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ABSTRACT

It is suggested on the basis of transition strength and excitation energy arguments, that the resonance at 18.5 MeV observed in the $^{90}Zr(^{3}He, t)^{90}Nb$ - experiment corres - ponds to a T = 4 isovector dipole state.

Among different isovector mode the excitation caracterized by the quantum numbers $\tau = 1$ and $\mu_\tau = -1,0$ and 1, only the giant dipole resonance ($\ell = 1$, $\sigma = 0$, $\tau = 1$, $\mu_\tau = 0$) is well established so far.

Recently, a giant Gamow-Teller resonance ($\ell=0$, $\sigma = 1$, $\tau = 1$, $\mu_\tau = -1$) has been observed in the $^{90}Zr(p,n)^{90}Nb$ reaction at incident proton energies of 35 and 45 MeV by Doering et al[1]. The resonance is centered around an excitation energy of 8.5 MeV in $^{90}Nb$ and has a full width of 4.2 MeV. Subsequent $^{90}Zr(^{3}He, t)^{90}Nb$ experiments at 130 MeV in Jülich[2,3] and to 80 MeV in Grenoble[4] have confirmed this result although in the latter experiment the resonance peak appears to be split into two components one of which at 7.2 MeV is of Gamow-Teller type and the other at 9.7 MeV of unknown multipolarity. Particularly interesting is the ($^{3}He$, t) experiment of Galonsky and the Jülich group[2,3] since they observed for the first time another broad bump at 18.5 MeV excitation energy. The new bump is as strongly populated as the one at 8.5 MeV. Since no angular distributions could be measured for this peak, questions were raised about its nature. Galonsky et al[2,3] discussed two possibilities, namely that it could be either the dipole non-spin-flip resonance (which they ruled out because of energy considerations), or the $|1^{+}, T = 5 >$ analog state of the giant magnetic dipole resonance (GMD) in $^{90}Zr$[5].*

* Note, however, that only 15% of possible M1-strength of the GMD has yet been found in the 8 to 9 MeV energy region in high resolution inelastic electron scattering experiments[6].

The interpretation of the two bumps at 8.4 MeV and 18.5 MeV as being the $|1^+, T = 4>$ antianalog and $|1^+, T = 5>$ analog states of the GMD in $^{90}Zr$ is, at first sight, quite attractive. There are, however, two problems with this interpretation. One is the rather large energy spliting of $\sim$ 11 MeV and the other the observed equal population of the 8.4 and 18.5 MeV peaks un-expected from a simple closed shell model for $^{90}Zr$[1]. The fact that both states are excited with comparable strength was attibuted by Galonsky et al[2,3] to the large amplitude of the $|(\pi g_{9/2})^2 0^+>$ configuration present in the ground state of $^{90}Zr$[7].

The purpose of this letter is twofold. Firstly, we show, on the basis of transition strength and energy splitting arguments, that the resonance observed at 18.5 MeV[2,3], can not be of Gamow-Teller nature, even when a large amplitude of the $|(\pi g_{9/2})^2 0^+>$ configuration is present in the ground state of $^{90}Zr$. Secondly, we suggest, from an estimate for the excitation energy that the above mentioned resonance is most likely an isovector dipole state.

Our first argument is connected with the relative transition strengths between the $|1^+, T = 4>$ antianalog and the $|1^+, T = 5>$ analog states of the GMD. It has been argued[2,3] that the large admixture of the $|(\pi g_{9/2})^2 0^+>$ configuration in the ground state wave function of $^{90}Zr$

$$|0^+> = a|(\pi p_{1/2})^2 0^+> + b|(\pi g_{9/2})^2 0^+> \qquad (1)$$

could result in an increase of transition strength to the $|1^+, T = 5>$ state relative to that of the $|1^+, T = 4>$ state. Here we investigate this point by constructing $|1^+>$ basis wave functions with good isospin for $^{90}$Zr and $^{90}$Nb which include the particular configurations under discussion. The basic idea in the construction of the basis vectors is to start from the $(\pi g_{9/2}\ \nu g_{7/2})\ 1^+$ configuration in the nucleus $^{90}_{51}Y_{39}$ shown in fig. 1a. From this $|1^+, T = 6, M_T = 6>$ state we obtain the analogs in $^{90}$Zr and $^{90}$Nb by successive application of $T_-$ lowering operator[8]. From the analog states we construct the antianalogs and then the additional basis vectors by the orthogonality requirement. In figs. 1b and c we show the configurations involved in the wave functions with good isospin for $^{90}$Zr and $^{90}$Nb, respectively. In table 1 we give the $|1^+>$ wave functions with good isospin for $^{90}$Zr. Beside the $|1^+, T = 5>_4 = |(\nu g_{7/2}\ \nu g^{-1}_{9/2})1^+, T = 5>$ state present also in the closed shell model we have now two other $|1^+, T = 5>$ states with one of them $|1^+, T = 5>_2$, having the $(\pi g_{7/2}\ \pi g_{9/2})1^+$ configuration as the main component.

Note that there is also a $|1^+, T = 6>$ wave function in $^{90}$Zr. In table 2 we list the analog and antianalog states of the $|1^+>$ states in $^{90}$Zr and also the additional basic $|1^+>$ states in $^{90}$Nb. It can be noticed that the $|1^+, T = 5>_2$ state has a large component of $|(\pi g_{7/2}\ \pi g_{9/2})1^+, (\pi g_{9/2}\ \nu g^{-1}_{9/2})0^+>$ which was expected by Galonsky et al[2,3] to enhance the transition strength to the $|1^+, T = 5>$ state. If one, however, calculates the transition matrix elements, one has to recouple, i.e., only the component $|(\pi g_{7/2}\ \nu g_{9/2}^{-1})1^+, (\pi g_{9/2}\ \pi g_{9/2})0^+ >$ out of $|(\pi g_{7/2}\ \pi g_{9/2})1^+, (\pi g_{9/2}\ g_{9/2}^{-1})0^+ >$ can be excited by acting with a one body operator onto the ground state of $^{90}Zr$. The overlap of these two configurations is equal to $-\sqrt{\frac{2}{2j(2j+1)}}$ where $j = 9/2$. Therefore the transition strength to the $|1^+, T = 5>_2$ state is reduced by a factor 1/45. We want to emphasize that these arguments are only valid under the assumption of a direct process, i.e., when only configurations $|1>$ and $|3>$ in fig. 1c can be directly excited. The transition strengths to different final states calculated under this assumption are listed in table 3.

Summing up all the particle transition strengths we obtain the following result for the total transition strength ratio between $|1^+, T = 5>$ - and $|1^+, T = 4>$ - states:

$$S(T=5):S(T=4)=(1+2b^2/15):9(1-\frac{b^2}{55}) \tag{2}$$

Note that this ratio is almost independent an the amplitude b of the $|(\pi g_{9/2})\ 0^+>$ - configuration in the ground state wave function of $^{90}Zr$ and always of the order 1 : 9. Consequently, this population ratio contradicts a $|1^+, T = 5>$ assignment for the resonance at 18.5 MeV in $^{90}Nb$ since the $^{90}Zr(^3He, t)$ - experiment gives <u>equal</u> population for the known $|1^+, T = 4>$ resonance at 8.4 MeV and the resonance at 18.5 MeV.

When the amplitude $b \neq 0$ there is also a $|1^+, T = 6>$ - state in $^{90}Nb$. Its transition strength, however, is always very small in comparison to that of the $|1^+, T = 4>$-states, namely

$$S(T = 6) : S(T = 4) = \frac{5}{27} b^2 : 9(1 - \frac{b^2}{55}) \qquad (3)$$

otherwise one could think of determining the amplitude b by measuring this ratio. A more favourable case for measuring this amplitude is the $|1^+, T = 6>$ to $|1^+, T = 5>$ transition strength ratio in $^{90}Zr$ given by the relation

$$S(T = 6) : S(T = 5) = b^2 : (15 + 2b^2) \qquad (4)$$

Our second argument is concerned with the isospin splitting of energies of the $1^+$ - states in $^{90}Nb$. In a Tamm-Dancoff approximation with a schematic multipole-multipole force and a degenerate model for the single particle energies the energy difference between collective states with isospin $T = T_o$ and $T = T_o - 1$ is given by[9]

$$\Delta E = \frac{V_1}{A} T_o + \kappa \left| \frac{T_o + 1}{T_o} S(T = T_o) - \right.$$

$$\left. - \frac{2T_o + 1}{2T_o - 1} S(T = T_o - 1) \right| \qquad (5)$$

In eq. (5), $T_o$ is the isospin of the target nucleus ground state, $V_1$ the symmetry potential ($V_1$ = 100 MeV) and $\kappa$ the coupling strength of the residual interaction. For the Gamow-Teller transitions discussed here the second term on the right hand side of eq. (4) has the value $-16(5-b^2)\kappa 27$ and then is almost independent on the amplitude b. Using the estimate $\kappa = 40/A$ MeV[10] for $1^+$ - collective states one finds an isospin energy splitting of $\Delta E \simeq 4.5$ MeV in $^{90}$Nb. Therefore one slhould expect the $|1^+, T = 5>$ - state to be 4.5 MeV above the $|1^+, T = 4>$ - state |at 8.4 MeV| that means in the energy region around 13 MeV, which is considerabey lewer than 18.5 MeV.

This means that the Gamow-Teller T = 5 state should be located at $\sim$ 13 MeV and not in the energy region where the second bumb has been observed[2,3].

Our third argument is related with the estimate of the excitation energy of the $|1^-, T = 4>$ state in $^{90}$Nb. Also here the residual interaction is approximated by a multipole multipole force and all single particle excitations are considered to be degenerate. However, as the ground state correlations are quite significant for the dipole excitation we use here the random-phase approximation (RPA)*. The corresponding expression, already derived by Bohr and Mottelson[10] reads

$$E(\mu_\tau = 1) = |\nu(1 + \zeta) + (1 + 2\zeta + \zeta^2\nu^2)^{1/2} \hbar\omega_0 \qquad (5)$$

where the parameter $\nu = (3N)^{1/3} - (3Z)^{1/3}$ measures the neutron

* It should be noted that the backward going graphs in the case of $\mu_\tau = -1$ transitions arise from the $\mu_\tau = 1$ particle-hole excitations and vise-versa.

excess, $\hbar\omega o = 41^{-1/3}$ MeV is the harmonic oscillator energy and

$$\zeta = \frac{1}{2} \{ [\frac{E(\mu_\tau = 0)}{\hbar\omega o}]^2 - 1 \} \qquad (6)$$

is a dimensionless quantity which depends on the excitation energy $E(\mu_\tau = 0)$ of the GDR in the target nucleus. Using the experimental value $E(\mu_\tau = 0) = 16.8$ MeV for $^{90}Zr$[11] we obtain $\zeta = 1.19$ which leads to an energy of $E(\mu_\tau = -1) = 24.9$ MeV measured with respect to the ground state of $^{90}Zr$.

The corresponding excitation energy in $^{90}Nb$ is then equal to 18 MeV, which is very close to the energy of the second peaks observed in the ($^3$He, t) experiment[2,3].

Finally we want to point out that after finishing this work a (p, n) - experiment was performed at the Indiana University cycltron[12] with the following results:

(i) an isovector dipole state was tentatively identified at an excitation energy of 17.9 ± 6 MeV;

(ii) Gamow-Teller states with isospin T = 5 and T = 4 have been observed at 13.2 MeV and 8.4 Mev, respectively, with the cross section ratio 1 : 8 MeV.

All results of this experiment confirm our theoretical estimates.

In summary, the present analysis, as well as the recent (p, n) experiment[12] strongly suggest that the resonance at 18.5 MeV in $^{90}Nb$ should be of dipole nature and not a Gamow-Teller state as it has been argeed by Galonsky et al[2,3].

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## TABLE CAPTIONS

Table 1. Wave functions for various $1^+$ states in $^{90}Zr$ involving the $2p_{1/2}$, $1g_{9/2}$ and $1g_{7/2}$ single particle orbits. The configurations labelled by $|a\rangle$ , $|b\rangle$, $|c\rangle$, and $|d\rangle$ are defined in fig. 1. Configuration $|e\rangle$ is a linear combination of $|e\rangle = \sqrt{\frac{45}{44}}|\bar{e}\rangle$ as given in fig. 1. Note that $|\bar{e}\rangle$ and $|c\rangle$ are nonorthogonal: $\langle c|\bar{e}\rangle = -\sqrt{\frac{1}{45}}$ .

Table 2. Wave functions for $1^+$ states in $^{90}Nb$ with configurations $|1\rangle$ through $|7\rangle$ as defined in fig. 1. Configurations $|8\rangle$, $|9\rangle$ and $|10\rangle$ correspond to the linear combinations

$$|8\rangle = \sqrt{\frac{45}{44}}\,|\bar{8}\rangle + \sqrt{\frac{1}{44}}\,|3\rangle,$$

$$|9\rangle = \sqrt{\frac{81}{77}}\,|\bar{9}\rangle + \sqrt{\frac{4}{77}}\,|6\rangle,$$

$$|10\rangle = \sqrt{\frac{45}{44}}\,|\overline{10}\rangle + \sqrt{\frac{1}{44}}\,|5\rangle,$$

respectively. The configurations $|\bar{8}\rangle$, $|\bar{9}\rangle$ and $|\overline{10}\rangle$ are shown in fig. 1. They have the following overlaps with configurations $|3\rangle$, $|6\rangle$ and $|7\rangle$: $\langle\bar{8}|3\rangle = \langle\overline{10}|5\rangle = -\sqrt{\frac{1}{45}}$ and $\langle\bar{9}|6\rangle = -\frac{2}{9}$.

Table 3. Transition strengths calculated for various $|0^+, {}^{90}Zr\rangle \to |1^+, {}^{90}Nb\rangle$ transitions expressed in terms of the square of the reduced matrix element $M = \langle n,\ell,j = \ell+1/2||\sigma||n,\ell,j = \ell - 1/2\rangle = -\left(8\ell\frac{\ell + 1}{2 + 1}\right)^{1/2}$, where $\ell = 4$.

TABLE 1

| State | $\|a\rangle$ | $\|b\rangle$ | $\|c\rangle$ | $\|d\rangle$ | $\|e\rangle$ |
|---|---|---|---|---|---|
| $\|1^+, T=6\rangle_1$ | 0 | $\sqrt{\frac{1}{12}}$ | $\sqrt{\frac{9}{12}}$ | $\sqrt{\frac{2}{12}}$ | |
| $\|1^+, T=5\rangle_2$ | 0 | $-\sqrt{\frac{121}{132}}$ | $\sqrt{\frac{9}{132}}$ | $\sqrt{\frac{2}{132}}$ | |
| $\|1^+, T=5\rangle_3$ | | 0 | $-\sqrt{\frac{2}{11}}$ | $\sqrt{\frac{9}{11}}$ | |
| $\|1^+, T=5\rangle_4$ | 1 | 0 | 0 | 0 | |
| $\|1^+, T=5\rangle_5$ | | | | | 1 |

TABLE 2

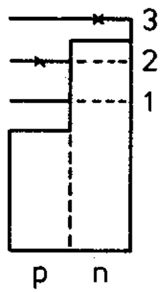
| State | $\|1\rangle$ | $\|2\rangle$ | $\|3\rangle$ | $\|4\rangle$ | $\|5\rangle$ | $\|6\rangle$ | $\|7\rangle$ | $\|8\rangle$ | $\|9\rangle$ | $\|10\rangle$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\|1^+,T=6\rangle_1$ | | | $\sqrt{\frac{9}{66}}$ | $\sqrt{\frac{2}{66}}$ | $\sqrt{\frac{18}{66}}$ | $\sqrt{\frac{36}{66}}$ | $\sqrt{\frac{1}{66}}$ | | | |
| $\|1^+,T=5\rangle_2$ | | | $-\sqrt{\frac{225}{330}}$ | $-\sqrt{\frac{50}{330}}$ | $\sqrt{\frac{18}{330}}$ | $\sqrt{\frac{36}{330}}$ | $\sqrt{\frac{1}{330}}$ | | | |
| $\|1^+,T=5\rangle_3$ | | | $-\sqrt{\frac{2}{110}}$ | $\sqrt{\frac{9}{110}}$ | $\sqrt{\frac{49}{110}}$ | $-\sqrt{\frac{32}{110}}$ | $\sqrt{\frac{18}{110}}$ | | | |
| $\|1^+,T=5\rangle_4$ | $\sqrt{\frac{1}{10}}$ | $\sqrt{\frac{9}{10}}$ | | | | | | | | |
| $\|1^+,T=5\rangle_5$ | | | | | | | | $\sqrt{\frac{1}{10}}$ | $\sqrt{\frac{7}{10}}$ | $\sqrt{\frac{2}{10}}$ |
| $\|1^+,T=4\rangle_6$ | | | | | $\sqrt{\frac{16}{90}}$ | $-\sqrt{\frac{2}{90}}$ | $-\sqrt{\frac{72}{90}}$ | | | |
| $\|1^+,T=4\rangle_7$ | | | $\sqrt{\frac{162}{990}}$ | $-\sqrt{\frac{729}{990}}$ | $\sqrt{\frac{49}{990}}$ | $-\sqrt{\frac{32}{990}}$ | $\sqrt{\frac{18}{990}}$ | | | |
| $\|1^+,T=4\rangle_8$ | $-\sqrt{\frac{9}{10}}$ | $\sqrt{\frac{1}{10}}$ | | | | | | | | |
| $\|1^+,T=4\rangle_9$ | | | | | | | | $-\sqrt{\frac{81}{90}}$ | $+\sqrt{\frac{7}{90}}$ | $\sqrt{\frac{2}{90}}$ |
| $\|1^+,T=4\rangle_{10}$ | | | | | | | | | $-\sqrt{\frac{2}{9}}$ | $\sqrt{\frac{7}{9}}$ |

TABLE 3

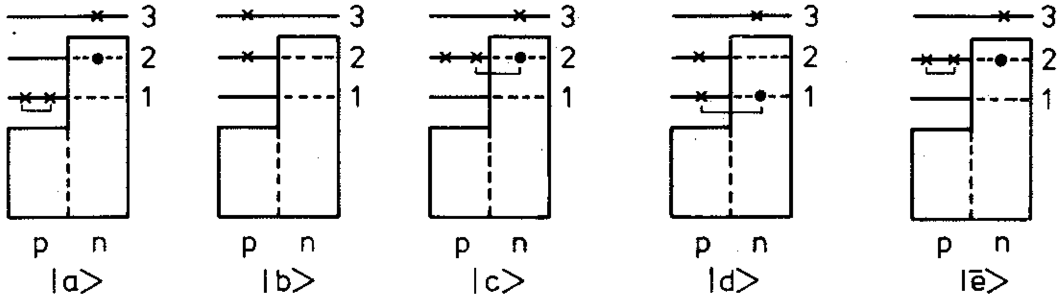
| Transition $\vert 0^+, {}^{90}Zr\rangle \rightarrow \vert 1^+, {}^{90}Nb\rangle$ | Transition Strength \|arbitrary units\| |
|---|---|
| $\vert 0^+ \rightarrow \vert 1^+, T = 6\rangle_1$ | $\frac{1}{99}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 5\rangle_2$ | $\frac{1}{198}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 5\rangle_3$ | $\frac{1}{7425}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 5\rangle_4$ | $\frac{1}{30}\ a^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 5\rangle_5$ | $\frac{22}{675}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 4\rangle_6$ | 0.0 |
| $\vert 0^+ \rightarrow \vert 1^+, T = 4\rangle_7$ | $\frac{1}{825}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 4\rangle_8$ | $\frac{3}{10}\ a^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 4\rangle_9$ | $\frac{22}{75}\ b^2M^2$ |
| $\vert 0^+ \rightarrow \vert 1^+, T = 4\rangle_{10}$ | 0.0 |

FIGURE CAPTION

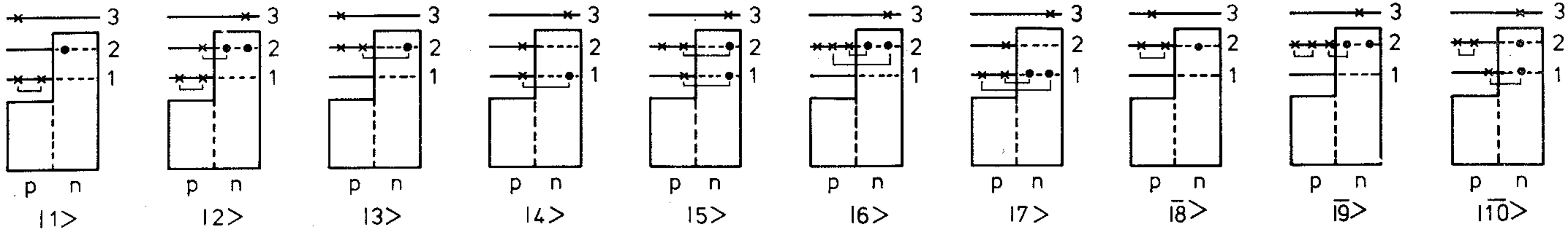
Fig. 1. Schematical representation of $1^+$ configurations involved in the $1^+$ wave functions of $^{90}Y$ (fig. 1a ), $^{90}Zr$ (fig. 1b) and $^{90}Nb$ (fig. 1c), res - pectively. The brackets indicate a coupling of the corresponding particles and (or) holes to an angular momentum $J = 0^+$ while the particles and holes with no special marking are coupled to $J = 1^+$. The single particle orbits labelled with 1, 2 and 3 belong to the $2p_{1/2}$, $1g_{9/2}$ and $1g_{7/2}$ orbits, respectively.




(a) $^{90}$Y




(b) $^{90}$Zr




(c) $^{90}$Nb