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preprint

IFUSP/P-253

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CP^{n-1} MODEL

by

E. Abdalla , M.C.B. Abdalla and M. Gomes
Instituto de Física , Universidade de São Paulo
São Paulo, Brasil

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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Instituto de Física, Universidade de São Paulo,
São Paulo, Brasil

ABSTRACT

We prove that the non-local charge anomaly of the CP^{n-1} model vanishes when fermions are coupled in a minimal or supersymmetric way.

JAN/1981

I. INTRODUCTION

In a recent paper[1] we proved that the CP^{n-1} model has a quantum anomaly in the conservation of the non-local quantum charge, which explains the departure from its classical analog, known to be classically integrable[2].

The classically integrability condition is known to be[3]:

$$\partial_\mu j_\nu - \partial_\nu j_\mu + 2\left[j_\mu . j_\nu \right] = 0 \qquad (I.1)$$

but due to quantum fluctuations the right hand side of (I.1) aquires the term $-\frac{2}{\pi} z_i z_j . F_{\mu\nu}$.

However there are reasons to suspect that when coupled to fermions in a minimal or supersymmetric way the equation (I.1) is again valid. These suspections root in the fact that in the supersymmetric model quarks are liberated[4] as well as in the case of minimal coupling[5]. For both cases there is a proposed exact S-matrix[5,6] verified in lowest non-trivial order.

In this paper we will verify that, indeed, in lowest order of the 1/n-expansion, the anomaly of the pure CP^{n-1} model is cancelled with a complementary one coming from the additional couplings considered.

In section II we are going to discuss in detail the case of the CP^{n-1} model minimally coupled to fermions. In section III we state the Feynmann rules for the supersymmetric CP^{n-1} model.

In section IV we write down the Wilson expansion for the product of two currents. Finally in sec. V we construct the conserved current for both cases.

II. MINIMUM COUPLING TO FERMIONS

We couple the CP^{n-1} model minimally to fermions by writting down the Lagrangian:

$$\mathcal{L} = D_\mu z \overline{D_\mu z} + \overline{\Psi} i (\partial\!\!\!/ - A\!\!\!/) \Psi \qquad \text{(II.1)}$$

where $D_\mu z = \partial_\mu z - A_\mu z$; $\bar{A}_\mu = - A_\mu$; $\bar{z} z = \frac{n}{2f}$

The Euler's equation for the A_μ field furnishes

$$A_\mu = \frac{f}{n} \left(\bar{z} \overleftrightarrow{\partial}_\mu z - i \overline{\Psi} \gamma^\mu \Psi \right) \qquad \text{(II.2)}$$

so that, classically, A_μ is not an independent field.

Associated with the linear transformations of the z_i fields, leaving $\bar{z}z$ unchanged, there are internal symmetry currents givenby

$$J^\mu_{ij} = z_i \overline{D_\mu z_j} - D_\mu z_i \bar{z}_j = j^\mu_{ij} - \frac{2if}{n} \left(\overline{\Psi} \gamma^\mu \Psi \right) z_i \bar{z}_j \qquad \text{(II.3)}$$

where $j^\mu_{ij} = z_i \overleftrightarrow{\partial}_\mu \bar{z}_j + \frac{2f}{n} \left(\bar{z} \overleftrightarrow{\partial}_\mu z \right) z_i \bar{z}_j$

As can be easily verified, these currents satisfy

$$[J^\mu, J^\nu]_{ij} = -\frac{1}{2} \bar{z} z (\partial_\mu J_\nu - \partial_\nu J_\mu)_{ij} + \frac{i}{2} \left[(z_i \bar{z}_j) \overleftrightarrow{\partial}_\nu (\overline{\Psi} \gamma^\mu \Psi) - \nu \leftrightarrow \mu \right] =$$

$$= -\frac{1}{2} \bar{z} z (\partial_\mu J_\nu - \partial_\nu J_\mu)_{ij} - \frac{i}{2} \left[\partial_\nu (z_i \bar{z}_j \overline{\Psi} \gamma^\mu \Psi) - (\nu \leftrightarrow \mu) \right] \qquad \text{(II.4)}$$

where, in the last equality, axial current conservation $\varepsilon_{\mu\nu} \partial^\mu (\bar{\psi} \gamma^\nu \psi) = 0$ was used. From (II.4) we see that the classical non-local charge

$$Q = \int dy_1 dy_2\, \varepsilon(y_1 - y_2) J_0(y_1, t) J_0(y_2, t) - \frac{n}{2f} \int \left(j_1 - \frac{4if}{n} \overline{\Psi} \gamma^1 \Psi z_i \bar{z}_j \right) dy \qquad \text{(II.5)}$$

is conserved, i.e., $\frac{dQ}{dt} = 0$.

In the quantum version of the model all these calculations must be reexamined. Using the path integral formalism the fermions can be trivially integrated out and we obtain the Feynman rules adequated to the 1/n expansion.

$$A_\mu \text{ - propagator} \longleftrightarrow \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\left[(p^2+4m^2)\,A(p)\right]^{-1} \tag{II.6a}$$

$$z \text{ - propagator} \longleftrightarrow \frac{1}{p^2+m^2} \tag{II.6b}$$

$$\alpha \text{ - propagator} \longleftrightarrow \left[A(p)\right]^{-1} \tag{II.6c}$$

$$A(p) = \frac{1}{2\pi}\left[p^2(p^2+4m^2)\right]^{-1/2} \log \frac{\sqrt{p^2+4m^2}+\sqrt{p^2}}{\sqrt{p^2+4m^2}-\sqrt{p^2}}$$

where α is the lagrange multiplier field added to to enforce the classical constraint $\bar{z}z = \frac{n}{2f}$. The mass m is a dynamical generated mass given by $m^2 = \mu^2 e^{-n/2f}$, where μ is the renormalization spot.

When compared with the pure CP^{n-1} case, we note that the A_μ field has lost its pole at $p^2 = 0$. Heuristically, this means that the partons (the quanta of the z_i fields) are liberated. In ref. (5) a factorized S-matrix which agrees in lower order with the one obtained by the use of (II.6) has been proposed. This strongly suggests the existence of quantum non-local conserved charges. Taking into account the anomaly in the pure CP^{n-1} case we conclude that a compensating anomaly coming from the minimal coupling must exist. In lowest order this can be verified by the study of the short distance product of the currents.

$$J_\mu(x+\varepsilon)\,J_\nu(x) - J_\nu(x)\,J_\mu(x+\varepsilon) = C^{\rho}_{\mu\nu} J_\rho + D^{\rho\sigma}_{\mu\nu}\,\partial_\sigma J_\rho + $$

$$+ C^{\rho\sigma}_{\mu\nu}\, z_i \bar{z}_j\, F_{\rho\sigma} + N_2\left[J_\mu(x+\varepsilon), J_\nu(x)\right] \tag{II.7}$$

$$C^{\rho}_{\mu\nu} = \frac{n}{2\pi}\left[\frac{-\delta_{\mu\nu}\varepsilon_\rho}{\varepsilon^2} + \frac{\delta_{\mu\rho}\varepsilon_\nu}{\varepsilon^2} + \frac{\varepsilon_{\nu\rho}\varepsilon_\mu}{\varepsilon^2} + 2\frac{\varepsilon_\mu\varepsilon_\nu\varepsilon_\rho}{(\varepsilon^2)^2}\right] \tag{II.8}$$

$$D^{\sigma\rho}_{\mu\nu} = \frac{n}{2\pi}\left[\left(\frac{\gamma}{2}+\frac{1}{4}\ln\frac{m^2\varepsilon^2}{4}\right)\left(\delta_{\mu\sigma}\delta_{\nu\rho}-\delta_{\nu\sigma}\delta_{\mu\rho}\right) + \frac{\delta_{\nu\sigma}\varepsilon_\mu\varepsilon_\rho}{2\varepsilon^2} - \frac{\delta_{\mu\nu}\varepsilon_\nu\varepsilon_\rho}{2\varepsilon^2}\right.$$

$$\left. + \frac{\delta_{\mu\rho}\varepsilon_\nu\varepsilon_\sigma}{2\varepsilon^2} + \frac{\delta_{\nu\rho}\varepsilon_\mu\varepsilon_\sigma}{2\varepsilon^2} - \frac{\delta_{\mu\nu}\varepsilon_\rho\varepsilon_\sigma}{2\varepsilon^2} + \frac{\varepsilon_\mu\varepsilon_\rho\varepsilon_\nu\varepsilon_\sigma}{(\varepsilon^2)^2}\right]$$

$$E^{\rho\sigma}_{\mu\nu} = \frac{n}{2\pi}\left[2\delta_{\mu\rho}\frac{\varepsilon_\nu\varepsilon_\sigma}{\varepsilon^2} + 2\delta_{\nu\sigma}\frac{\varepsilon_\mu\varepsilon_\rho}{\varepsilon^2}\right]$$

where the symbol N_2 denotes the normal product defined by Zimmermann making the minimum number of subtractions necessary to render the product of currents at the same point well defined.

In contra-distinction to the pure CP^{n-1} case we find that this normal product gives an additional contribution. From (II.4) we have

$$N\left[J_\mu, J_\nu\right] = -\frac{1}{2} N\left[z_i\bar z_j\,\partial_\nu\left(z_k\overleftrightarrow{\partial}_\mu\bar z_k + 2\bar z_k z_k A_\mu\right) - (\nu\leftrightarrow\mu)\right] \tag{II.9}$$

Now, the graphs contributing to the right hand side (r.h.s.) of (II.9) have the structure shown in fig. 1. where in fig. 1b the detached subgraph of fig. 1a can not occur. The momentum space expression for the detached subgraph of fig. 1a is

$$I_{\mu\nu} = -\frac{in}{2}(2q-p)_\nu\left(\frac{q_\mu q_\rho}{q^2}-\delta_{\mu\rho}\right)F(q)\left(\frac{q_\rho q_\sigma}{q^2}-\delta_{\rho\sigma}\right)\frac{1}{F+\frac{1}{\pi}}\, j_\sigma(q) - (\mu\leftrightarrow\nu) \tag{II.10}$$

where p is the total momentum entering at V. Thus

$$I_{\mu\nu} = -\frac{in}{2}(2q-p)_\nu j_\mu(q) - \frac{in(2q-p)_\nu}{2\pi}\left(\frac{q_\mu q_\sigma}{q^2}-\delta_{\mu\nu}\right)\frac{1}{F+\frac{1}{\pi}}\, j_\sigma(q) - (\mu\leftrightarrow\nu) \tag{II.11}$$

where current conservation $q^\mu j_\mu(q) = 0$ has been used.

The first term in (II.11) cancels against the contributions from the graphs of fig. 1b. The contribution from the second term, on the other hand is, in coordinate space, equal to

$$-\frac{1}{\pi} z_i \bar{z}_j \left(\partial_\nu A_\mu - \partial_\mu A_\nu\right) - \frac{i}{2}\partial_\nu\left[z_i \bar{z}_j \left(\bar{\Psi}\gamma^\mu\Psi\right)\right] - (\mu \leftrightarrow \nu) \quad \text{(II.12)}$$

Thus

$$N_2\left[J_\mu, J_\nu\right] = \frac{n}{\pi} z_i \bar{z}_j F_{\mu\nu} - \frac{in}{2}\left[\partial_\nu \left(z_i \bar{z}_j \bar{\Psi}\gamma^\mu\psi\right) - (\mu \leftrightarrow \nu)\right] \quad \text{(II.13)}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Thus defining $Q = \lim Q_\delta$ where

$$Q_\delta = \frac{1}{n}\int_{|y_1-y_2|\geq\delta} dy_1 dy_2\, \varepsilon(y_1-y_2)\, J_0(y_1,t)\, J_0(y_2,t) - \frac{Z}{n}\int J_1 dy + i\int \bar{\Psi}\gamma^1\psi\, z_i \bar{z}_j\, dy \quad \text{(II.14)}$$

$$Z = \frac{n}{2\pi} \ln(\mu\delta)$$

one obtains $\frac{dQ}{dt} = 0$ up to the order considered.

III. FEYNMAN RULES FOR THE SUPERSYMMETRIC CP^{n-1} MODEL

Supersymmetry couples the CP^{n-1} to the Gross-Neveu chiral model[4]. The formal Lagrangian density describing this coupling is given by

$$\mathcal{L} = D_\mu z \overline{D_\mu z} + \bar{\Psi}\left(\partial\!\!\!/ - \gamma_\mu \bar{z}\partial_\mu z\right)\psi + \frac{f}{2n}\left[(\bar{\Psi}\psi)^2 + (\bar{\Psi}\gamma^5\psi)^2 - (\bar{\Psi}\gamma_\mu\psi)^2\right] \quad \text{(III.1)}$$

where $D_\mu z = \partial_\mu z - \frac{2f}{n}\left(\bar{z}\partial_\mu z\right) z$

$$\bar{z}z = \frac{n}{2f} \quad ; \quad \bar{\Psi}z = \bar{z}\Psi = 0$$

The $1/n$ expansion of this model was extensively discussed in ref. (4). For completeness we repeat the main arguments. The generating functional for the euclidian Green-functions is given by

$$Z(J,\bar{J},\eta,\bar{\eta}) = \int \mathcal{D}z\, \mathcal{D}\bar{z}\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi} \prod_x \left[\delta\left(|z|^2 - \frac{n}{2f}\right) \delta(\bar{\psi}z)\, \delta(\bar{z}\psi) \right] \times$$

$$\times \exp\left\{ -S + \int d^2x \left[\bar{J}z + \bar{z}J + \bar{\eta}\psi + \bar{\psi}\eta \right] \right\} \tag{III.2}$$

where S is the action.

The constraints and the quartic interaction can be eliminated by introducing auxiliary fields α , λ_μ , $\bar{c}$, c , ϕ and π in the following way

$$\prod_x \left[\delta\left(|z|^2 - \frac{n}{2f}\right) \delta(\bar{\psi}z)\, \delta(\bar{z}\psi) \right] \exp \int d^2x \left\{ -\frac{f}{2n} \left[(\bar{z}\overleftrightarrow{\partial}_\mu z - \bar{\psi}\gamma_\mu\psi \right]^2 + \right.$$

$$\left. + \frac{f}{2n} \left[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2 \right] \right\} = \int \mathcal{D}_\alpha\, \mathcal{D}_c\, \mathcal{D}_{\bar{c}}\, \mathcal{D}_{\lambda_\mu}\, \mathcal{D}\phi\, \mathcal{D}\pi \exp \int d^2x \left\{ \frac{i}{\sqrt{n}}\alpha\left(|z|^2 - \frac{n}{2f}\right) \right.$$

$$+ \frac{i}{\sqrt{n}}(\bar{c}\bar{z}\psi + \bar{\psi}zc) + \frac{i}{\sqrt{n}}\lambda_\mu\left((\bar{z}\overleftrightarrow{\partial}_\mu z) - \bar{\psi}\gamma_\mu\psi \right) - \left(m^2 + \frac{1}{n}\lambda_\mu\lambda_\mu \right)|z|^2 +$$

$$+ \frac{1}{\sqrt{n}}\left(\phi\bar{\psi}\psi + \pi\bar{\psi}\gamma_5\psi \right) - \frac{1}{2f}\left(\phi\phi + \pi\pi\right). \tag{III.3}$$

Using (III.3) it's straightforward to integrate out the ψ and z fields to obtain an effective action, which is expandable in a power series of $1/\sqrt{n}$. The saddle point condition implies mass transmutation and that ϕ has a non vanishing vacuum expectation value.

The renormalization of f is the same as in the case without fermions, which by the way do not affect asymptotic freedom[4]. We have now for the quadratic part of S :

$$S^{(2)} = \int d^2x\, d^2y \left\{ \frac{1}{2}\alpha(x)\Gamma^{\alpha}(x-y)\,\alpha(y) + \frac{1}{2}\lambda_\mu(x)\,\Gamma^{\lambda}_{\mu\nu}(x-y)\,\lambda_\nu(y) \right.$$

$$+ \frac{1}{2}\phi(x)\,\Gamma^{\phi}(x-y)\,\phi(y) + \frac{1}{2}\pi(x)\,\Gamma^{\pi}(x-y)\,\pi(y) + \lambda_\mu(x)\,\Gamma^{\lambda\pi}_{\mu}(x-y)\,\pi(y)$$

$$\left. + \bar{c}(x)\,\Gamma^{\bar{c}c}(x-y)\,c(y) \right\} \qquad \text{(III.4)}$$

where

$$\Gamma^{\alpha}(p) = A(p) \qquad \text{(III.5)}$$

$$\Gamma^{\lambda}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) p^2 A(p) \qquad \text{(III.6)}$$

$$\Gamma^{\phi}(p) = (p^2 + 4m^2)\, A(p) \qquad \text{(III.7)}$$

$$\Gamma^{\pi}(p) = p^2 A(p)\, A(p) \qquad \text{(III.8)}$$

$$\Gamma^{\lambda\pi}_{\mu}(p) = -2\,\varepsilon_{\mu\nu}\, p_\nu\, m\, A(p) \qquad \text{(III.9)}$$

$$\Gamma^{\bar{c}c}(p) = \left(i\frac{\not p}{2} - m\right) A(p) \qquad \text{(III.10)}$$

$$A(p) = \int \frac{d^2q}{(2\pi)^2} \left\{ (q^2+m^2)\left((p+q)^2+m^2\right) \right\}^{-1}$$

$$= \frac{1}{4\pi}\left[p^2(p^2+4m^2)\right]^{-1/2} \log \frac{\sqrt{p^2} + \sqrt{p^2+4m^2}}{\sqrt{p^2} - \sqrt{p^2+4m^2}} \qquad \text{(III.11)}$$

The propagators read:

$$D^{z\bar{z}}(p) = (p^2 + m^2)^{-1} \qquad \text{(III.12)}$$

$$D^{\psi\bar{\psi}}(p) = (i\not{p} - m)^{-1} \qquad \text{(III.13)}$$

$$D^{\alpha}(p) = \left[A(p)\right]^{-1} \qquad \text{(III.14)}$$

$$D^{\lambda}(p) = \Gamma^{\bar{\pi}}(p)\left[\Gamma^{\lambda}(p)\,\Gamma^{\bar{\pi}}(p) + p^2\left(\Gamma^{\lambda\bar{\pi}}(p)\right)^2\right]^{-1} \qquad \text{(III.15)}$$

$$D^{\phi}(p) = \left[(p^2 + 4m^2)\,A(p)\right]^{-1} \qquad \text{(III.16)}$$

$$D^{\bar{\pi}}(p) = \Gamma^{\lambda}(p)\left[\Gamma^{\lambda}(p)\,\Gamma^{\bar{\pi}}(p) + p^2\left(\Gamma^{\lambda\bar{\pi}}(p)\right)^2\right]^{-1} \qquad \text{(III.17)}$$

$$D^{\lambda\bar{\pi}}(p) = \Gamma^{\lambda\bar{\pi}}(p)\left[\Gamma^{\lambda}(p)\,\Gamma^{\bar{\pi}}(p) + p^2\left(\Gamma^{\lambda\bar{\pi}}(p)\right)^2\right]^{-1} \qquad \text{(III.18)}$$

$$D^{c\bar{c}}(p) = \left[\Gamma^{c\bar{c}}(p)\right]^{-1} \qquad \text{(III.19)}$$

where

$$\Gamma^{\lambda}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Gamma^{\lambda}(p) \qquad \text{(III.20)}$$

$$\Gamma^{\lambda\bar{\pi}}_{\mu}(p) = \varepsilon_{\mu\nu}\, p_\nu\, \Gamma^{\lambda\bar{\pi}}(p) \qquad \text{(III.21)}$$

The graphical notation for these propagators and for the vertices are shown in fig. 2.

IV. SHORT DISTANCE EXPANSION FOR THE PRODUCT OF TWO "ISOSPIN" CURRENTS

Associated with the isospin rotations there is a set of conserved currents j_μ^{ij} given by

$$j_\mu^{ij} = z_i \overleftrightarrow{D}_\mu \bar{z}_j + \bar{\psi}_j \gamma^\mu \psi_i \tag{IV.1}$$

which are conserved. In ref. (7) it was shown how these currents can be used, at the classical level, to construct, non-trivial, non-local conserved charges. In the quantum version we need, similarly to do what was done in section II, to study the short distance behaviour of the product of two such currents. The most general expansion for this product is

$$\begin{aligned}
\left[j_\mu(x+\varepsilon), j_\nu(x)\right] = {} & N\left[j_\mu(x+\varepsilon), j_\nu(x)\right] + C_{\mu\nu}^{\rho} j_\rho^{ij} + D_{\mu\nu}^{\sigma\rho} \partial_\sigma j_\rho^{ij} + E_{\mu\nu}^{\rho\sigma} z_i z_j F_{\rho\sigma} \\
& + 2m G_{\mu\nu}^{\rho\sigma} \varepsilon_{\rho\sigma} \bar{\psi}_j \gamma_5 \psi_i + H_{\mu\nu}^{\rho\sigma} \varepsilon_{\rho\sigma} \pi \bar{\psi}_j \psi_i + I_{\mu\nu}^{\rho\sigma} \varepsilon_{\rho\sigma} \phi \bar{\psi}_j \gamma_5 \psi_i \\
& + J_{\mu\nu}^{'5} (z_i \bar{\psi}_j \gamma_5 c - \psi_i \bar{z}_j \gamma_5 \bar{c}) + 2 C_{\mu\nu}^{'\rho} \bar{\psi}_j \gamma_\rho \psi_i + L_{\mu\nu} \partial_\sigma (\bar{\psi}_j \gamma_\rho \psi_i) \\
& + A_{\mu\nu}^{\rho} \bar{\psi}_j \overleftrightarrow{D}_\rho \psi_i + B_{\mu\nu} z_i \bar{z}_j \bar{\psi} \overleftrightarrow{\not{\partial}} \psi + O_{\mu\nu}^{\rho} \partial_\rho (z_i \bar{z}_j) \phi + \\
& + J_{\mu\nu}^{\rho} (z_i \bar{\psi}_j \gamma_\rho \bar{c} - \psi_i \bar{z}_j \gamma_\rho c) + J_{\mu\nu}^{'\rho} (z_i \bar{\psi}_j \gamma_\rho c - \psi_i \bar{z}_j \gamma_\rho \bar{c}) + \\
& + P_{\mu\nu}^{\rho\sigma} \varepsilon_{\rho\sigma} z_i \bar{z}_j \pi + J_{\mu\nu} (z_i \bar{\psi}_j \bar{c} - \psi_i \bar{z}_j c) + R_{\mu\nu}^{\rho} z_i \bar{z}_j \bar{\psi} \overleftrightarrow{\partial}_\rho \psi \\
& + J_{\mu\nu}' (z_i \bar{\psi}_j c - \psi_i \bar{z}_j \bar{c}) + Q_{\mu\nu}^{\rho\sigma} \varepsilon_{\rho\sigma} z_i \bar{z}_j \phi \pi
\end{aligned} \tag{IV.2}$$

Higher order terms (quadrilinear in the ψ's and z's) have not been considered. We observe now the following facts.

(a) Making a charge conjugation transformation, the left hand side of the above equation changes sign, and interchanges i and j . So, by C-invariance we have that $B_{\mu\nu}$, $J'^{\rho}_{\mu\nu}$, $J^{\rho}_{\mu\nu}$, $O^{\rho}_{\mu\nu}$ are zero.

(b) $A^{\rho}_{\mu\nu}$ and $R^{\rho}_{\mu\nu}$ vanish as $\varepsilon \to 0$. This is so because they are not most logarithimically divergent and it is impossible to construct a third rank tensor (using only $\delta_{\mu\nu}$ and the ε's) having such property.

(c) The $P^{\rho\sigma}_{\mu\nu}$ and $Q^{\rho\sigma}_{\mu\nu}$ contributions come from the subtraction at zero momentum of the graph of fig. (1), but this is readly seen to be zero when antisymmetrized in μ , ν .

(d) $J_{\mu\nu}$ and $J'_{\mu\nu}$ are antisymmetric in μ , ν , thus proportional to $\varepsilon_{\mu\nu}$ what is impossible by P-invariance.

With the above remarks the eq. IV.1 can be re-written as follows:

$$\left[j_\mu , j_\nu \right] = C^{\rho}_{\mu\nu} j_\rho + D^{\sigma\rho}_{\mu\nu} \partial_\sigma j_\rho + \mathcal{E}^{\rho\sigma}_{\mu\nu} z_i \bar{z}_j F_{\rho\sigma} +$$

$$+ 2m\, g^{\rho\sigma}_{\mu\nu} \varepsilon_{\rho\sigma} \bar{\Psi}_j \gamma_5 \psi_i + H^{\rho\sigma}_{\mu\nu} \varepsilon_{\rho\sigma} \pi \bar{\Psi}_j \psi_i +$$

$$+ F^{\rho\sigma}_{\mu\nu} \varepsilon_{\rho\sigma} \phi \bar{\Psi}_j \gamma_5 \psi_i + 2\, C'^{\rho}_{\mu\nu} \bar{\Psi}_j \gamma_\rho \psi_i +$$

$$+ L^{\sigma\rho}_{\mu\nu} \partial_\sigma (\psi_j \gamma_\rho \psi_i) + N^{\sigma\rho}_{\mu\nu} \partial_\sigma (z_i \bar{z}_j \pi)$$

$$+ J'^{5}_{\mu\nu} (z_i \bar{\Psi}_j \gamma_5 c - \psi_i \bar{z}_j \gamma_5 \bar{c}) \tag{IV.3}$$

The calculations involving only z and A_μ fields are in the ref. (1).

The contribution for the term $\pi \bar{\Psi}_j \psi_i$ comes from fig.(3) which is

$$(\text{fig.}3) = \pi \bar{\Psi}_{j\alpha} \psi_{i\beta} \int \gamma_\mu \frac{1}{ik-m} \gamma_5 \frac{1}{ik-m} \gamma_\nu e^{ik\epsilon} d^2k$$

$$(\text{f.}3) = 2\pi \bar{\Psi}_{j\alpha} \psi_{i\beta} \varepsilon_{\mu\nu} K_0(m\epsilon) \tag{IV.4}$$

Analogouslly for the term $\phi \bar{\Psi}_j \gamma_5 \psi_i$ that comes from fig. 4 we have:

$$(\text{fig.}4) = \phi \bar{\Psi}_{j\alpha} \psi_{i\beta} \int \gamma_\mu \frac{1}{ik-m} \frac{1}{ik-m} \gamma_\nu e^{ik\varepsilon} d^2k \tag{IV.5}$$

$$(\text{fig. }4) = -2\phi\,\bar\psi_{j\alpha}\,\psi_{i_\beta}\,K_0(m\varepsilon)\,\varepsilon_{\mu\nu}\,\gamma_{5_{\alpha\beta}} +$$

$$+\ \bar\psi_j\,4m^2\varepsilon_{\mu\nu}\gamma_5\int d^2k\,\frac{e^{ik\varepsilon}}{(k^2+m^2)^2}\,\psi_i\,\phi \qquad \text{(IV.6)}$$

the second (constant) contribution will be included in the finite part of the buble in order to normalize it exactly in the same way as the inverse ϕ-propagator and implement the cancelation shown in fig.5 in the finite part. The contribution to the Wilson-expantion is only the first term. So far for this term.

For the $\bar\psi_j\gamma_5\psi_i$ and $\bar\psi_j\gamma_\rho\psi_i$ that come from fig. 6 we have as result

$$(\text{fig.}6) = 2m\,\varepsilon_{\mu\nu}K_0(m\varepsilon)\bar\psi_j\gamma_5\psi_i - \frac{2}{\varepsilon^2}\bar\psi_j\gamma_\rho\psi_i\left(\delta_{\mu\nu}\varepsilon^\rho - \delta_{\mu\rho}\varepsilon^\nu - \delta_{\nu\rho}\varepsilon_\mu\right) \qquad \text{(IV.7)}$$

The first term contributes to $G^{\rho\sigma}_{\mu\nu}$ and the second one to $C^{\rho}_{\mu\nu} + K^{\rho}_{\mu\nu}$. The term corresponding to fig. 7a is given by:

$$(\text{fig.}7a) = z_i\bar\psi_j c\int d^2k\,e^{ik\varepsilon}\,\frac{ik_\mu}{k^2+m^2}\,\frac{i\not k+m}{k^2+m^2}\,\gamma_\nu - (\mu\leftrightarrow\nu)$$

$$(\text{fig.}7a) = 4z_i\bar\psi_j c\,\varepsilon_{\mu\nu}K_0(m\varepsilon) + 2\varepsilon_{\mu\nu}\gamma_5 z_i\bar\psi_j c \qquad \text{(IV.8)}$$

where the second term goes with the finite part which is not normalized to zero in order to be proportional to the inverse c propagator.

The fig 7b is completely analogous.

Gathering the results (IV.4), (IV.6), (IV.7), we get the divergence of the axial current.

$$2\epsilon_{\mu\nu} K_0(m\epsilon)\partial_\lambda(\bar{\psi}_\lambda \gamma_\lambda \gamma_5 \psi_i) = 2\epsilon_{\mu\nu} K_0(m\epsilon)\partial_\lambda(\bar{\psi}_j \gamma_\sigma \psi_i)\epsilon^{\lambda\sigma} \tag{IV.9}$$

This contribution can be included in $L^{\sigma\rho}_{\mu\nu}$.

We are finally left with

$$[j_\mu, j_\nu] = C^\rho_{\mu\nu} j^c_\rho + 2 C'^\rho_{\mu\nu} i_\rho + D^{\sigma\rho}_{\mu\nu} \partial_\sigma j^c_\rho + $$
$$+ 2 D'^{\sigma\rho}_{\mu\nu} \partial_\sigma i_\rho + E^{\rho\sigma}_{\mu\nu} z_i \bar{z}_j F_{\rho\sigma} \tag{IV.10}$$

where j^c_ρ is the pure CP^{n-1} current and $i_\rho = \bar{\psi}_j \gamma_\rho \psi_i$. Neverthless note that the field A_μ which is in j^c_ρ although having exactly the same Feynman rules as in the pure CP^{n-1} model is the total A_μ , that contains also $\bar{\psi} \gamma_\mu \psi$. As a consequence the terms $z_i \bar{z}_j \bar{\psi} \gamma_\rho \psi$ and $\partial_\sigma(z_i z_j \psi \gamma_\rho \psi)$ are already included in what we called j^c_ρ and $\partial_\sigma j^c_\rho$.

The coeficients $C^\rho_{\mu\nu}$, $D^{\rho\sigma}_{\mu\nu}$, $E^{\sigma\rho}_{\mu\nu}$ are known from pure CP^{n-1} calculations, and given by (II.8).

$$C^\rho_{\mu\nu} = \frac{n}{2\pi}\left[\frac{-\delta_{\mu\nu}\epsilon_\rho}{\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\nu}{\epsilon^2} + \frac{\delta_{\nu\rho}\epsilon_\mu}{\epsilon^2} + \frac{2\epsilon_\mu\epsilon_\nu\epsilon_\rho}{(\epsilon^2)^2}\right] \tag{IV.11a}$$

$$D^{\sigma\rho}_{\mu\nu} = \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \frac{1}{4}\ln\frac{m^2\epsilon^2}{4}\right)\left(\delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\nu\sigma}\delta_{\mu\rho}\right) + \frac{\delta_{\nu\sigma}\epsilon_\mu\epsilon_\rho}{2\epsilon^2}\right.$$
$$\left. - \frac{\delta_{\mu\nu}\epsilon_\nu\epsilon_\rho}{2\epsilon^2} - \frac{\delta_{\mu\nu}\epsilon_\rho\epsilon_\sigma}{2\epsilon^2} + \frac{\delta_{\mu\nu}\epsilon_\nu\epsilon_\sigma}{2\epsilon^2} + \frac{\delta_{\nu\rho}\epsilon_\mu\epsilon_\sigma}{2\epsilon^2} + \frac{\epsilon_\mu\epsilon_\nu\epsilon_\rho\epsilon_\sigma}{(\epsilon^2)^2}\right] \tag{IV.11b}$$

$$E^{\sigma\rho}_{\mu\nu} = \frac{n}{2\pi}\left[2\delta_{\mu\nu}\frac{\epsilon_\nu\epsilon_\sigma}{\epsilon^2} + 2\delta_{\nu\sigma}\frac{\epsilon_\mu\epsilon_\rho}{\epsilon^2}\right] \tag{IV.11c}$$

and from the second term of eq. (IV.7) $C^{\rho'}_{\mu\nu}$ is given by:

$$C'^{\rho}_{\mu\nu} = \frac{n}{\pi}\left[\frac{-\delta_{\mu\nu}\epsilon_\rho}{\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\nu}{\epsilon^2} + \frac{\epsilon_{\nu\rho}\epsilon_\mu}{\epsilon^2} \right] \tag{IV.11d}$$

We could calculate direct $D'^{\sigma\rho}_{\mu\nu}$ from 1/N expansion. However given $C'^{\rho}_{\mu\nu}$ we can achieve enough information about $D'^{\sigma\rho}_{\mu\nu}$ from PT conservation, locality and current conservation.

$$\begin{aligned} D'^{\sigma\rho}_{\mu\nu} = {} & \frac{1}{2} C'^{\rho}_{\mu\nu}\,\epsilon^\rho + D_1\,\epsilon^\rho(\epsilon_\mu\delta_\nu^\sigma - \epsilon_\nu\delta_\mu^\sigma) + \\ & + D_2\,\epsilon^\sigma(\epsilon_\mu\delta_\nu^\sigma + \epsilon_\nu\delta_\mu^\sigma) + D_3'(\delta_\mu^\rho\delta_\nu^\sigma - \delta_\nu^\rho\delta_\mu^\sigma) \end{aligned} \tag{IV.12}$$

where $-z^2 D_1' + D_3' = \frac{1}{4\pi} \ln \mu^2 z^2$

This results are enough to define the quantum non-local charge which will be shown to be conserved.

V. CONSTRUCTION AND CONSERVATION OF THE QUANTUM NON-LOCAL CHARGE

We define a cut off supersymmetric quantum version of the non-local charge as

$$\begin{aligned} Q_\delta = {} & \frac{1}{n}\int_{|y_1-y_2|\geq\delta} dy_1\,dy_2\,\epsilon(y_1-y_2)\,j_0^{ik}(t,y_1)\,j_0^{kj}(t,y_2) - \frac{2\delta}{n}\int dy\,(j_1^{ij}(t,y) + 2\,i_1^{ij}(t,y)) \\ & + i\int dy\,(z_i\bar{z}_j\bar{\psi}\gamma^1\psi)(t,y) \end{aligned} \tag{V.1}$$

That this charge is well defined in the $\delta \to 0$ limit can be seen using

$$[j_0, j_0]\Big|_{t_1=t_2=t} = \frac{-n}{2\pi(y_1-y_2)}\,j_1^C(t,y_2) - \frac{2n}{2\pi(y_1-y_2)}\,i_1(t,y_2) + \mathcal{O}(|y_1-y_2|^{-0}) \tag{V.2}$$

and choosing $Z_\delta = \frac{n}{2\pi} \ell n\, \mu\delta$ it's readly seen that the charge is well defined.

It remains to be proved that with this choice Q_δ is conserved.

$$\frac{dQ}{dt} = \frac{1}{n}\int dy_1\, dy_2\, \epsilon(y_1 - y_2)\left[\frac{\partial}{\partial y_1} j_1^{ik}(t,y_1)\, j_0^{kj}(t,y_2) + j_0^{ik}(t,y_1)\frac{\partial j_1}{\partial j_0}(t,y_2)\right]$$

$$- \frac{Z_\delta}{n}\int dy\, \frac{\partial}{\partial t}\left(j_1^c + 2i_1\right) + i\int dy\, \frac{\partial}{\partial t}\left(z_i \bar{z}_j \bar{\Psi}\gamma^1\psi\right) \qquad (V.3)$$

and integrating in one of the variables we have:

$$\frac{dQ}{dt} = \frac{1}{n}\int dy \left[\left(j_1^{ik}(t,y+\delta) + j_1^{ik}(t,y-\delta)\right) j_0^{kj}(t,y) - \left(j_1^{kj}(t,y+\delta) + \right.\right.$$

$$\left.\left. + j_1^{kj}(t,y-\delta)\right) j_0^{ik}(t,y) - Z_\delta \frac{\partial}{\partial t}\left(j_1^c + 2i_1\right) + i\frac{\partial}{\partial t}\left(z_i \bar{z}_j \bar{\Psi}\gamma^1\psi\right)\right] \quad (V.4)$$

For the comutator $[j_0 j_1]$ we have

$$[j_0, j_1] = N[j_0, j_1] + D_{01}^{\sigma\rho}\partial_\sigma j_\rho + 2D_{01}^{\sigma\rho}\partial_\sigma i_\rho + \epsilon_{01}^{\rho\sigma} z_i \bar{z}_j F_{\rho\sigma}. \qquad (V.5)$$

Inserting(V5) in(V.4), $\partial_0 j_1$ and $\partial_0 i_1$ cancel imediately leaving (V4) as

$$\frac{dQ}{dt} = \frac{2}{n}\int dy\; N[j_0, j_1] + \frac{2}{\pi}\int dy\, z_i \bar{z}_j F_{01} \qquad (V.6)$$

$$+ i\int dy\, \frac{\partial}{\partial t}\left(z_i \bar{z}_j \bar{\Psi}\gamma^1\psi\right)$$

Now a straightforward calculation, similar to that done in section II shows that $N(j_0, j_1) = -\frac{e}{2} z_i \bar{z}_j F_{01} - \frac{i}{2}\frac{\partial}{\partial t}(z_i \bar{z}_j \bar{\psi}\gamma^1\psi)$ up to the order considered. So

$$\frac{dQ}{dt} = 0 \tag{V.7}$$

ACKNOWLEDGEMENTS

It is our pleasure to thank A.J. da Silva for useful discussions. This work of M.C.B. Abdalla was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), whereas E. Abdalla and M. Gomes were partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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FIGURE CAPTIONS

1) Graphs contributing to the r.h.s. of (II.9).

2) Feynman rules for the supersymmetric CP^{n-1} model.

3) The coefficient of the term $\pi \bar{\psi}_j \psi_i$ in the short distance product of two currents, comes from this graph.

4) Graph which generates the coefficient of the term $\bar{\psi}_j \psi_i \phi$

5) Mechanism of the cancellation of the finite part of the above graphs. The finite part of the bubble is norma - lized as to cancel the ϕ propagator.

6) This graph gives contributions to the coefficients of $\bar{\psi}_j \gamma_5 \psi_i$ and $\bar{\psi}_j \gamma_\rho \psi_i$.

7) Graphs producing the coefficients of the terms $\bar{\psi}_j z_i C$ and $\bar{z}_j \psi_i \bar{C}$

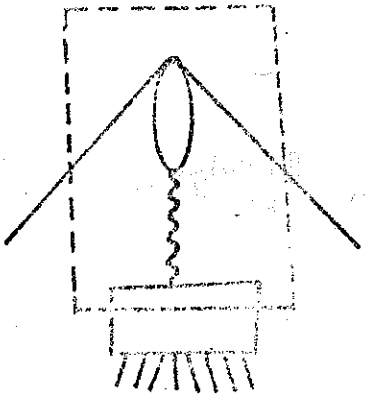


Fig. 1a

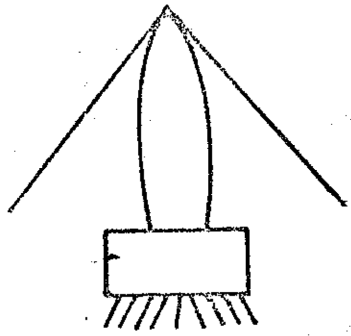


Fig. 1b

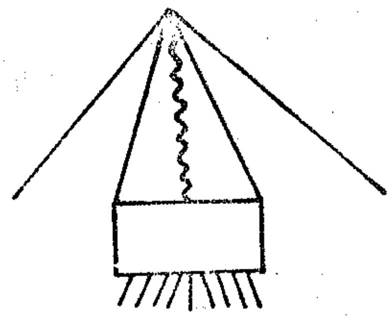


Fig. 1b

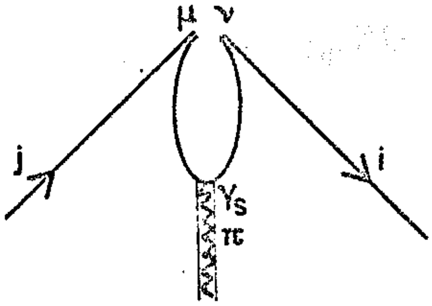


Fig. 3

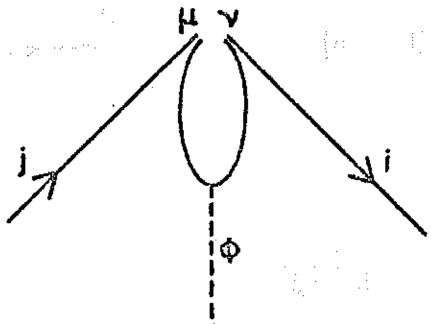


Fig. 4

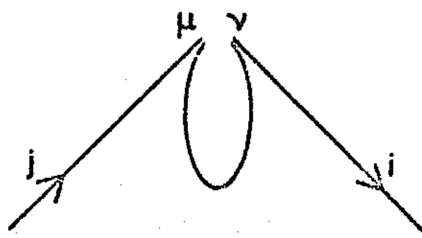


Fig. 6

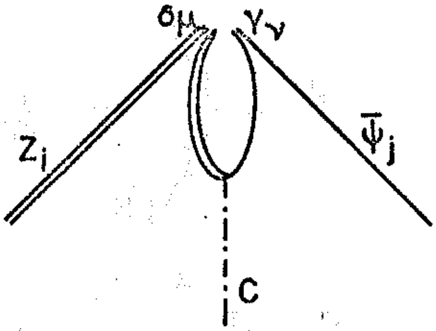


Fig. 7a

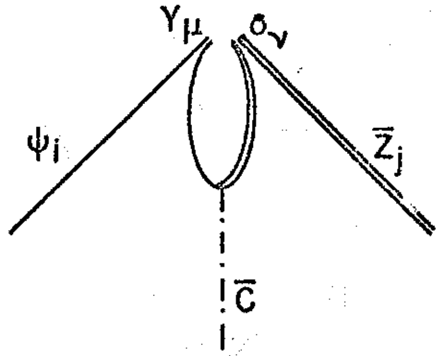


Fig. 7b

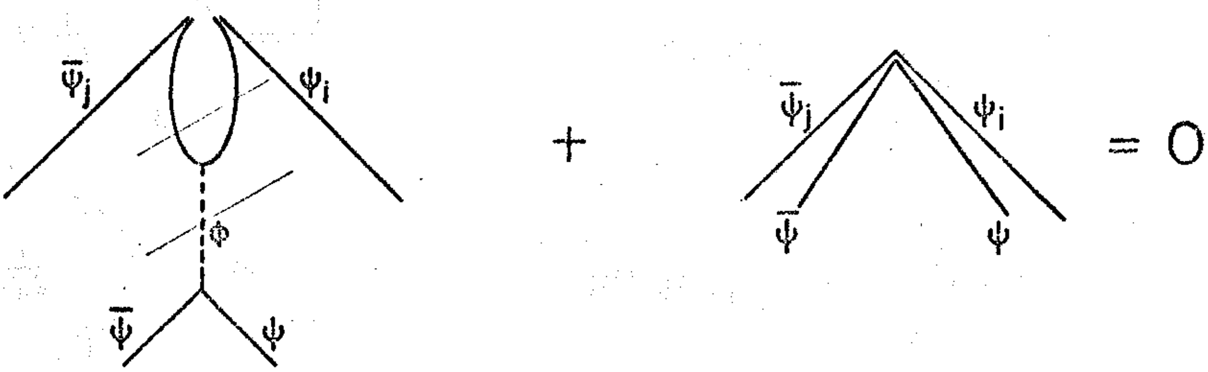
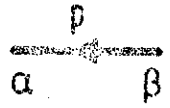
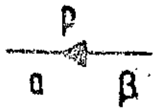


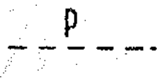
Fig. 8



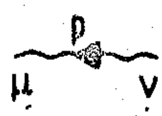
$= \delta_{\alpha\beta}\, D^{z\bar{z}}(p)$



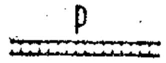
$= \delta_{\alpha\beta} \quad D^{\psi\bar{\psi}}(p)$



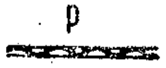
$= D^{\alpha}(p)$



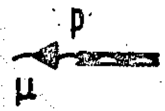
$= \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D^{\lambda}(p)$



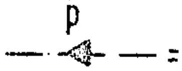
$= \quad D^{\varphi}(p)$



$= D \quad (p)$

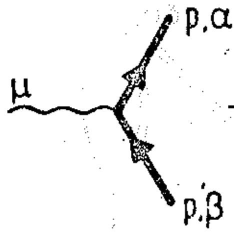


$= \epsilon_{\mu\nu}\, p_\nu\, D^{\lambda\varphi}(p)$

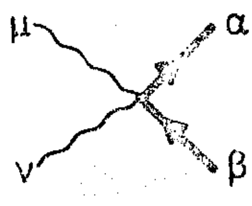


$= \quad D^{c\bar{c}}(p)$

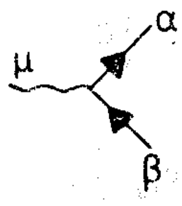
$= \frac{1}{\sqrt{n}}\, \delta_{\alpha\beta}$



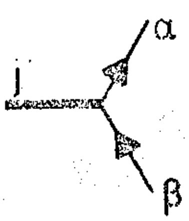
$= -\frac{1}{\sqrt{n}} (p_\mu + p'_\mu)\, \delta_{\alpha}$



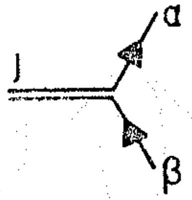
$= -\frac{1}{n}\, \delta_{\mu\nu}\, \delta_{\alpha\beta}$



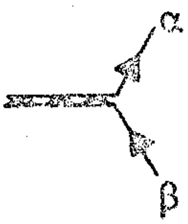
$= -\frac{ie}{\sqrt{n}}\, \gamma_\mu\, \delta_{\alpha\beta}$



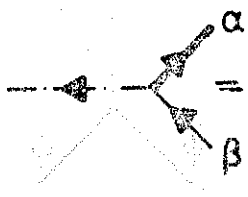
$= \frac{1}{\sqrt{n}}\, \delta_{\alpha\beta}$



$\frac{1}{\sqrt{n}}\, \gamma_5\, \delta_{\alpha\beta}$



$= \frac{1}{\sqrt{n\,\bar{k}}}\, \gamma_5\, \delta_{\alpha\beta}$



$= \frac{1}{\sqrt{n}}\, \delta_{\alpha\beta}$