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ON CHARGE - EXCHANGE GAMOW - TELLER AND DIPOLE RESONANCES IN ^{90}Zr

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ABSTRACT

The charge-exchange resonances in ^{90}Zr are discussed within the framework of a simple model. Recent experimental results are confronted with the corresponding theoretical estimates.

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1. INTRODUCTION

Among the different isovecter modes of excitation, only the giant electric dipole (GED) resonance with quantum numbers $\kappa=1$, $\sigma=0$, $\lambda=1$, $\tau=1$ and $\mu_\tau=0$ is well established*. Additional information on nuclear collective excitation may be obtained from the study of the charge exchange modes ($\tau=1$, $\mu_\tau=\pm1$) using reaction processes such as (p,n) (^{3}He,t), (π^+,π^o), etc., and the corresponding charge conjugate processes.

In a nucleus with N=Z, the ground state isospin is $T_o=0$ and the charge exchange modes are related to the ($\tau=1,\mu_\tau=0$) - excitation by isobaric invariance. As a consequence,

i) the vibrational frequencies are given by the simple relation[1)]

$$E(\tau=1,\mu_\tau=\pm1) = E(\tau=1,\mu_\tau=0) - \mu_\tau \Delta E_{Coul} \qquad (1.1)$$

where ΔE_{Coul} is the Coulomb energy displacement, and

ii) the transition strengths, for a given multipole operater $M(\lambda,\tau=1,\mu_\tau)$,

$$S(\lambda,\tau=1,\mu_\tau) \equiv B(\lambda,T=1) = \frac{|\langle I_f=\lambda,T_f=1|||M(\lambda,\tau=1)|||I_i=0,T_i=0\rangle|^2}{3(2\lambda+1)} \qquad (1.2)$$

are independent of μ_τ. Here, the matrix element is reduced both in spin and isospin.

In nuclei with $T_o>>1$ the situation is entirely different

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* The notation is the same as in ref. 1). The quantum numbers κ, σ,λ,τ and μ stand for the orbital angular momentum, the spin, the total angular momentum ($\vec{\lambda}=\vec{\kappa}+\vec{\sigma}$), the isospin and the third component of the isospin respectively. Throught this paper, the unnecessary quantum numbers will be always omitted.

even in the zeroth order approximation. The neutron excess, or equivalently, the Pauli principle, causes a reduction in the number of proton hole-neutron particle excitations and a simultaneous increase of excitations of the type neutron hole-proton particle. Furthermore, a particle-hole operator acting on the ground state of a nucleus with $T_o \neq 0$ may give rise to states with isospin $T=T_o+1$, $T=T_o$, T_o+1 and $T=T_o-1$, T_o, T_o+1 for $\mu_\tau=1$, $\mu_\tau=0$ and $\mu_\tau=-1$, respectively. Both the energies and the transition strengths depend now on the orientation of the corresponding states in isospace.

The first forbidden charge exchange collective excitations ($\kappa=1;\sigma=0,\ 1;\lambda=0,1,2;\tau=1,\mu_\tau=\pm1$) in the lead region, were discussed theoreticaly in ref. [2]. Experimental evidences on these states as well as on allowed Gamow-Teller (GT) mode ($\kappa=0;\sigma=1;\lambda=1;\tau=1;\mu_\tau=-1$) in ^{208}Bi were also reported quite recently[3].

The nucleus which has more attention received, with respect to the charge-exchange collective states, experimentally is ^{90}Zr. A GT resonance has been observed in the ^{90}Zr(p,n)^{90}Nb reaction at incident proton energies of 35 and 45 MeV by Doering et al[4]. The resonance is centered around an energy of 14.4 MeV in ^{90}Nb and has a full width of 4.2 MeV*. Subsequent ^{90}Zr(^{3}He,t)^{90}Nb experiments at 130 MeV in Julich[5,6] and at 80 MeV in Grenoble[5] have confirmed these results although in the latter experiment the resonance peak appears to be split into two components, one of which at 14.1 MeV is of GT type and the other at 16.6 MeV of unknown multipolarity. Particularly interesting is the (^{3}He,t) experiment of Galonsky and the Julich group[6,7] since they observed for the first time another broad bump at 25.4 MeV excitation energy. The new bump

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* All the energies are measured with respect to the ground state of ^{90}Zr, which is 6.9 MeV more bound than the ground state of ^{90}Nb.

is as strongly populated as the one at 14.4 MeV. Since no angular distribution could be measured for this peak, questions were raised about its nature. Galonsky et al[6,7] discussed two possibilities, namely that it could be either a giant vector dipole (GVD) excitation ($\kappa=1,\sigma=0,\lambda=1,\tau=1,\mu_\tau=-1$) or the isobaric analog of the giant magnetic dipole (GMD) resonance in ^{90}Zr[8], whose quantum numbers are $\kappa=0$ $\sigma=1,\lambda=1,\tau=1,\mu_\tau=0$. They ruled out, however, the first possibility because of energy considerations. Finally, quite recently a new (p,n) - experiment was performed at the Indiana University Cycloton[9] with the following results:

i) two GT states with isospin T=4 and T=5 have been observed at 15.6 and 20.3 MeV, respectively, with the cross section ratio 1: 8.3, and

ii) a GVD state was tentatively identified at an excitation energy of 24.8±0.6 MeV.

A few theoretical results on the charge exchange collective states in ^{90}Zr have been presented recently by Osterfeld and the author of the present paper[10].

The aim of this work is twofold:

1) to point out a simple way of estimating the energies and the distribution of the transition strengths for the charge-exchange collective states, and

2) to perform numerical estimates for the GT and GVD modes in ^{90}Nb and confront them with existing experimental results.

As the GED and GVD (GMD and GT) modes differ only in the projection of isospin μ_τ, in what follows both will be labeled only by the quantum number $\sigma=0$ ($\sigma=1$).

2. THEORY

2.1) Transition Strengths

The total transition strength for a given multipolarity λ and the orientation in isospace μ_τ is defined as

$$S(\lambda,\mu_\tau) = \sum_T s(\lambda,T,\mu_\tau) \tag{2.1}$$

where

$$s(\lambda,T,\mu_\tau) = (T_o T_o\ 1\mu_\tau|T,T_o+\mu_\tau)^2 B(\lambda,T) \tag{2.2}$$

are the partial transition strengths to different members of the isospin triplet, and

$$B(\lambda,T) = \frac{|\langle I_f=\lambda,T_f=T||\mathcal{M}(\lambda,\tau=1)|||I_i=0,T_i=T_o\rangle|^2}{(2\lambda+1)\ (2T+1)} \tag{2.3}$$

are the transition probabilities reduced both in angular momentum and isospin.

In the weak coupling model discussed by Fallieros et al.[11] the $B(\lambda,T)$ values are independent of T and the relative excitation strengths for different final states are given simply by the geometrical factor (Clebsch-Gordon coefficients) displayed in relation (2.2). Such an approximation is, however, only valid when the neutron orbitals from the neutron excess region do not participate in the excitation process; the corresponding phonon is considered to be a definite entity and rotates freely in isospin space[12].

A simple way to estimate the unperturbed strengths $S^{(0)}(\lambda,\mu_\tau)$ is based on the use of the recipe of Macfarlane [13] and French[14] (monopole sum rule). After dividing convenientely the shell model orbitals into the filled (f), the valence (v) and empty

(e) orbitals, and in such a way that no transition of the type $v \to v$ or $f \to e$ is possible, one has

$$S^{(0)}(\lambda,\mu_\tau) = \frac{1+|\mu_\tau|}{6(2\lambda+1)} \sum_v \left[\frac{\langle N_v^p(\mu_\tau)\rangle}{(2j_v+1)} \sum_e \langle j_v|||M(\lambda,\tau{=}1)|||j_e\rangle^2 + \frac{\langle N_v^h(\mu_\tau)\rangle}{(2j_v+1)} \sum_f \langle j_f|||M(\lambda,\tau{=}1)|||j_v\rangle^2 \right] \tag{2.4}$$

Here, the single particle matrix elements are reduced with respect to both spin and isobaric spin. $\langle N_v^p(\mu_\tau)\rangle$ is the expectation value, in the target state, of the number of active particles for the excitation $(\tau=1,\mu_\tau)$ while $\langle N_v^h(\mu_\tau)\rangle$ denotes the corresponding mean number of holes.

Knowing the total strengths $S(\mu_\tau)$, the B(T) - values and the partial strengths $s(T,\mu_\tau)$ are obtained from expressions (2.2) and (2.3)*. Explicitely,

$$\begin{aligned} s(T=T_o+1,\mu_\tau=1) &= B(T=T_o+1) \\ &= S(\mu_\tau=1)\ , \end{aligned} \tag{2.5a}$$

$$\begin{aligned} s(T=T_o+1,\mu_\tau=0) &= \frac{1}{T_o+1} B(T=T_o+1) \\ &= \frac{1}{T_o+1} S(\mu_\tau=1)\ , \end{aligned} \tag{2.5b}$$

$$\begin{aligned} s(T=T_o,\mu_\tau=0) &= \frac{T_o}{T_o+1} B(T=T_o) \\ &= S(\mu_\tau=0) - \frac{1}{T_o+1} S(\mu_\tau=1)\ , \end{aligned} \tag{2.5c}$$

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* The quantum number λ is omitted.

$$s(T=T_o+1,\mu_\tau=-1) = \frac{1}{(2T_o+1)(T_o+1)} B(T=T_o+1)$$
$$= \frac{1}{(2T_o+1)(T_o+1)} S(\mu_\tau=1), \tag{2.5d}$$

$$s(T=T_o,\mu_\tau=-1) = \frac{1}{T_o+1} B(T=T_o)$$
$$= \frac{1}{T_o} S(\mu_\tau=0) - \frac{1}{T_o(T_o+1)} S(\mu_\tau=1), \tag{2.5e}$$

$$s(T=T_o-1,\mu_\tau=-1) = \frac{2T_o-1}{2T_o+1} B(T=T_o-1)$$
$$= S(\mu_\tau=-1) - \frac{1}{T_o} S(\mu_\tau=0) +$$
$$+ \frac{1}{T_o(2T_o+1)} S(\mu_\tau=1). \tag{2.5f}$$

One should notice that due to the Pauli principle

$$S(\mu_\tau=-1) \geq S(\mu_\tau=0) \geq S(\mu_\tau=1) \tag{2.6}$$

and consequentely,

$$B(T=T_o-1) \geq B(T=T_o) \geq B(T=T_o+1) \tag{2.7}$$

This means that the strength $s(T=T_o+1,\mu_\tau=0)$ is reduced with respect the strength $s(T=T_o,\mu_\tau=0)$ not only by the geometrical suppression factor $1/T_o$, but also by the dynamical suppression factor $B(T=T_o+1)/B(T=T_o)$. A similar comment is pertinent for the three $\mu_\tau=-1$ partial strengths.

The total unperturbed strengths are related as

$$2S^{(0)}(\mu_\tau=0) = S^{(0)}(\mu_\tau=1) + S^{(0)}(\mu_\tau=-1) \tag{2.8}$$

while for the $\sigma=0$ mode the relation holds[1)]

$$S(\mu_\tau=-1) - S(\mu_\tau=1) = T_o/\pi \, \langle r^2 \rangle_{n.exc.} \tag{2.9}$$

where $\langle r^2 \rangle_{n.exc.}$ is the mean square radius in the neutron excess region.

2.1 Nuclear Model

The energies of the collective states will be estimated at the expense of using a very schematic force of the form

$$H = -\frac{1}{2}\chi \sum_{\mu_\tau} M^+(\tau=1,\mu_\tau) M(\tau=1,\mu_\tau) \tag{2.10}$$

where χ is the coupling constant and

$$M(\tau=1,\mu_\tau) = \begin{cases} \sum_{i=1}^{A} \vec{\sigma}(i)\tau_{\mu_\tau}(i) & \text{for } \sigma=1 \text{ modes} \\ \sum_{i=1}^{A} r_i \vec{Y}_1(i)\tau_{\mu_\tau}(i) & \text{for } \sigma=0 \text{ modes} \end{cases} \tag{2.11}$$

Furthermore, a degenerate model for the single-particle energies is assumed. Then the unperturbed energies of a state with isospin T and the third component of isospin $M_T=T_o+\mu_\tau$ read[1)]

$$e^{(0)}(T,\mu_\tau) = \varepsilon + \frac{V_1}{2A}\left[T(T+1) - T_o(T_o+1)-2\right] - \mu_\tau \Delta E_{Coul} \tag{2.12}$$

where ε represents the average single-particle excitation energy and V_1 is the symmetry potential ($V_1 \approx 100$ MeV).

In the Tamm-Dancoff approximation (TDA) the transition strengths are not affected by the residual interaction ($\delta(T,\mu_\tau)\equiv\delta^{(0)}(T,\mu_\tau)$) while the perturbed excitation energies are given by

$$e(T,\mu_\tau) = e^{(0)}(T,\mu_\tau) + \chi B^{(0)}(T) \tag{2.13}$$

or explicitly

$$\begin{aligned}
e(T=T_o+1,\mu_\tau=1) &= \varepsilon + U_o - E_{Coul} + \chi B^{(0)}(T=T_o+1)\\
e(T=T_o+1,\mu_\tau=0) &= \varepsilon + U_o + \chi B^{(0)}(T=T_o+1)\\
e(T=T_o,\mu_\tau=0) &= \varepsilon - \frac{U_o}{T_o} + \chi B^{(0)}(T=T_o)\\
e(T=T_o+1,\mu_\tau=-1) &= \varepsilon + U_o + \Delta E_{Coul} + \chi B^{(0)}(T=T_o+1)\\
e(T=T_o,\mu_\tau=-1) &= \varepsilon - \frac{U_o}{T_o} + \Delta E_{Coul} + \chi B^{(0)}(T=T_o)\\
e(T=T_o-1,\ =-1) &= \varepsilon - \frac{T_o+1}{T_o} U_o + \Delta E_{Coul} + \chi B^{(0)}(T=T_o-1)
\end{aligned} \tag{2.14}$$

where

$$U_o = \frac{V_1 T_o}{A} . \tag{2.15}$$

Characterizing the difference between the strengths $S^{(0)}(\mu_\tau=1)$ and $S^{(0)}(\mu_\tau=-1)$ by the parameter ν,

$$S^{(0)}(\mu_\tau=\pm 1) = S^{(0)}(\mu_\tau=0)(1\mp\nu) \tag{2.16}$$

the energy splittings

$$\begin{aligned}
D(T=T_o+1) &= e(T=T_o+1,\mu_\tau=0) - e(T=T_o,\mu_\tau=0)\\
&= e(T=T_o+1,\mu_\tau=-1) - e(T=T_o,\mu_\tau=-1)
\end{aligned} \tag{2.17a}$$

and

$$D(T=T_o-1) = e(T=T_o,\mu_\tau=-1) - e(T=T_o-1,\mu_\tau=-1) \tag{2.17b}$$

take a very simple form

$$D(T=T_o+1) = \frac{T_o+1}{T_o} U \tag{2.18a}$$

$$D(T=T_o-1) = U \tag{2.18b}$$

where

$$U = U_o - \chi\nu\, S^{(0)}(\mu_\tau=0) \tag{2.19}$$

In the random phase approximation (RPA) the unperturbed excitation energies for given orientation in the isospace μ_τ, are written as

$$E^{(0)}(\mu_\tau) = \sum_T (T_o T_o 1 \mu_\tau | T_o, T_o+\mu_\tau) e^{(0)}(T,\mu_\tau) \tag{2.20}$$

or

$$E^{(0)}(\mu_\tau) = \varepsilon + \mu_\tau(U_o - \Delta E_{Coul}) \tag{2.20'}$$

After introducing the residual interaction the corresponding energies and transition strengths read[1)]

$$E(\mu_\tau=0) \equiv K_o = \left[\varepsilon\ (\varepsilon + 2\chi S^{(0)}(\mu_\tau=0))\right]^{1/2} \tag{2.21a}$$

$$S(\mu_\tau=0) = \varepsilon\ \ S^{(0)}(\mu_\tau=0) K_o^{-1} \tag{2.21b}$$

$$E(\mu_\tau=\pm 1) = K + \mu_\tau(U - \Delta E_{Coul}) \tag{2.21c}$$

$$S(\mu_\tau=\pm 1) = S^{(0)}\left[(\varepsilon + \chi\nu^2 S^{(0)}(\mu_\tau=0))K^{-1} - \mu_\tau \nu\right] \tag{2.21d}$$

where

$$K = \left[K_o^2 + (\chi\nu S^{(0)}(\mu_\tau=0))^2\right]^{1/2} \tag{2.22}$$

In the RPA the isospin energy splittings, defined in (2.17a) and (2.17b) are given by

$$D(T=T_o+1) = \frac{T_o+1}{T_o}(U+K-K_o) \tag{2.23a}$$

and

$$D(T=T_o-1) = U - \frac{2T_o+3}{2T_o-1}(K-K_o) \tag{2.23b}$$

As $K>K_o$ it means that while the energy difference $D(T=T_o+1)$ is increased by the ground-state correlations associated with the coupling between the $\mu_\tau=+1$ and $\mu_\tau=-1$ particle-hole excitations, the spliting $D(T=T_o-1)$ is decreased by the same effect.

It should be noted that when varying χ the quantity

$$S(\mu_\tau=1) - S(\mu_\tau=-1) = S^{(0)}(\mu_\tau=1) - S^{(0)}(\mu_\tau=-1) \tag{2.24}$$

is constant. This is an important difference with respect to the $\mu_\tau=0$ case, where the oscillator sum $E(\mu_\tau=0)\ S(\mu_\tau=0) = \varepsilon S(\mu_\tau=0)$ is constant and thus $S(\mu_\tau=0)$ decreases as χ grows.

In order to estimate in the RPA the energies $e(T,\mu_\tau)$ of the different isospin components, for a given mode of excitation $(\tau=1,\mu_\tau)$, it will be assumed that a relation similar to (2.16a) also holds for the perturbed energies, namely that

$$E(\mu_\tau) = \sum_T (T_o T_o 1\mu_\tau | T_o, T_o+\mu_\tau)\ e(T,\mu_\tau) \tag{2.25}$$

Then, combining this last expression with the relations

$$\begin{aligned} e(T=T_o+1,\mu_\tau=\pm1) &= e(T=T_o+1,\mu_\tau=0) \mp \Delta E_{Coul} \\ e(T=T_o,\mu_\tau=-1) &= e(T=T_o,\mu_\tau=0) + \Delta E_{Coul} \end{aligned} \tag{2.26}$$

which arise from the isobaric invariance, one has

$$\begin{aligned} e(T=T_o+1,\mu_\tau=1) &= E(\mu_\tau=1) \\ e(T=T_o+1,\mu_\tau=0) &= E(\mu_\tau=1) + \Delta E_{Coul} \end{aligned} \tag{2.27}$$

$$
\begin{aligned}
e(T=T_o,\mu_\tau=0) &= \frac{1}{T_o}\left[(T_o+1)E(\mu_\tau=0)-E(\mu_\tau=1)-\Delta E_{Coul}\right] \\
e(T=T_o+1,\mu_\tau=-1) &= E(\mu_\tau=1)+2\Delta E_{Coul} \\
e(T=T_o,\mu_\tau=-1) &= \frac{1}{T_o}\left[(T_o+1)E(\mu_\tau=0)-E(\mu_\tau=1)+\right. \\
&\quad \left.+(T_o-1)\Delta E_{Coul}\right] \\
e(T=T_o-1,\mu_\tau=-1) &= \frac{1}{T_o(2T_o-1)}\{(2T_o+1)\left[T_oE(\mu_\tau=-1)-\right. \\
&\quad \left.-E(\mu_\tau=0)\right]+E(\mu_\tau=1)-(2T_o-1)\Delta E_{Coul}\}.
\end{aligned}
\tag{2.27}
$$

Analogously the partial transition strengths $s(T,\mu_\tau)$ are obtained from the total transition strengths $S(\mu_\tau)$ by making use of the relations (2.5).

3. NUMERICAL ESTIMATES

In evaluating the strengths $S(\mu_\tau)$ for the $\sigma=0$ modes we assume that in the ground state of ^{90}Zr all the levels up to and including the $1g_{9/2}$ subshell are occupied by neutrons, and that for protons the $1g_{9/2}$ level is completely empty. Futhermore, if the radial wave-functions are approximated by those of an harmonic oscillator, from (2.4) we obtain

$$
\begin{aligned}
S^{(0)}(\mu_\tau=1) &= 80 \\
S^{(0)}(\mu_\tau=0) &= 135 \\
S^{(0)}(\mu_\tau=-1) &= 190
\end{aligned}
\tag{3.1}
$$

in units of $b^2/4\pi$, where $b=A^{1/6}$ fm$=2.12$ fm is the length parameter. The relations (2.4) then lead to the result (in the same units)

$$
\begin{aligned}
&s^{(0)}(T=6,\mu_\tau=1)=80 \\
&s^{(0)}(T=6,\mu_\tau=0)=40/3 \\
&s^{(0)}(T=5,\mu_\tau=0)=365/3
\end{aligned}
\tag{3.2}
$$

$$\mathcal{s}^{(0)}(T=6,\mu_\tau=-1)=40/33$$
$$\mathcal{s}^{(0)}(T=5,\mu_\tau=-1)=803/33 \qquad (3.2)$$
$$\mathcal{s}^{(0)}(T=4,\mu_\tau=-1)=1809/11$$

For the discussion of the $\sigma=1$ modes we will assume that the wave function of the target state is of the form

$$|{}^{90}Zr;0^+\rangle = a|(2p_{1/2})^2 0\rangle + b|(1g_{9/2})^2 0\rangle$$

with $a^2+b^2=1$. Furthermore, we consider only the $1g_{9/2}\to 1g_{7/2}$ single-particle transition, as it is the most relevant one with respect to the available experimental information[3-7]. The expressions (2.4) and (2.5) give now*

$$S^{(0)}(\mu_\tau=1) = \frac{b^2}{15} M^2$$

$$S^{(0)}(\mu_\tau=0) = \frac{5+b^2}{30} M^2$$

$$S^{(0)}(\mu_\tau=-1) = \frac{1}{3} M^2 .$$

and

$$\mathcal{s}^{(0)}(T=6,\mu_\tau=1) = \frac{b^2}{15} M^2$$

$$\mathcal{s}^{(0)}(T=6,\mu_\tau=0) = \frac{b^2}{90} M^2$$

$$\mathcal{s}^{(0)}(T=5,\mu_\tau=0) = \frac{15+2b^2}{90} M^2 \qquad (3.5)$$

$$\mathcal{s}^{(0)}(T=6,\mu_\tau=-1) = \frac{b^2}{990} M^2$$

$$\mathcal{s}^{(0)}(T=5,\mu_\tau=-1) = \frac{15-2b^2}{450} M^2$$

$$\mathcal{s}^{(0)}(T=4,\mu_\tau=-1) = \frac{9(55-b^2)}{1650} M^2$$

---

* As we do not consider the transition $1g_{9/2}\to 1g_{9/2}$ the sum rule for for the $\sigma=1$ mode

$$S(\mu_\tau=-1)-S'(\mu_\tau=-1) = N-Z$$

is clearly not fulfilled.

where

$$M = \frac{1}{\sqrt{6}} \langle 1\, g_{9/2} ||| (\kappa=0, \sigma=1, \lambda=1, \tau=1) ||| 1g_{7/2} \rangle$$

$$= \langle 1g_{9/2} || \sigma || 1g_{7/2} \rangle = -\sqrt{\frac{160}{9}} \qquad (3.6)$$

The single-particle strengths given by (3.5) were also derived in ref.[10] but in a more complicated way. There one first constructs the complete shell-model basis with good isospin for the final states, which means to include two-particles-two-holes configurations for the $\mu_\tau=0$ excitations and the configurations of the type two-particles-two-holes and three-particles-three-holes for the $\mu_\tau=-1$ mode. After having done this one evaluates the single particle transition probabilities for the operator $(\kappa=0, \sigma=1, \lambda=1, \tau=1)$ between the final states $|I^\pi=1^+, T=4,5,6\rangle$ and the initial states given by (3.3).

For the $\sigma=0$ particle-hole energies the usual estimate[1)]

$$\varepsilon \equiv \hbar\omega_o = 41A^{-1/3} \text{ MeV} = 9.15 \text{ MeV} \qquad (3.7)$$

will be employed. The corresponding energy for the $\sigma=1$ mode was obtained from experimental data, namely

$$\begin{aligned} \varepsilon &= \varepsilon(1g_{7/2}) - \varepsilon(1g_{9/2}) \\ &= S_n(^{90}Zr) - S_n(^{91}Zr) + \bar{\varepsilon}(1g_{7/2}) - \bar{\varepsilon}(2d_{5/2}) \qquad (3.8) \\ &= (11.98 - 7.20 + 2.84 - 0.10)\text{MeV} = 7.62 \text{ MeV} \end{aligned}$$

where the symbol S_n stands for the neutron separation energy[16)] and $\bar{\varepsilon}(1j)$ are controid single-particle energies as measured in the $^{90}Zr(d,p)$ reaction study[17)]. For the coupling constant we will use the estimates given in ref.[1)]:

$$\chi = \frac{\pi V_1}{A\langle r^2\rangle} \approx 3.64\, V_1 A^{-5/3} \text{ fm}^{-2} = 0.201 \text{ MeV fm}^{-2}$$

$$(\langle r^2\rangle = \frac{3}{5}(1.2A^{1/3})^2 \text{fm}^2), \qquad (3.9)$$

for the $\sigma=0$ mode, and

$$\chi \simeq 40/A \text{ MeV} = 0.44 \text{ MeV} \qquad (3.10)$$

for the $\sigma=0$ mode.

Finally, the Coulomb energy diplacement was taken to be

$$\begin{aligned}\Delta E_{Coul} &= \Delta B + E_x(IA) \\ &= (6.9 + 5.1) \text{ MeV} = 12.0 \text{ MeV}\end{aligned} \qquad (3.11)$$

where ΔB is the difference in binding energy between the ground states of ^{90}Zr and ^{90}Nb[16] and $E_x(IA)$ is the excitation energy of the isobaric analog state in ^{90}Zr, as measured in ref. 5). It should be noted that the above result for ΔE_{Coul} agrees with the one obtained from the Fermi gas expression for the Coulomb energy, namely[7],

$$\begin{aligned}E_{Coul} &= \frac{3}{5}\frac{Z^2e^2}{R}\left[1 - 5\left(\frac{3}{16\pi Z}\right)^{2/3}\right] \\ &\simeq 0.70\frac{Z^2}{A^{1/3}}\left[1 - 0.76Z^{-2/3}\right]\text{MeV}\end{aligned} \qquad (3.12)$$

$$(R_c = 1.25\ A^{1/3} \text{ fm};\ A > 40)$$

Numerical results for excitation energies and transition strengths are listed in Table 1. The locations of the $\sigma=1$ resonances are also shown in fig. 1.

3.1) <u>$\sigma=1$ modes</u>

As the ground state correlations are very small in this case, the TDA and RPA give very similar results. Therefore, we will concern ourselves only with the first one. Both the energies and the transition strengths are almost independent of the amplitude of the $|(2g_{9/2})^2 0\rangle$ - configuration in the ground state wave-function of ^{90}Zr, except, of course for the $\mu_\tau=1$ component which does not exist when b=0.

On the basis of the present theoretical estimate, the $\sigma=1$ resonance with isospin T=5 in ^{90}Zr should be located at an excitation energy of 8 MeV. However only 15% of the possible $\sigma=1$ strength has been found in this energy region, by means of high resolution, inelastic electron scattering experiments[10].

The transition strength ratio between the $I^\pi=1^+$ states with isospin T=5 and T=4 in ^{90}Nb is

$$s(T=5,\mu_\tau=-1):s(T=4,\mu_\tau=-1) = (1+\frac{2b^2}{15}):9(1-\frac{b^2}{55}) \quad (3.13)$$

and consequentely always of the order of 1:9. This population ratio contradicts a $I^\pi=1^+$, T=5 assignment for resonance at 25.4 MeV, since the ^{90}Zr (^{3}He,t) experiment[5] gives equal population for the known $I^\pi=4^+$, T=5 state at 15.4 MeV and the bump seen at 25.4 MeV. In addition, in the experiment the two bumps are separated by ≈ 10 MeV, while our estimate for D (T=4) is only 4.3 MeV. The present theoretical estimates for the $\sigma=1$ states agree, both in the cross section ratio and the excitation energies, with the recent experiment of Goodman et al.[9].

When the amplitude b≠0 there is also a $I^\pi=1^+$, T=6 state in ^{90}Nb. Its transition strength, however, is always very weak in comparison with that of the $I^\pi=1^+$, T=4 state, namely,

$$s(T=6,\mu_\tau=-1):s(T=4,\mu_\tau=-1) = \frac{5}{27}b^2:\ 9(1-\frac{b^2}{55}) \quad (3.14)$$

otherwise, one could think of determining the amplitude b by measuring this ratio. A more favorable case for measuring this amplitude is the transition strength ratio between $I^\pi=1^+$, T=5 and $I^\pi=1^+$, T=6 states in ^{90}Zr given by the relation

$$\delta(T=6,\mu_\tau=0):\delta(T=5,\mu_\tau=0) = b^2:(15+2b^2) \qquad (3.15)$$

3.2) <u>σ=0 modes</u>

The ground state correlations are quite importat for σ=0 modes of excitations. As a consequence, in the RPA the locations of all 1^- states are appreciably lower and the corresponding transition strengths significantly weaker than in the TDA.

The theoretical estimate, within the RPA, of the excitation energy $e(T=5,\mu_\tau=0)$=15.7 MeV, agrees well with the measured value which is centred at around 16.7 MeV[20].

There are a few experimental evidences that the energy difference D(T=6) is of the form (2.16a) with[21]

$$U \approx (55 \pm 15) \frac{T_o}{A} \text{ MeV}$$

which for ^{90}Zr gives D(T=6)=(3.7 ± 1.0) MeV. Our estimate is significantly smaller. It should be mentioned, however, that the quantity $D(T=T_o+1)$ depends in a very sensitive way on the mean square radius in the excess region. Namely, the relation (2.19) may be rewritten in the form

$$U = \frac{V_1 T_o}{A} \left(1 - \frac{\langle r^2\rangle_{n.exc.}}{2\langle r^2\rangle}\right) \qquad (3.17)$$

Assuming that $\langle r^2\rangle_{n.exc.} = \langle r^2\rangle$, as was done in ref. 22), one would have agreement with (3.16), but then the sum rule (2.9) would not be fulfilled any more. With the harmonic oscillator wave functions $\langle r^2\rangle_{n.exc.} = 5.5\, A^{1/3} = 24.65\ \text{fm}^2$ while from (3.9) $\langle r^2\rangle = 17.35\ \text{fm}^2$.

The theoretical estimate, within the RPA, for the energy $e(T=4,\mu_\tau=-1)$=26.8 MeV, is only a few MeV higher that the third peak observed at (24.8±0.6) MeV in the ^{90}Zr(p,n)^{90}Nb reaction[9], or that of the second bump seen by Galonsky et al[5] at ≈25.4 MeV and suggested to be a $I^{\pi}=1^{+}$, T=5 state.

It should be very difficult to observe experimentally the $I^{\pi}=1^{-}$, T=5 resonance as it lies very close to its T=4 partner and as its strength is relatively weak.

FIGURE CAPTION

Fig. 1 - Estimated excitation energies of the $\sigma=0$ charge-exchange resonances in ^{90}Zr. The thickness of the lines that begin in the ground state of ^{90}Zr represent the transition strengths. The symbol T_- stands for the isospin lowering operator which connects the isobaric analog states.

TABLE 1 - Theoretical estimates for the excitation energies, measured with respect to the ground state of ^{90}Zr and transition strengths of the $\sigma=1$ and $\sigma=0$ charge-exchange resonances in ^{90}Zr, evaluated according to the TDA and RPA approaches.

	$\sigma=1$				$\sigma=0$	
	TDA		RPA		TDA	RPA
	$b^2=0$	$b^2=0.5$	$b^2=0$	$b^2=0.5$		
$S(\mu_\tau=1)$	0	0.59	0	0.43	28.52	11.54
$S(\mu_\tau=0)$	2.96	3.26	2.55	2.77	48.14	27.27
$S(\mu_\tau=-1)$	5.93	5.93	5.93	5.79	67.76	50.78
$E(\mu_\tau=1)$	-	1.3	-	1.3	8.4	6.2
$E(\mu_\tau=0)$	8.8	9.0	8.7	8.9	18.8	16.2
$E(\mu_\tau=-1)$	16.6	16.6	16.6	16.5	29.1	26.9
$s(T=6,\mu_\tau=1)$	0	0.59	0	0.43	28.52	11.54
$s(T=6,\mu_\tau=0)$	0	0.10	0	0.07	4.75	1.92
$s(T=5,\mu_\tau=0)$	2.96	3.16	2.55	2.70	43.39	25.35
$s(T=6,\mu_\tau=-1)$	0	0.01	0	0.01	0.43	0.17
$s(T=5,\mu_\tau=-1)$	0.59	0.63	0.51	0.54	8.68	5.07
$s(T=4,\mu_\tau=-1)$	5.33	5.28	5.42	5.24	58.65	45.51
$e(T=6,\mu_\tau=1)$	-	1.3	-	1.3	8.4	6.2
$e(T=6,\mu_\tau=0)$	-	13.3	-	13.3	20.4	18.2
$e(T=5,\mu_\tau=0)$	8.0	8.1	8.0	7.9	18.5	15.7
$e(T=6,\mu_\tau=-1)$	-	25.3	-	25.3	32.4	30.2
$e(T=5,\mu_\tau=-1)$	20.0	20.1	20.0	19.9	30.5	27.7
$e(T=4,\mu_\tau=-1)$	15.7	15.7	15.7	15.7	28.9	26.8

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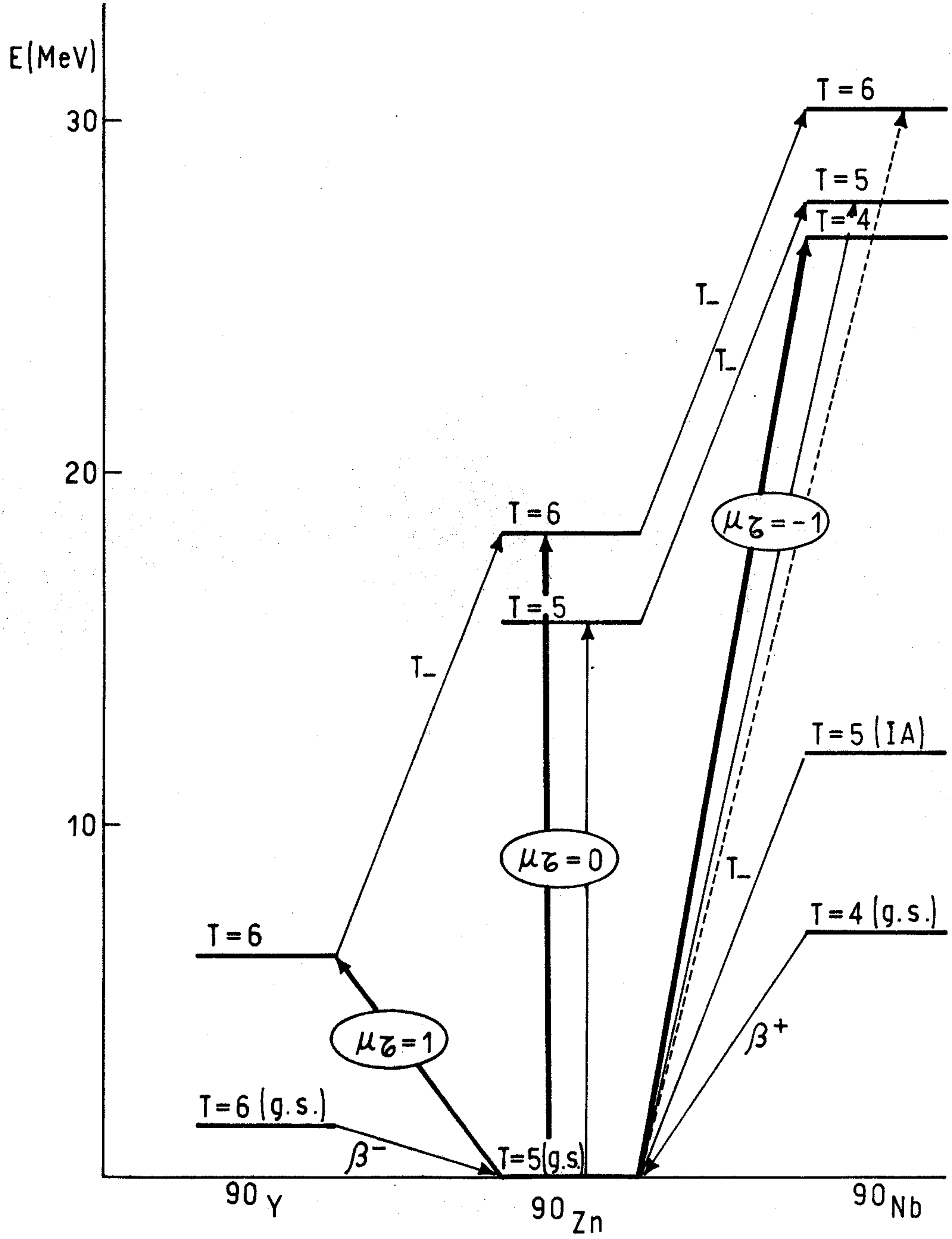

E(MeV)
30
20
10
T = 6
T = 5
T = 4
T₋
T₋
T = 6
T = 5
$\mu_\tau = -1$
T₋
T = 5 (IA)
$\mu_\tau = 0$
T₋
T = 4 (g.s.)
T = 6
$\mu_\tau = 1$
β^+
T = 6 (g.s.)
β^-
T = 5 (g.s.)
90 Y
90 Zn
90 Nb