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ON THE EXACT S-MATRIX FROM CP^{n-1} AND SU(n) CHIRAL THIRRING MODEL

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ON THE EXACT S-MATRIX FROM CP^{n-1} AND SU(n) CHIRAL THIRRING MODEL

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ABSTRACT

We calculate the S-matrix of CP^{n-1} and SU(n) Thirring model perturbatively up to 2 loops. The calculation shows striking similarities, but the S-matrix has some deviations from the expected exact one.

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INTRODUCTION

The SU(n) chiral Thirring model and CP $^{n-1}$ model have been recently extensively studied[1)2)] and many interesting results were discovered. For the first model, the class II S-matrix[3)] was suspect to represent the exact S-matrix of the model. Recently Lowenstein and Andrei[4)] succeded in diagonalizing the Hamiltonian The model is plagued with infrared divergences[5)], due to the spontaneous break of symmetry. However some authors managed in 1/n expanding the S-matrix in lowest non-trivial order, which was proved to be the class II one, with bound states modification. Recently one has proved that the infrared divergences came from a spin 1/n particle and the physical particles of the theory have spin $\frac{1}{2} - \frac{1}{2n}$ [6)]. Moreover, each particle is a bound state of the remaining anti-particles. The CP $^{n-1}$ model was proved to describe confined particles. Both models[7)8)] are assymptotically free, and exhibit, at classical level an infinite number of conserved currents. Then, one suspected that the models presented fatorization, and were a couple of models, analogous to non-linear σ and Gross-Neveu models. Although the CP^{n-1} model presents confinement, we suppose, due to asymptotic freedom, to be licit to speak of an S-matrix at very high energies, using perturbation theory. In a recent paper[9)] we proved the non-existence of pair production in this model. This procedure shows striking similarities, and some surprising departs from S-matrix, which we discuss at the end of this paper.

This paper is divided as follows: in section I we expand the proposed S-matrix describing its properties, in section II we present the SU(n) model in perturbation theory, in section III the CP^{n-1} model is discussed in the same framework and in section IV we discuss the results.

THE S-MATRIX

Two dimensional theories, without particle production are known to possess very simple S-matrices. Indeed, these theories are known to be of soliton type, that is, the scattering changes only a phase of the solution.

The S-matrix is entirely defined in terms of the $2\to 2$ S-matrix, if the matrix presents factorization, which under very general arguments[10)] is equivalent to no particle production. Because of energy-momentum conservation, in two dimensions, a $2\to 2$ scattering is such that the set of inicial momenta is equal to the set of outgoing momenta and the S-matrix is given by

$$ {}^{out}\langle P_\beta(\tilde p_1) . A_\delta(\tilde p_2) \mid P_\alpha(p_1)\, A_\gamma(p_2)\rangle^{in} = $$

$$ = {}_{\alpha\gamma}F_{\beta\delta}(\theta)\, \delta(\tilde p_1^1 - p_1^1)\, \delta(\tilde p_2^1 - p_2^1) - {}_{\alpha\gamma}B_{\delta\beta}(\theta)\, \delta(\tilde p_1^1 - p_2^1)\, \delta(p_2^1 - \tilde p_1^1) \qquad (1) $$

and

$$ {}^{out}\langle P_\beta(\tilde p_1)\, P_\delta(\tilde p_2) \mid P_\alpha(p_1)\, P_\gamma(p_2)\rangle^{in} = $$

$$ = {}_{\alpha\gamma}S_{\beta\delta}(\theta)\, \delta(\tilde p_1^1 - p_1^1)\, \delta(\tilde p_2^1 - p_2^1) - {}_{\alpha\gamma}S_{\delta\beta}(\theta)\, \delta(\tilde p_1^1 - p_2^1)\, \delta(\tilde p_2^1 - p_1^1) \qquad (2) $$

where P_α is a particle and A_α an antiparticle belonging to the

fundamental symmetry group of the theory and $p^0_\alpha = m\,\mathrm{ch}\,\theta_\alpha$, $p^1_\alpha = m\,\mathrm{sh}\,\theta_\alpha$.
If the group is U(n), Berg et all[11] showed that

$$ {}_{\alpha\gamma}F_{\beta\delta}(\theta) = t_1(\theta)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} + t_2(\theta)\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} \tag{3} $$

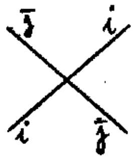
$$ {}_{\alpha\gamma}B_{\delta\beta}(\theta) = r_1(\theta)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} + r_2(\theta)\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} \tag{4} $$

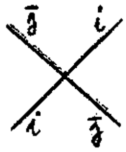
$$ {}_{\alpha\gamma}S_{\beta\delta}(\theta) = u_1(\theta)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} + u_2(\theta)\,\delta_{\alpha\delta}\,\delta_{\gamma\beta} \tag{5} $$

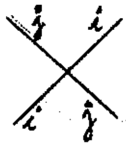
and using general arguments of field theory they found that there were 5 classes of minimal S-matrix satisfying all requirements, its meant the one with the minimum number of singularities and zeros on the physical sheet. These S-matrices depend on the function $S(i\pi\varphi, \lambda)$ as follows.

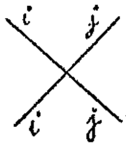
$$ f(i\pi\varphi, \lambda) = \frac{\Gamma(\frac{1}{2}+\frac{1}{2}\varphi)\,\Gamma(\frac{1}{2}+\frac{1}{2}\lambda-\frac{1}{2}\varphi)}{\Gamma(\frac{1}{2}-\frac{1}{2}\varphi)\,\Gamma(\frac{1}{2}+\frac{1}{2}\lambda+\frac{1}{2}\varphi)} \tag{6} $$

We will be interested in a class where the amplitudes $r_1(\theta)$ and $r_2(\theta)$ vanish. The only one is the class II, given by:

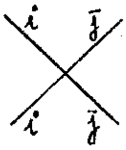
$t_1(\theta) =$  $= f(\theta, \lambda)$ (7a)

$t_2(\theta) =$  $= -\lambda \dfrac{i\pi}{i\pi-\theta}\, t_1(\theta)$ (7b)

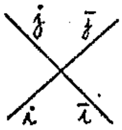
$u_1(\theta) =$  $= t_1(i\pi-\theta)$ (7c)

$$u_2(\theta) =$$ 

$$= -\lambda \frac{i\pi}{\theta} u_1(\theta) \tag{7d}$$

$$r_1(\theta) =$$ 

$$= 0 \tag{7e}$$

$$r_2(\theta) =$$ 

$$= 0 \tag{7f}$$

We can obtain the high energy expansion of the S-matrix as a series in $1/\theta$, beginning on the assymptotic expansion of the Γ function. We have:

$$f(\theta, \tfrac{i}{n}) = \frac{\Gamma\left(\frac{1}{2} + \frac{\theta}{2\pi i}\right) \Gamma\left(\frac{1}{2} + \frac{1}{n} - \frac{\theta}{2\pi i}\right)}{\Gamma\left(\frac{1}{2} - \frac{\theta}{2\pi i}\right) \Gamma\left(\frac{1}{2} + \frac{1}{n} + \frac{\theta}{2\pi i}\right)} = X e^{Y} \tag{8}$$

$$X = 1 + \mathcal{O}\left(\frac{1}{\theta^3}\right) \tag{8a}$$

$$Y = \frac{i\pi}{n} - \frac{2\pi i}{\theta n^2} + \mathcal{O}\left(\frac{1}{\theta^3}\right) \tag{8b}$$

we get then:

$$f = e^{\frac{i\pi}{n}} \left[1 + \frac{ig^2}{\pi} \ln \frac{s}{\mu^2} - \frac{ig^3 n}{2\pi^2} \left(\ln \frac{s}{\mu^2} \right)^2 \right] \tag{9}$$

where we used

$$\frac{1}{\Theta} = \frac{ng}{2\pi}\left[1 - \frac{ng}{2\pi}\ln\frac{s}{\mu^2} + \left(\frac{ng}{2\pi}\ln\frac{s}{\mu^2}\right)^2\right] \tag{10}$$

THE SU(n) MODEL IN PERTURBATION THEORY

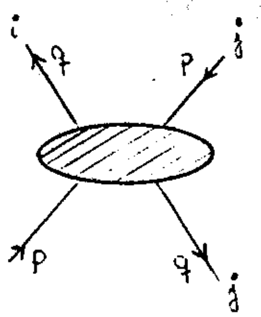
The model is defined by the Lagrangian density

$$\mathcal{L}_0 = \bar{\Psi} i \partial\!\!\!/ \Psi + \frac{g^2}{2}\left[\left(\bar{\Psi}\Psi\right)^2 - \left(\bar{\Psi}\gamma_5\Psi\right)^2\right] \tag{11}$$

The model is assymptotically free, and presents an infinite number of conservation laws. The model presents also a chiral symmetry, which is spontaneously broken, bringing about terrible infrared problems, because this is a two dimensional theory[5).

We circunvent this problem by giving the Ψ small mass μ^2. We expect the theory to yield mass transmutation, so that the theory should depend on an appropriate combination of μ^2 and g. In all the cases infrared divergences do not appear, we put $\mu^2=0$.

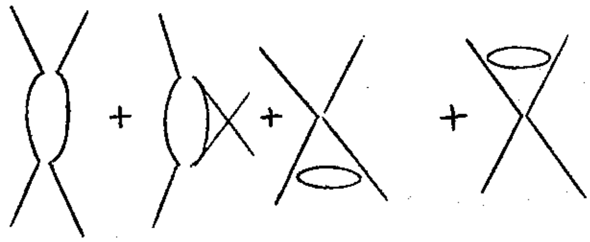
The graphical structure, up to 3^{rd} order perturbation theory is given by fig1 (1-47). The amplitude

$r_1(\theta) =$ 

was calculated, and we had as a result $r_1(\theta) = 0$

Analogously $r_2(\theta) = 0$

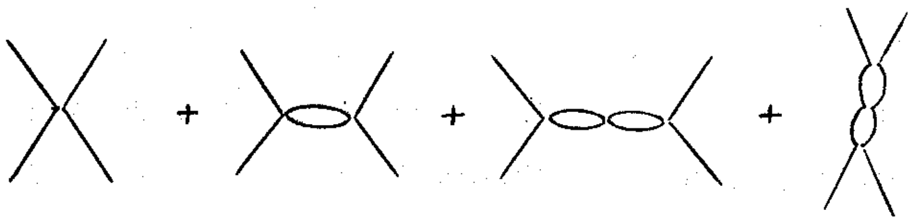
These results by them own show that the only candidate should be the class II of ref. (11), and they came about because of the chiral interaction, as a cancelation graf by graph of the $(\bar{\Psi}\gamma_5\Psi)^2$ and $(\bar{\Psi}\Psi)^2$ pieces, so that each graph contributing to these amplitudes is equal to zero. For the $u_1(\theta)$ we have similar cancelation in the majority of the graphs, and the only surviving ones are:

$u_1(\theta) = 1 +$ 

The result is given by:

$$u_1(\theta) = 1 + \frac{ig^2}{\pi}\ln\frac{s}{\mu^2} - \frac{ig^3 n}{2\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2 \qquad (12)$$

For $u_2(\theta)$ the non-vanishing contributions come from the graphs.

$u_2(\theta) =$ 

And $u_2(\theta)$ is given by:

$$u_2(\theta) = ig\left(1 - \frac{ng}{2\pi}\ln\frac{-s}{\mu^2} + \left[\frac{ng}{2\pi}\ln\frac{-s}{\mu^2}\right]^2 + \frac{g^2}{4\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2\right) \qquad (13)$$

It is interesting to note that the first 3 graphs contributing to this amplitude are the ones expect from the 1/n expansion of the theory whereas the last one would be only expected to 3rd order.

THE CP^{n-1} MODEL IN PERTURBATION THEORY

The Lagrangian is given by

$$\mathcal{L} = \overline{D_\mu z_i}\; D^\mu z_i \tag{14}$$

$$D_\mu = \partial_\mu + ig A_\mu \tag{14a}$$

$$A_\mu = \frac{i}{2}\left(\bar{z}\, \partial_\mu z - z\, \partial_\mu \bar{z} \right) \tag{14b}$$

with the constraint $\bar{z}_i z_i = 1$

This model is also assimptotically free, and presents an infinite number of conservations lows. It was shown, by 1/n expanding the model, that it describes partons, confined by a topological Coulomb force. Because of assymptotic freedom, we expect however that the perturbation theory S-matrix has any meaning. We will not pursue this point any more, but suppose the validity of the argument.

Again, in order to circunvent the infrared problems associated with the Goldstone boson, we give the Z-particle a small mass μ^2, and the theory is supposed to yield mass transmutation, analogous to the one discussed in the SU(n) model.

The graphs, up to 3^{rd} order are in fig.1 (1-55). First of all we treat $r_1(\theta)$ and $r_2(\theta)$ In this case all the graphs are equal to zero, and we have

$$r_1(\theta) = r_2(\theta) = 0 \tag{15}$$

For $u_1(\theta)$ we have many contributions, but many cancelations among diferents graphs. The nett result is

$$u_1(\theta) = 1 + ig - \frac{ig^2}{2\pi} \ln\frac{s}{\mu^2} - \frac{ig^3 n}{2\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2 - \frac{5ig^3}{4\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2 \qquad (16)$$

For $u_2(\theta)$ the result is

$$u_2(\theta) = ig\left(1 + \frac{ng}{2\pi}\ln\frac{s}{\mu^2} + \left(\frac{ng}{2\pi}\ln\frac{s}{\mu^2}\right)^2 - \frac{3ng^2}{4\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2 - \frac{3g^2}{4\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2\right) \qquad (17)$$

CONCLUSION

For both models, we verified that the amplitudes $r_1(\theta)$ and $r_2(\theta)$ vanish identically. This means that a possible S-matrix must be the one of class II SU(n) symmetric S-matrix. This is the most stribing similarity between the two models.

However, this similarity fails, as we go closer to the other amplitudes. For the amplitude $\overset{SU(n)}{u_1}(\theta)$, we have:

$$\overset{SU(n)}{u_1}(\theta) = 1 + \frac{ig^2}{\pi}\ln\frac{s}{\mu^2} - \frac{ig^3 n}{2\pi^2}\left(\ln\frac{s}{\mu^2}\right)^2 \qquad (18)$$

Which is the expansion of the exact S-matrix amplitude, if we let the factor $e^{i\pi/n}$ aside. But this factor should come from the strange spin character of the theory: the physical fermions must carry spin 1/2 - 1/2n. This anomalous spin must also explain the terms of the S-matrix expansion, which are proporcional to the powers of 1/n, not expected even at lower energies.

At high energies they do not contribute. The exact S-matrix fails for the amplitude $u_2(\theta)$ which should be

$$u_2(\theta) = \frac{2i\pi}{n\theta}\, u_1(\theta) \tag{19}$$

Now concerning the $u_3(\theta)$ amplitude for CP^{n-1} model we got (16) which fails to reproduce the expected result already at first order. However, as we expect for the CP^{n-1} model a class II S-matrix, the same factor $e^{i\pi/n}$ should appear in the $u_3(\theta)$ amplitude, and we could speculate about the spin character of the z-particles.

We did not calculate, for the CP^{n-1} case the scattering of bound states. However we hope, in the future to have some more ideas about this problem, and about the instanton background effect.

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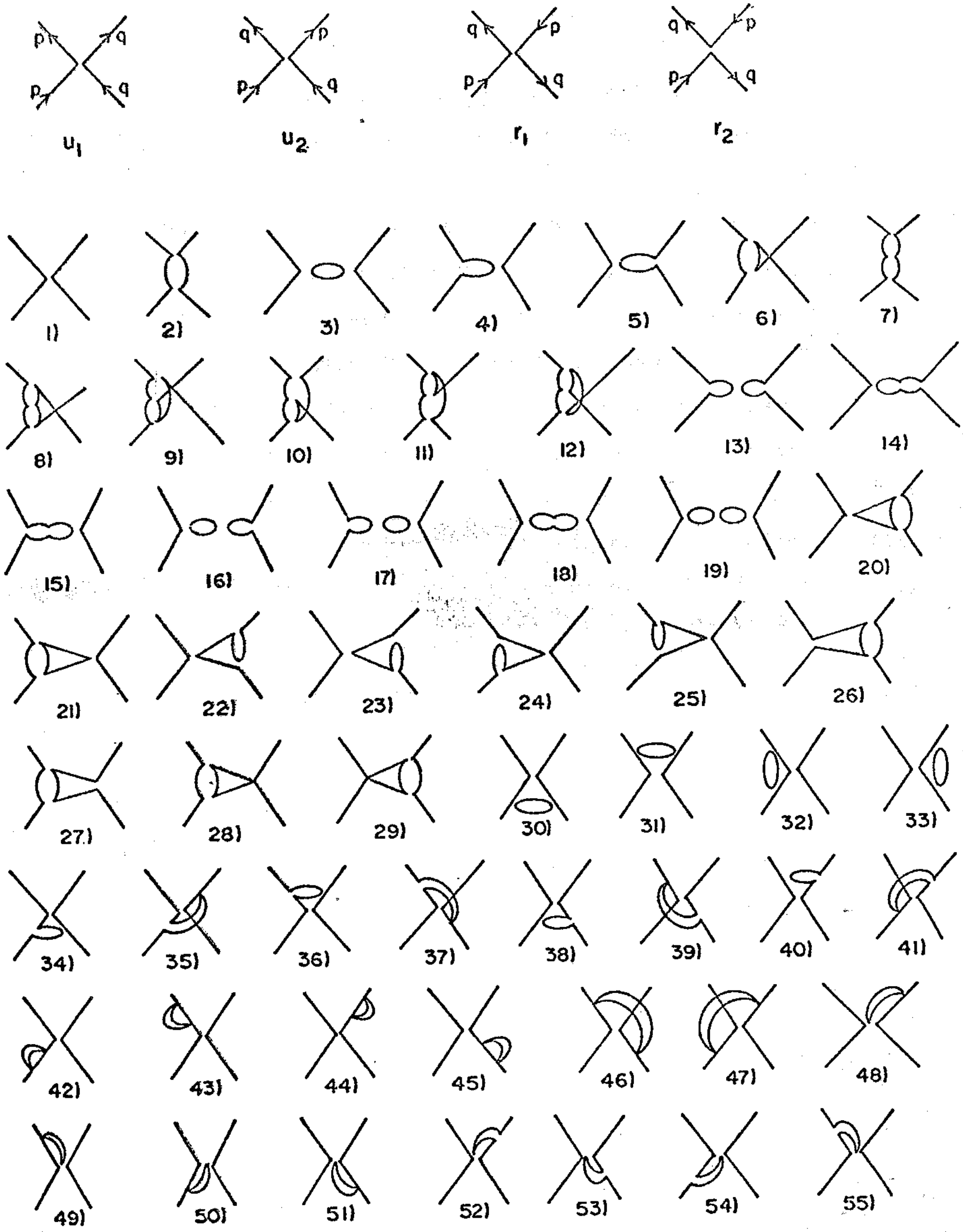
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