p is explained in the text.



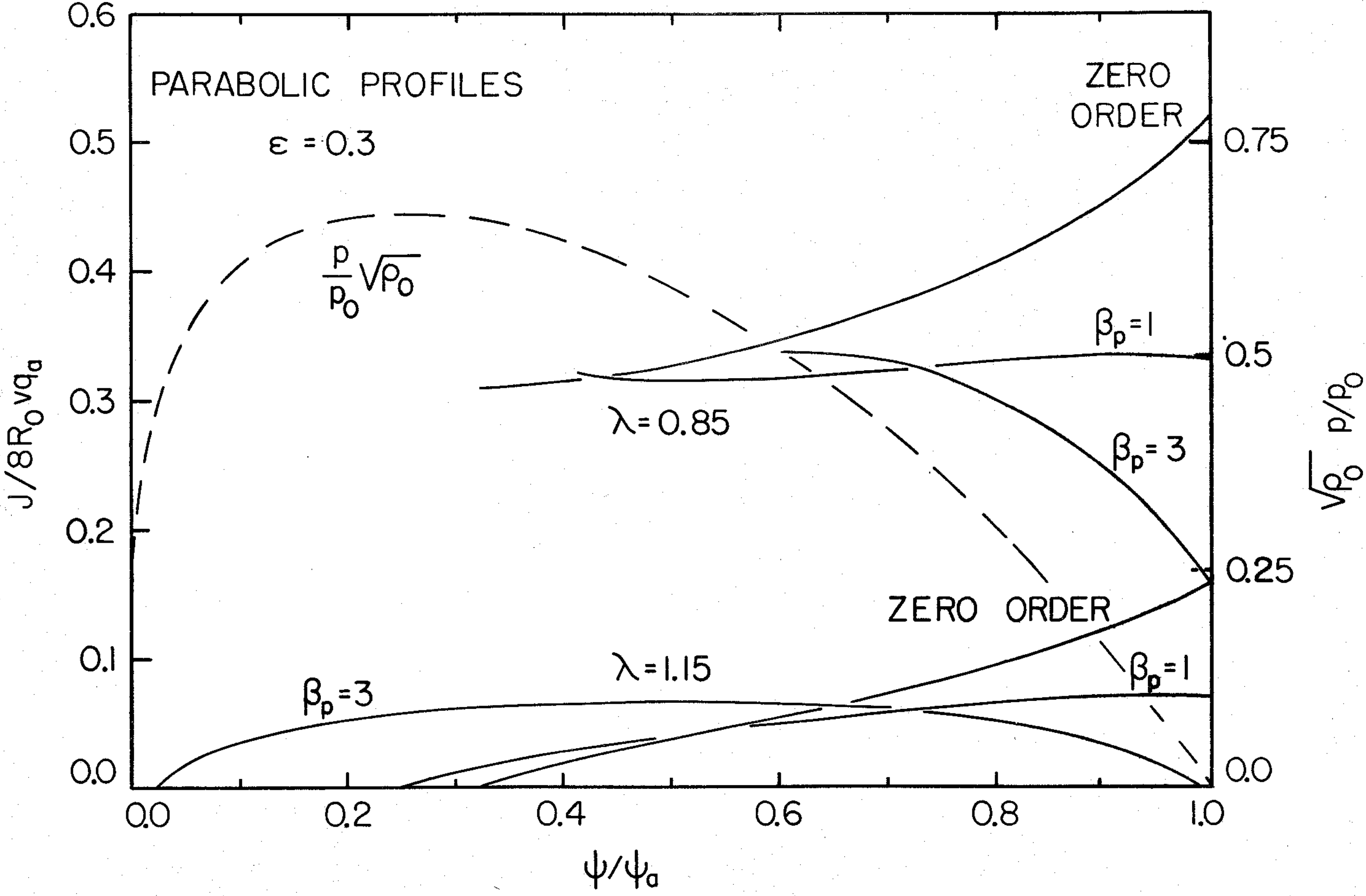
ζ
PLASMA BOUNDARY
GEOMETRIC AXIS
R
ρ
ρ'
θ
θ'
R_o
Δ
ρ_o
MAGNETIC SURFACE
$\psi = \psi'$
CENTRE OF
$\psi = \psi'$


PARABOLIC PROFILES
ε = 0.3
ZERO ORDER
p/p0 and j/j0
ψ/ψa
p/p0 √ρ0
βp = 1
βp = 2
βp = 3
J/8R0 vqa
√ρ0 p/p0
0.0
0.1
0.2
0.3
0.4
0.6
0.8


QUARTIC PROFILES
ε = 0.3
ZERO ORDER
$\frac{p}{p_0}$ and $\frac{j}{j_0}$
$\frac{p}{p_0}\sqrt{\rho_0}$
$\beta_p = 1$
$\beta_p = 2$
$\beta_p = 3$
$J/8R_0 v q_a$
$\sqrt{\rho_0}\ p/p_0$
ψ/ψ_a



OFF-AXIS MAXIMUM
ε = 0.3
ZERO ORDER
$\beta_p = 1$
$\beta_p = 2$
$\beta_p = 3$
$\beta_p = 3$
$\frac{p}{p_0}\sqrt{\rho_0}$
$J/8R_0\,vq_a$
$\sqrt{\rho_0}\,p/p_0$
ψ/ψ_a
0.0
0.1
0.2
0.3
0.4
0.6
0.8
1.0
2.0
3.0
$\frac{j}{j_0}$
$\frac{p}{p_0}$
0
0.5
1.5



PARABOLIC PROFILES
ε = 0.3
$\frac{p}{p_0}\sqrt{\rho_0}$
ZERO ORDER
$\beta_p = 1$
$\beta_p = 3$
$\lambda = 0.85$
ZERO ORDER
$\lambda = 1.15$
$\beta_p = 3$
$\beta_p = 1$
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$J/8R_0 vq_a$
0.0
0.2
0.4
0.6
0.8
1.0
ψ/ψ_a
0.75
0.5
0.25
0.0
$\sqrt{\rho_0}\ p/p_0$