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EXCHANGE-CURRENT CONTRIBUTION TO THE PION-DEUTERON SCATTERING LENGTH

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Chiral symmetry, implemented by means of effective Lagrangians, is used to evaluate the exchange-current contribution to the pion-deuteron scattering length. It is shown that this approximate symmetry is responsible for partial cancellations, yielding an overall contribution of about 10% of the total scattering length.

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Field theory may prove to be an important tool in the study of pion-deuteron scattering, in particular as far as contributions of pion-production amplitudes are concerned. The main problem in such an approach is that it is not possible to use ordinary perturbation theory on calculations of strong processes. However, the interactions of low-energy pions can be well described by means of effective Lagrangians based on chiral symmetry.[1]

Chiral symmetry predicts that in the unphysical limit in which the pion is soft, i.e. its four-momentum vanishes, the outcome of a scattering process depends only on the isospin of the target[2]. In particular, when the target is a deuteron, the exact chiral symmetric limit corresponds to a vanishing amplitude.

The possibility of studying π-d interactions using chiral symmetry is an attractive one, for in it all the nice features of a covariant field theory are present. Moreover, the nature of this approach is such that amplitudes in agreement with low-energy theorems are often the result of large cancellations between amplitudes that in isolation do not exhibit such an agreement. One such cancellation has been found in the evaluation of exchange-current contributions to the π-d scattering length, the destructive interference occurring between the $\pi N \to \pi\pi N$ and $\pi\pi \to \pi\pi$ amplitudes[3]. The possibility of this kind of cancellations could not be easily grasped without the use of chiral symmetry.

In the present work the exchange-current contribution to the process $\pi NN \to \pi NN$, for pions at rest, is calculated under the assumption that the nucleon-nucleon interaction is due to exchange of pions. This amplitude is evaluated by means of effective Lagrangians which are approximately chiral invariant and is denoted by $T_e(q)$, where q is the momentum of the exchanged pion. One subsequently uses $T_e(q)$ between deuteron wave-functions to obtain a_e, the contribution of the exchange-current to $a_{\pi d}$. Thus

$$a_e = \frac{m_\pi}{(1+m_\pi/M_d)(2\pi)^7}\int d^3Q\, d^3q\; \Psi^*(Q-q/2)\, T_e(q)\, \Psi(Q+q/2) \qquad (1)$$

where m_π, M_d and Ψ are respectively the pion mass, the deuteron mass and the deuteron wave-function.

The amplitude $T_e(q)$ is assumed to be dominated by the processes in Fig.1. The vertices for these processes are derived from the following effective Lagrangians, describing the interactions of pions with pions[4], nucleons[4], nucleons and deltas[5], deltas[6].

$$\mathcal{L}_{\pi\pi} = \frac{1}{4f_\pi^2}\left\{\frac{1}{2}(1-\xi)(\partial_\mu\Phi^2)^2 - \xi\Phi^2(\partial_\mu\Phi)^2 + \left(\frac{3}{2}\xi-1\right)\frac{1}{2}m_\pi^2\Phi^4\right\} \qquad (2)$$

$$\mathcal{L}_{\pi NN} = -\frac{1}{4f_\pi^2}(\bar{N}\gamma^\mu\boldsymbol{\tau}N)(\Phi\times\partial_\mu\Phi) + g_{\pi NN}(\bar{N}\gamma^\mu\gamma_5\boldsymbol{\tau}N)\left[\left(1-\xi\frac{\Phi^2}{4f_\pi^2}\right)\partial_\mu\Phi - \frac{1}{4f_\pi^2}(\xi-1)\Phi\,\partial_\mu\Phi^2\right] \qquad (3)$$

$$\mathcal{L}_{\pi N\Delta} = i\bar{g}_{\pi N\Delta}(\bar{\Delta}^\mu\mathbf{M}\gamma^\mu\gamma_5 N)(\partial_\mu\Phi\times\partial_\nu\Phi) + g_{\pi N\Delta}(\bar{\Delta}^\mu\mathbf{M}N)\partial_\mu\Phi + \text{h.c.} \qquad (4)$$

$$\mathcal{L}_{\pi\Delta\Delta} = g_{\pi\Delta\Delta}\left[\bar{\Delta}^\mu(\gamma_\lambda g_{\mu\nu} - g_{\mu\lambda}\gamma_\nu - \gamma_\mu g_{\lambda\nu} + \gamma_\nu\gamma_\lambda\gamma_\mu)\gamma_5\mathbf{T}\Delta^\nu\right]\partial^\lambda\Phi \qquad (5)$$

The symbols Φ, N and Δ denote respectively the pion, nucleon and delta fields. $\boldsymbol{\tau}$, $\mathbf{M}$ and $\mathbf{T}$ are matrices that combine two nucleons, one nucleon and one delta and two deltas into isospin 1 states. The parameter ξ is determined by the group

transformation properties of the chiral symmetry breaking term in the Lagrangian; for instance, it assumes the values 0 or -2 when this term transforms according to the (1/2,1/2) or (1,1) representations of SU(2)xSU(2)[4,7]. The constant f_π describes the pion decay and the value adopted for it is 85 MeV[8]. The axial πNN coupling constant is $g_{\pi NN}$ = 0.996 m_π^{-1}[5]. The vector $\pi N\Delta$ coupling constant $\bar{g}_{\pi N\Delta}$ is related to the $\gamma N\Delta$ coupling constant by $\bar{g}_{\pi N\Delta} = g_{\gamma N\Delta}/4f_\pi^2$[9]; the value of $g_{\gamma N\Delta}$ is taken to be 0.30 m_π^{-1}[10]. The axial $\pi N\Delta$ coupling constant is $g_{\pi N\Delta}$ = 1.84 m_π^{-1}[5]. Finally, $g_{\pi\Delta\Delta}$ can be evaluated using symmetry SU(4) to be $g_{\pi\Delta\Delta}$ = 1.2 $g_{\pi NN}$[11].

One represents by $T_n(q)$ the contribution to $T_e(q)$ of the process n in Fig.1, when the nucleons are free, non-relativistic and have zero total isospin. Thus one writes

$$T_e(q) = \sum_{n=1}^{10} T_n(q) = \frac{1}{2m_\pi}\,\frac{\sigma^{(1)}\cdot q\;\; \sigma^{(2)}\cdot q}{(q^2+m_\pi^2)}\sum_{n=1}^{10} t_n(q) \qquad (6)$$

Here $\sigma^{(i)}$ is the expectation value of the spin operator in the fermion line i. The use of eqs(2-5) and Faynman rules yield the following values for $t_n(q)$

$$t_1 = \frac{g_{\pi NN}^2}{f_\pi^2}\left[m_\pi^2(1+5/2\,\xi)/(q^2+m_\pi^2) + (2-5\xi)\right] \qquad (7)$$

$$t_2 = -\frac{g_{\pi NN}^2}{f_\pi^2}(2-5\xi) \qquad (8)$$

$$t_3 + t_4 = -\frac{g_{\pi NN}^2}{f_\pi^2}(2m_\pi^2/m_N^2) \qquad (9)$$

$$t_5 = -g_{\pi NN}^4(3m_\pi^2/2m_N^2) \qquad (10)$$

$$t_6 + t_7 = g_{\pi NN}^2\, g_{\pi N\Delta}^2\,(16\, m_\pi^2/9\, m_N m_\Delta) \qquad (11)$$

$$t_8 + t_9 = - g_{\pi NN}\, g_{\pi N\Delta}\, \bar{g}_{\pi N\Delta} \left[16\, m_\pi^2 (2 m_N + m_\Delta)/9 M_\Delta^2 \right] \tag{12}$$

$$t_{10} = - g_{\pi NN}\, g^2_{\pi N\Delta}\, g_{\pi\Delta\Delta} \left[40\, m_\pi^2 (4 m_N + 3 m_\Delta)/27\, m_\Delta^3 \right] \tag{13}$$

One notes that all but the first two amplitudes are proportional to m_π^2 . However, a partial cancellation occurs between the amplitudes t_1 and t_2 , their sum being proportional to m_π^2 . This is due to the fact that $T_e(q)$ vanishes in the limit of exact SU(2)xSU(2) and this limit is formally achieved by letting $m_\pi \to 0$.

When the Humberston and Wallace wave-function[12], containing a d-state probability of 6.953%, is used in eq.(1) one has

$$a_e = -\left(0.00018\, \frac{g^2_{\pi NN}}{f_\pi^2}\, (1+5/2\,\xi) + 0.00527 \sum_{n=3}^{10} t_n\right) m_\pi^3 \tag{14}$$

This result is strongly dependent upon the d-wave content of the deuteron wave-function, for only about 10% of its value come from the diagonal s-s term in eq.(1). The inclusion of πNN form factors can also produce significant changes in these results, reducing them up to 50%, as shown in Ref.(13).

When the numerical values of the masses and coupling constants are used in the above equation one obtains

$$a_e = (0.0035 - 0.0012\,\xi)\, m_\pi^{-1} \tag{15}$$

The single largest contribution to this value comes from diagram 10 and corresponds to $a_{10} = 0.0025\, m_\pi^{-1}$. The importance of this diagram may also prove to be considerable to dynamical calculations of π-d scattering away from threshold, due to the presence of two delta propagators.The coupling constant $g_{\pi\Delta\Delta}$ is a source of uncertainty in the calculation, for its SU(4) value can be as much as 30% inaccurate[14].

The result of eq.(15) is of the same order of magnitude and sign as the contributions of processes other than single and double scatterings to $a_{\pi d}$, as calculated by Myhrer[15].

One notes that the value of the parameter ξ plays an important role in eq.(15). For $\xi = -2$ the exchange-current contribution to $a_{\pi d}$ is about 10% of the value measured in pionic atoms by Bailey et.al.[16] and therefore can correspond to observable effects if the precision of measurements is increased.

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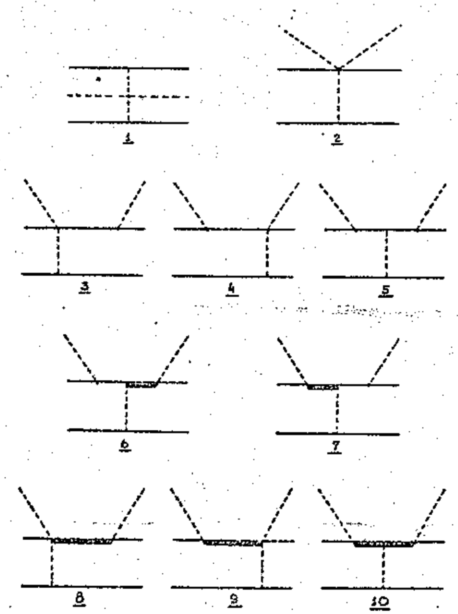

Figure Caption

Fig.1. Exchange-current contributions to $\pi NN \to \pi NN$. Pions are represented by broken lines, nucleons by full ones and deltas by thick ones.