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ABSTRACT

The exact S-matrix of a two-dimensional supersymmetric Z(N) model is proposed.

The analitic S-matrix program has in the last couple of years been applied with great sucess to two-dimensional problems and there exists by now an impressive list of models, whose exact S-matrix is known(1). In this paper we want to add one more exact solution: the S-matrix of a model which we call supersymmetric Z(N) model. The reason for this name comes from the fact that in this model there is no reflection, antiparticles are bound states of (N-1) particles and its relation to the usual Z(N) model(2) is the same as the relation between the supersymmetric σ-model and the usual σ-model(3). Namely the supersymmetric boson fermion S-matrix is the product of two factors, one being the S-matrix of the usual Z(N) model, whereas the other one comes from the supersymmetrization.

The model contains asymptotic states consisting of a degenerate multiplet containing a fermion f, a boson b and their antiparticles $\bar{f}$ and $\bar{b}$. Their scattering amplitudes for vanishing reflection are

$$S|b_1 b_2\rangle = A(\phi_{12})\ |b_2 b_1\rangle \tag{1a}$$

$$S|f_1 f_2\rangle = \hat{B}(\phi_{12})\ |f_2 f_1\rangle \tag{1b}$$

$$S|b_1 f_2\rangle = C(\phi_{12})\ |f_2 b_1\rangle + D(\phi_{12})\ |b_2 f_1\rangle \tag{1c}$$

$$S|b_1 \bar{b}_2\rangle = A(1-\phi_{12})\ |\bar{b}_2 b_1\rangle + D(1-\phi_{12})\ |\bar{f}_2 f_1\rangle \tag{1d}$$

$$S|f_1 \bar{f}_2\rangle = \hat{B}(1-\phi_{12})\ |\bar{f}_2 f_1\rangle + D(1-\phi_{12})\ |\bar{b}_2 b_1\rangle \tag{1e}$$

$$S|b_1 \bar{f}_2\rangle = C(1-\phi_{12})\ |\bar{f}_2 b_1\rangle \tag{1f}$$

where $\phi_{12} = \phi_1 - \phi_2$ and ϕ_i are the rapidity variables $P_i^0 = m\,\mathrm{ch}(i\pi\phi)$, $P_i^1 = m\,\mathrm{sh}(i\pi\phi)$ and where we have already used crossing symmetry to write the last three amplitudes. The unitarity equations are

$$A(\phi)\ A(-\phi) = 1 \tag{2a}$$

$$\hat{B}(\phi)\ \hat{B}(-\phi) = 1 \tag{2b}$$

$$C(1-\phi)\ C(1+\phi) = 1 \tag{2c}$$

$$C(\phi)\ C(-\phi) + D(\phi)\ D(-\phi) = 1 \tag{2d}$$

$$C(\phi)\ D(-\phi) + D(\phi)\ C(-\phi) = 0 \tag{2e}$$

Together with the factorization equations, which may be obtained in the usual way, they have the following minimal (without poles) solution

$$A_{min}(\phi) = \frac{\sin[\mu(\phi+\lambda)]}{\sin(\mu\phi)}\ C_{min}(\phi) \tag{3a}$$

$$B_{min}(\phi) = -\frac{\sin[\mu(\phi-\lambda)]}{\sin(\mu\phi)}\ C_{min}(\phi) \tag{3b}$$

$$D_{min}(\phi) = \frac{\sin[(\mu\phi)]}{\sin[(\mu\phi)]}\ C_{min}(\phi) \tag{3c}$$

$$C_{min}(\phi) = \hat{Y}_o(\phi) = \prod_{\ell=0}^{\infty} \frac{\Gamma(\frac{\phi}{2}+\frac{\lambda}{2}+\ell)\Gamma(1-\frac{\phi}{2}+\ell)}{\Gamma(1-\frac{\phi}{2}+\frac{\lambda}{2}+\ell)\ \Gamma(\frac{\phi}{2}+\ell)} \prod_{\ell=1}^{\infty} \frac{\Gamma(\frac{\phi}{2}-\frac{\lambda}{2}+\ell)\ \Gamma(1-\frac{\phi}{2}+\ell)}{\Gamma(\frac{1}{2}-\frac{\phi}{2}-\frac{\lambda}{2}+\ell)\ \Gamma(\frac{\phi}{2}+\ell)} \tag{3d}$$

Replacing $C_{min}(\phi)$ by

$$C(\phi) = \frac{\sin\frac{\pi}{2}(\phi+\lambda)}{\sin\frac{\pi}{2}(\phi-\lambda)}\ C_{min}(\phi) \tag{4}$$

with $\mu = \frac{\pi}{2}$ we introduce a pole at $\phi = \lambda = \frac{2}{N}$ in the boson-boson scattering amplitude $A(\phi)$ (and consequently into the combination $C(\phi) + D(\phi)$) in analogy with the usual $Z(N)$ model[2].

The mass spectrum is the same as in the usual $Z(N)$ model. In particular, there is a $(N-1)$ particle bound state at the same mass as the fundamental particles.

One of the consequences of a $Z(N)$ symmetry for particles without internal degrees of freedom is that antiparticles are bound states of $(N-1)$ particles. We now show that our amplitudes support the following identification

$$\bar{f} = \prod_{i=1}^{N-1} b_i \tag{5a}$$

$$\begin{aligned}\bar{b} &= f_1 b_2 b_3 \ldots b_{N-1} + b_1 f_2 b_3 \ldots b_{N-1} + \ldots + b_1 b_2 b_3 \ldots b_{N-2} f_N \\ &\equiv \sum_{\alpha=1}^{N-1} |\alpha\rangle \quad .\end{aligned} \tag{5b}$$

This means that the antifermion is a bound state of (N-1) bosons and the antiboson is a bound state of (N-2) bosons and one fermion.

In terms of our S-matrix amplitudes equs. (5) mean that, if in the N particle scattering amplitude we project (N-1) particles onto the pole of mass m, we have to reproduce the corresponding particle-antiparticle amplitude. Using the notation $\bar{f}\left[b_1 b_2 \ldots b_{N-1}\right]$ to indicate that we have gone to the pole of the (N-1) particles inside the bracket to obtain an antifermion (and omitting the outside letter for bosons, since only the linear combination equ. (5b) is the antiboson), we obtain

$$S|\bar{f}[b_1 \ldots b_{N-1}] b_N\rangle = \prod_{i=1}^{N-1} A_{iN} |b_N \, \bar{f}[b_1 \ldots b_{N-1}]\rangle \tag{6a}$$

$$\begin{aligned}S|\bar{f}[b_1 \ldots b_{N-1}] f_N\rangle = &\prod_{i=1}^{N-1} C_{iN} |f_N \, \bar{f}[b_1 \ldots b_{N-1}]\rangle \; + \\ &+ \sum_{\alpha=1}^{N-1} u_\alpha |b_N \, [b_1 \ldots f_\alpha \ldots b_{N-1}]\rangle\end{aligned} \tag{6b}$$

$$\begin{aligned}S|\bar{b} \sum_{\alpha=1}^{N-1} (b_1 \ldots f_\alpha \ldots b_{N-1}) \, b_N\rangle = &\sum_{\alpha=1}^{N-1} v_\alpha |b_N \, [b_1 \ldots f_\alpha \ldots b_{N-1}]\rangle + \\ &+ \sum_{i=1}^{N-1} \prod_{j=1}^{N-1} P_{ij} |f_N \, \bar{f}[b_1 \ldots b_{N-1}]\rangle\end{aligned} \tag{6c}$$

$$S|\bar{b} \sum_{\alpha=1}^{N-1} (b_1 \ldots f_\alpha \ldots b_{N-1}) \; f_N\rangle = \sum_{\alpha=1}^{N-1} w_\alpha |f_N [b_1 \ldots f_\alpha \ldots b_{N-1}]\rangle \tag{6d}$$

where $u_\alpha = \prod_{j=1}^{N-1} M_{\alpha j}$, $v_\alpha = \sum_{i=1}^{N-1} \prod_{j=1}^{N-1} N_{ij}^{(\alpha)}$, $w_\alpha = \sum_{i=1}^{N-1} \prod_{j=1}^{N-1} R_{ij}^{(\alpha)}$

(7)

and the (N-1) x (N-1) matrices M, N and R are given by

$$M_{ij} = \left\{ C_{jN} \text{ for } i+j<N-2;\ D_{jN} \text{ for } i+j=N-1;\ A_{jN} \text{ for } i+j>N-2 \right\} \tag{8a}$$

$$N_{ij}^{(\alpha)} = \begin{cases} A_{jN} & \text{for } (i=1, j\neq\alpha),\ (i>1, j>\alpha) \\ C_{\alpha N} & \text{for } (i=1, j=\alpha) \\ D_{jN} & \text{for } (i=j,\ 2\leq j\leq\alpha),\ (i\neq 1, j=\alpha) \\ C_{jN} & \text{otherwise} \end{cases} \tag{8b}$$

$$P_{ij} = \{A_{jN} \text{ for } i-j>0;\ D_{jN} \text{ for } i=j;\ C_{jN} \text{ for } j-i>0\} \tag{8c}$$

and $R_{ij}^{(\alpha)}$ is a (N-α) x (N-1) matrix given by

$$R_{ij}^{(\alpha)} = \begin{cases} \hat{B}_{\alpha N} & \text{for } (i=1, j=\alpha) \\ D_{jN} & \text{for } (2\leq i\leq N-1, j=\alpha),\ (j>\alpha, j-i=1) \\ A_{jN} & \text{for } (j>\alpha, j-i>2) \\ C_{jN} & \text{otherwise} \end{cases} \tag{8d}$$

In the equs (6)-(8) the argument of the amplitudes with indices (jN) is

$$\phi_j = \phi + n_j \frac{\lambda}{2}, \quad n_j = \begin{cases} +(N-2), N-4, \ldots +1, -1, \ldots -(N-2) & \text{for N odd} \\ N-2, N-4, \ldots 2, 0, -2, \ldots -(N-2) & \text{for N even} \end{cases} \tag{9}$$

Using the identity

$$\prod_{j=1}^{N-1} Y_o'(\phi_j) = \hat{Y}_o(1-\phi) \tag{10}$$

where

$$Y_o'(\phi) = \frac{\sin\frac{\pi}{2}(\phi+\lambda)}{\sin\frac{\pi}{2}\phi}\ \hat{Y}_o(\phi) \tag{11}$$

one sees that no extra terms appear on the r.h.s. of equ. (6d) and that

$$\prod_{i=1}^{N-1} A_{iN}(\phi_i) = C(1-\phi) \quad . \tag{12a}$$

$$\prod_{i=1}^{N-1} C_{iN}(\phi_i) = \hat{B}(1-\phi) \tag{12b}$$

$$u_1 = u_2 = \ldots = u_{N-1} = D(1-\phi) \tag{12c}$$

$$v_1 = v_2 = \ldots = v_{N-1} = A(1-\phi) \tag{12d}$$

$$\sum_{i=1}^{N-1} \prod_{j=1}^{N-1} P_{ij} = D(1-\phi) \tag{12e}$$

$$w_1 = w_2 = \ldots = w_{N-1} = C(1-\phi) \quad . \tag{12f}$$

This proves our identification exhibited in equ. (5). This identification raises the same problem of intermediate statistics for b and f as in the chiral Gross-Neveu[4] model and since at the present time we are unable to handle field witn intermediate statistics directly, we adopt the same procedure as expounded in ref. 4.

Finally, to complete the picture one would like to construct explictly the super algebra of our model. We only note that for $N = 2$ our amplitudes equs.(3) do not support the super algebra proposed in ref. 5, since our amplitude $D(\phi)$ is non-vanishing.

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