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A NOTE ON LORENTZ TRANSFORMATION AND PSEUDO-RAPIDITY DISTRIBUTIONS

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ABSTRACT

It is shown that although rapidity and pseudo-rapidity are almost equivalent variables, their difference may in practice become quite remarkable. Non Lorentz invariance of pseudo-rapidity distributions may cause appearance of strange effects at first sight, such as deformation of a perfectly symmetric particle distribution into an asymmetric one when going to another frame.

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One of the most widely used coordinate in describing the particles produced in high-energy collisions is the rapidity

$$y = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}} . \tag{1}$$

As is well known, the usefulness of this variable rests on its additivity under Lorentz transformation along $p_{\parallel}$ axis. A related variable which, under the experimental point of view, is even more useful than y is the pseudo-rapidity

$$\eta = -\ln \tan \frac{\theta}{2} = \frac{1}{2} \ln \frac{p + p_{\parallel}}{p - p_{\parallel}} . \tag{2}$$

These variables appear to be almost equivalent, especially for large particle energies. Thus, for many purposes one might use y or else η without making distinction. For instance, one might experimentally measure $d\sigma/d\eta$ and compare it with the theoretically predicted $d\sigma/dy$. However, in the central region where the particles are sufficiently non-relativistic, η distribution may substantially deviate from y distribution. This difference has indeed been observed and discussed by several authors[1-3] in the past, but since it has sometimes been ignored by subsequent workers, we think opportune reconsidering it here.

In one of these works[4], Koshiba considers ISR experiment of Pisa-Stony Brook collaboration[5], in which a 15.4 GeV proton beam collides with a 26.6 GeV proton beam and the inclusive $d\sigma/d\eta$ is measured. He observes that $d\sigma/d\eta$ is asymmetric in the ISR frame (around $\eta' = \eta - 0.274 = 0$) and argues that since in the C.M. frame the distribution is manifestly symmetric, Lorentz transformation may be violated in hadronic processes. In a more recent paper[6], he comes again to raise a doubt about the validity of

Lorentz transformation, this time by plotting the width of y and η distributions, measured in several experiments, as function of the incident energy (see Fig. 1). He remarks that there is a definite discontinuity both in the absolute value and the slope of the curve, when one goes from the data with stationary targets to those obtained in the ISR. Thus, Lorentz transformation would be broken down.

Our opinion is that both of the arguments above are perfectly correct, provided that every quantity is referred to the rapidity. But, given that all the ISR data used[5,7] are $d\sigma/d\eta$ instead of $d\sigma/dy$, a more careful analysis is needed before arriving at such a drastic conclusion.

We start by parametrizing the inclusive distribution as

$$E\frac{d\sigma}{d\vec{p}} = A\left\{\exp\left[-\beta(y-\bar{y})^2\right] + \exp\left[-\beta(y+\bar{y})^2\right]\right\} e^{-\alpha\sqrt{p_T^2+m^2}}, \quad (3)$$

where α is fixed[8] to be $\alpha = 7.1\ \mathrm{GeV}^{-1}$ and the yet unknown parameters A, β and $\bar{y}$ are determined by fitting $d\sigma/d\eta$ given by Ref. 7. We have also interpolated them to $\sqrt{s}=40.5$ GeV, which corresponds to the C.M. energy when the incident beams are 15.4 GeV + 26.6 GeV protons. Once all the parameters are fixed, we can compute now everything we need from $Ed\sigma/d\vec{p}$ above.

In Fig. 2, we compare $d\sigma/dy$, $d\sigma/d\eta$ (in the C.M. frame) and $d\sigma/d\eta_{lab}$ thus calculated at $\sqrt{s}=40.5$ GeV. One immediately notices that $d\sigma/d\eta$ is strongly depressed in the central region as compared to $d\sigma/dy$. In addition, as the total number of particles must obviously be conserved, it suffers a stretching toward high-η regions. These are precisely what have been pointed out in earlier works[1,2]. It is clear that this change of shape in the particle distribution causes an overestimation of the full width at the half maximum, when $d\sigma/d\eta$ is directly used instead of

$d\sigma/dy$. We have plotted in Fig. 1 Δy and $\Delta\eta$ computed in this way at each ISR energy, together with the other points which appear in Fig. 1 of Ref. 6). One sees that the discontinuities mentioned before disappear as soon as Δy (rather than $\Delta\eta$) are correctly plotted.

In Fig. 2, one also sees that, as to the shape, $d\sigma/d\eta_{lab}$ is almost the same to $d\sigma/dy$ and it is shifted toward large-η values. This is the reason why the pseudo-rapidity width as measured in the target frame is essentially equal to the rapidity width.

Let us now turn to the question of $d\sigma/d\eta$ in an asymmetric collision. As became clear from the discussion above, central region where the particle distribution is depressed means central in each reference frame, where non-relativistic effects become important, and it has nothing to do with the center of mass of the system. Then, it is evident that if the collision is observed in a frame which is moving with respect to C.M., in general $d\sigma/d\eta$ does become asymmetric[3]. In order to elucidate this statement, we have shifted the y distribution in Fig. 2 by an amount $y'-y=0.274$, obtaining $\frac{1}{\sigma}\frac{d\sigma}{d\eta}$ shown in Fig. 3. This shift corresponds to going from C.M. frame to one where the incident-particle energies are respectively 15.4 GeV and 26.6 GeV. The same effect which has been exhibited by Pisa-Stony Brook experiment[5] is clearly seen there.

One may also try to compare the invariant cross sections $E d\sigma/d\vec{p} = F(\sqrt{s}, y, p_T)$, say for a fixed value of p_T. This amounts just to rewriting y in terms of η in the corresponding expression. Namey,

$$y = th^{-1}(v\, th\, \eta) \quad , \quad v = \frac{p}{E} \quad , \tag{4}$$

which has the following limiting values

$$y \underset{\eta \to 0}{\simeq} \vartheta \eta$$

and (5)

$$y \underset{\substack{\eta \to \pm\infty \\ p_T \text{ fixed}}}{\simeq} \eta \mp \frac{1}{2} \ln\left[1 + \left(\frac{m}{p_T}\right)^2\right] .$$

The second of these relations means that, even for an ultrarelativistic particle, there is a finite difference between its y and η, which increases as p_T decreases. This implies that, if the invariant cross section is factorized as the one given by eq. (3), η distribution may become wider than the corresponding y distribution by any predetermined amount, provided that we take a sufficiently small value of p_T (here we are ignoring the existence of a boundary due to energy conservation, which must indeed be considered in an actual problem).

In short, our conclusion is that the appearance of a phenomenon when described in terms of η may look quite different from that in terms of y and, as far as the existing data are concerned, Lorentz transformation does give a correct account of hadronic processes, if it is properly applied. The parametrization given by eq. (3) is quite reasonable and we believe it represents the main features of the actual inclusive distributions. However, a word of remark is appropriate here. That although the detailed results are evidently model dependent, the basic conclusions are nevertheless independent of the particular parametrization which has been chosen.

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FIGURE CAPTIONS

FIG. 1 - Full widths at the half maximum of rapidity (Δy) and pseudo-rapidity ($\Delta\eta$) distributions in $pp \to$ hadrons, as function of C.M. energy squared. ■ : Δy with stationary targets (taken from Fig. 1 of Ref. 6)). □ : $\Delta\eta$ measured in target frame (taken from Ref. 6)). ● : $\Delta\eta$ measured in C.M. frame; the data are from Ref. 7) - see the text for details. ○ : Δy corresponding to the same data above. The straight lines are drawn to guide the eyes over $\Delta\eta$'s and Δy's respectively.

FIG. 2 - Single-particle y, η(C.M.) and η_{lab} distributions at $\sqrt{s}$ = 40.5 GeV as explained in the text. The positions of the half-maxima and the corresponding full widths are also indicated.

FIG. 3 - Single-particle η distribution at $\sqrt{s}$ = 40.5 GeV, as seen from a frame in which the incident-proton energies are respectively E_1=26.6 GeV moving rightward and E_2= =15.4 GeV moving leftward. The curve has been computed in the way explained in the text. Experimental points[7] shown for comparison are those of symmetrical collisions with the indicated total energies.

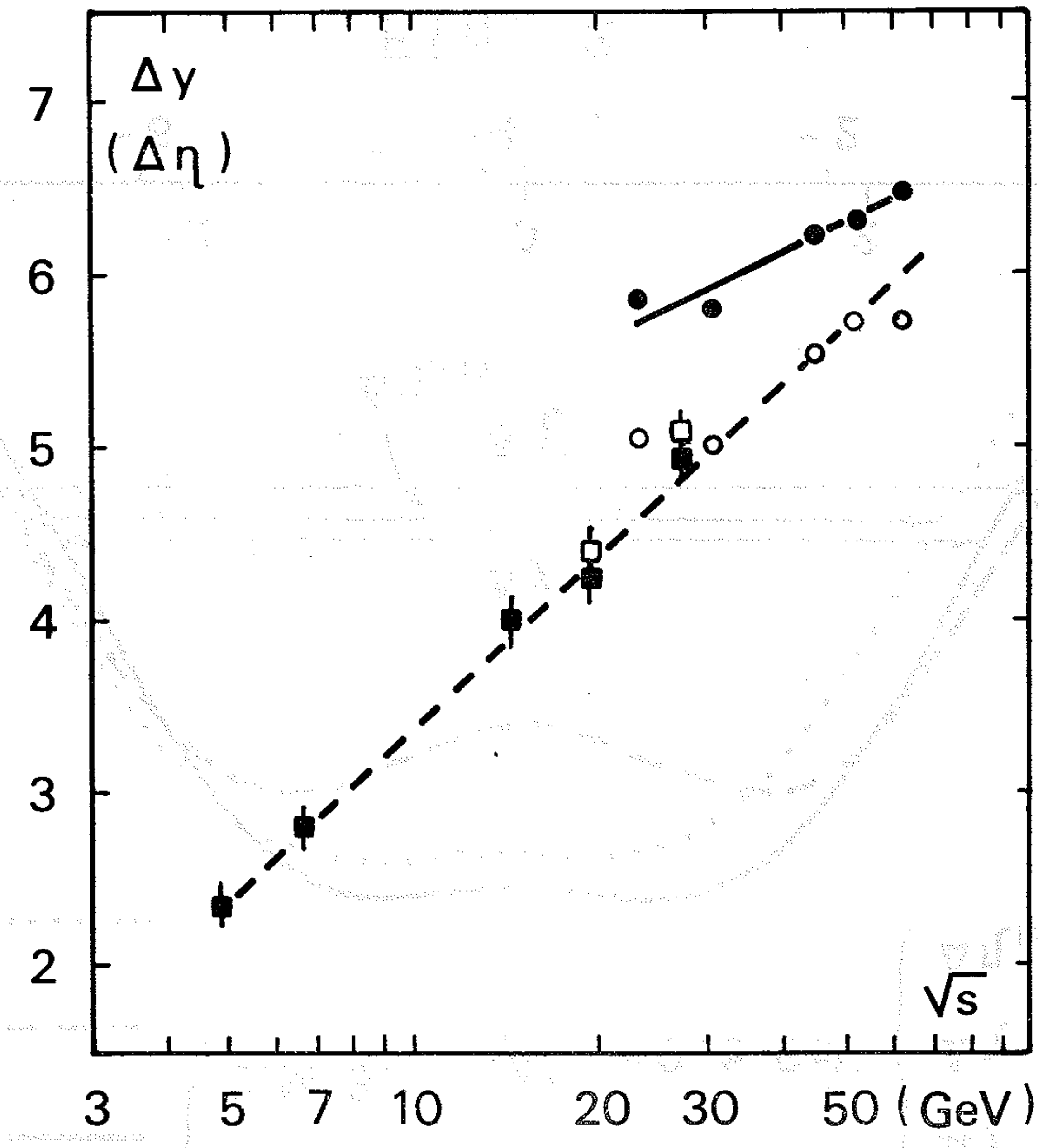
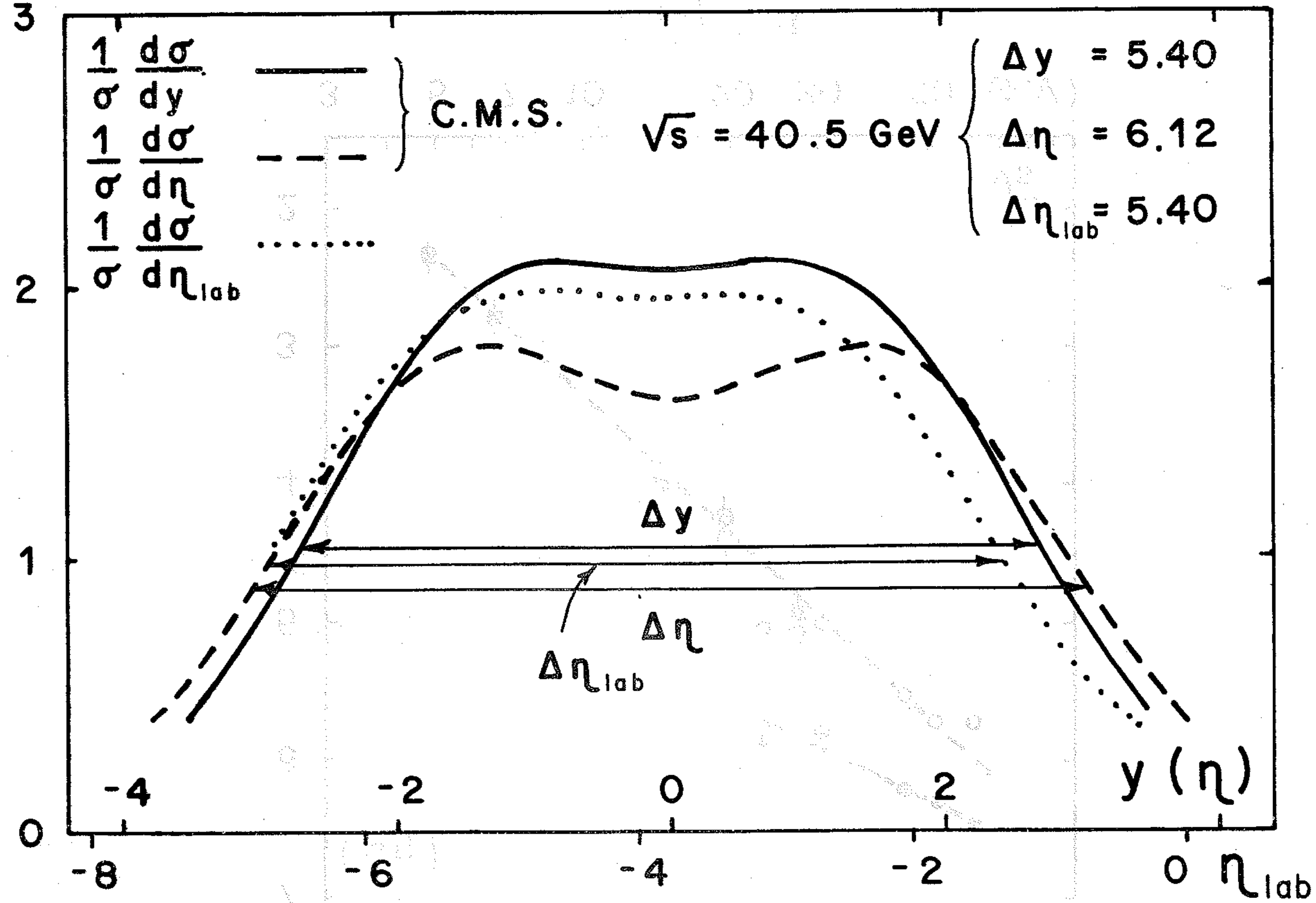


Fig. 1



$\frac{1}{\sigma}\frac{d\sigma}{dy}$
$\frac{1}{\sigma}\frac{d\sigma}{d\eta}$
C.M.S.
$\frac{1}{\sigma}\frac{d\sigma}{d\eta_{lab}}$
$\sqrt{s}$ = 40.5 GeV
Δy = 5.40
$\Delta\eta$ = 6.12
$\Delta\eta_{lab}$ = 5.40
Δy
$\Delta\eta$
$\Delta\eta_{lab}$
y (η)
η_{lab}

Fig. 2

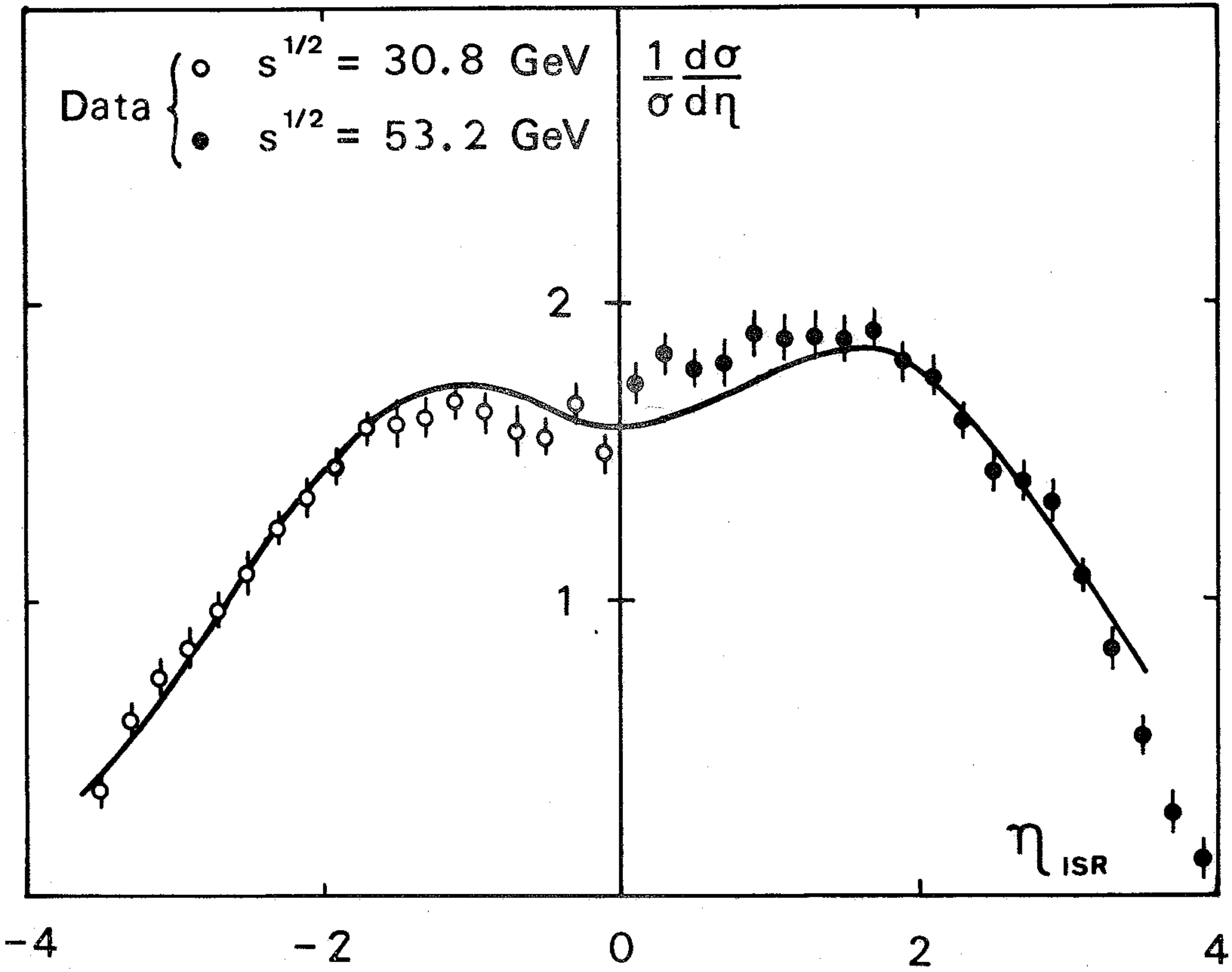

Data
$s^{1/2} = 30.8$ GeV
$s^{1/2} = 53.2$ GeV
$\frac{1}{\sigma}\frac{d\sigma}{d\eta}$
2
1
−4
−2
0
2
4
η_{ISR}

Fig. 3