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by

H. Fleming
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01000 São Paulo, Brasil

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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ON A BROKEN-SYMMETRIC THEORY OF GRAVITY

H. Fleming

Instituto de Física, Universidade de São Paulo,
C.P. 20516, 01000 São Paulo, Brasil

ABSTRACT

A theory of gravity recently proposed by Zee is examined. The propagation of the special scalar field introduced by this theory is studied in cosmological models, and some problems are pointed out, connected with the possibility of a time-dependent vacuum expectation value for this scalar field.

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In a recent paper[1] Zee proposed a new theory of gravity that has the interesting feature of obtaining Newton's coupling constant in terms of the vacuum expectation value of some scalar field.

The action is

$$S = \int d^4x \sqrt{-g} \left[- \frac{1}{2} \varepsilon \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + L_W \right] \tag{1}$$

where ϕ is the above-mentioned scalar field, ε is a dimensionless coupling constant taken to be of order $\lesssim 1$, $V(\phi)$ describes the self-interaction of the ϕ , and L_W is the lagrangian of the "rest of the world", which may or may not contain ϕ .

In this note we want to call attention to the fact that, in general, v , the vacuum expectation value of ϕ , will not be a constant, and that this fact may lead to unpleasant consequences. On the other hand, it allows one to relax Zee's assumption that $V(\phi=v)=0$. To exemplify, let us consider a very simple situation. We start by assuming that L_W does not contain ϕ , and that the matter described by L_W is much more abundant than that described by ϕ . We can, therefore, suppose that ϕ propagates in a space-time whose metrical properties are essentialy determined by L_W , that is, under the action of an external gravitational field. The relevant action is, therefore,

$$S = \int d^4x \sqrt{-g} \left[- \frac{1}{2} \varepsilon\phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \tag{2}$$

where R and $g\mu\nu$ are now given functions. We will take the metric to be that of the open Friedmann model[2], whose line element is

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2 - \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{3}$$

where $dt = a(\eta)d\eta$, t being the proper time.
We take $V(\phi) = \frac{\lambda}{12}\phi^4$. Then, the ϕ lagrangian is

$$L\phi = \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\varepsilon\phi^2 R - \frac{\lambda}{12}\phi^4\right] \tag{4}$$

Suppose now that

$$<0|\phi(x)|0> = v(\eta)$$

which is the most general functional dependence compatible with the spatial homogeneity of the Friedmann space-time. By redefining the fields through

$$\phi(x) = \zeta(x) + v(\eta) \tag{5}$$

one gets, from (4),

$$\begin{aligned} L\phi = \sqrt{-g}\Big[& \frac{1}{2} g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta - \frac{1}{2}\varepsilon\zeta^2 R - \frac{\lambda}{2} v^2\zeta^2 - \frac{\lambda v}{3}\zeta^3 - \frac{\lambda}{12}\zeta^4 + \\ & + g^{\mu\nu}\partial_\mu v\partial_\nu\zeta - \zeta(v\varepsilon R - \frac{\lambda}{3} v^3) + \\ & + \frac{1}{2} g^{\mu\nu}\partial_\mu v\partial_\nu v - \frac{1}{2}\varepsilon v^2 R - \frac{\lambda v^4}{12}\Big] \end{aligned} \tag{6}$$

As ζ is <u>defined</u> as having zero vacuum expectation value, the tadpole terms in (6) must cancel, giving

$$\Box v(\eta) + \varepsilon R\, v(\eta) + \frac{\lambda}{3} v^3(\eta) = 0 \tag{7}$$

We take now $\varepsilon = \frac{1}{6}$ and insert in (7) the value of R,

$$R = -\frac{6}{a^3}(a - \ddot{a}) \tag{8}$$

getting the equation

$$\ddot{v} + 2\frac{\dot{a}}{a}\dot{v} + (\frac{\ddot{a}}{a} - 1)v + \frac{\lambda a^2}{3}v^3 = 0 \tag{9}$$

where the dots stand for η-differentiation. By introducing the variable f through

$$v = \sqrt{\frac{3}{\lambda}}\ \frac{f}{a} \tag{10}$$

the following equation is obtained

$$\ddot{f} - f + f^3 = 0 \tag{11}$$

which is a particular case fo Duffing's equation. It has been studied recently by Frolov, Grib and Mostepanenko(3), who showed that the solution $f=0$ is unstable, the real vacua corresponding to the solutions $f=\pm 1$.

Following Zee we demand that the usual gravitational equations be recovered when ζ is put equal to zero. This is obtained when the additional condition

$$\frac{1}{2}g^{\mu\nu}\partial_\mu v\partial_\nu v - \frac{\lambda v^4}{12} = 0 \tag{12}$$

is satisfied. For the stable solutions of (11) this gives

$$\frac{\dot{a}^2}{a^2} = \frac{1}{2} \tag{13}$$

which has the solution

$$a(\eta) = a_0 \, e^{\frac{1}{\sqrt{2}}\eta} \tag{14}$$

a_o being a constant. This completes the specification of the model, as $v(\eta)$ is now computable:

$$v(\eta) = \sqrt{\frac{3}{\lambda}} \, \frac{1}{a(\eta)} = \sqrt{\frac{3}{\lambda}} \, \frac{e^{-\frac{1}{\sqrt{2}}\eta}}{a_0} \, . \tag{15}$$

Zee's basic relation,

$$G_N = \frac{1}{16\pi} \left(\frac{1}{\frac{1}{2}\varepsilon \, v^2} \right)$$

now gives

$$G_N = \frac{\lambda a_0^2}{36 \, . \, 16\pi} \, e^{\sqrt{2}\,\eta}$$

that is,

$$G_N = \frac{\lambda t^2}{1152\pi} \tag{16}$$

where use was made of $\frac{dt}{d\eta} = a(\eta)$.

So, in the open Friedmann model, the "Newton constant" is observed to grow with time, contrary to all speculations on the variation of the gravitational coupling. This is, in fact, characteristic of most theories defined on a curved space-time, where the term proportional to the scalar curvature plays a role similar to the "wrong sign" mass terms in models of the Goldstone type(4). As the curvature is large at the initial stages of the evolution of the Universe, so is $v(\eta)$, and this implies a decreasing G_N for an expanding Universe. The problem remains essentially unchanged if one tries, following Zee, to enforce a chosen vacuum expectation value by using a potential of the type $\lambda(\phi^2-w^2)^2$, where w is a given number. One can then show that the stable solutions are

$$v(\eta) = \sqrt{\frac{3}{\lambda a^2} + w^2}$$

and the problem remains. Also, the results are not characteristic of the particular value we have chosen for ε . In Ref. (5) it is argued that only for very small values of ε one could expect the absence of a time-dependent vacuum expectation value.

The situation is very different in the case of the closed Friedmann model. It has been shown recently(5) that, for models analogous to those treated above, no time-dependent vacuum expectation values for ϕ appear, so that, in this case, Zee's ideas lead to the usual results.

Apparently, therefore, Zee's theory of gravitation favors the closed Friedmann model, as compared to the open one. More detailed studies are, of course, necessary.

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