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ABSTRACT

We propose a description of collective states in self-conjugate nuclei, both odd-odd and even-even, in terms of an interacting isoscalar p-boson model. Within this model, two limiting cases can be identified with the anharmonic vibrator and axial rotor limits of the classical geometrical description.

The interacting boson model (IBM) has been very successful in describing collective states in nuclei[1). In this model one may describe the low-energy collective motion in the even-even nuclei as excitation of an interacting-boson system which consists mainly s-bosons and d-bosons. It is the purpose of this letter to communicate the possibility of extending this elegant model to include isospin and thus widening its applications even to light nuclei, both odd-odd and even-even.

In IBM, the s-boson and d-boson can be considered as typifying the correlated nucleon pairs coupled to angular momentum zero and two as a result of the pairwise interactions among the valence nucleons outside of a core. For example, the interaction between the nucleous in the s-boson can be thought of as pairing type. In view of the two existing components of pairing correlations in nuclei, namely, T=1 pairing and T=0 pairing, we are proposing a possible interpretation of both odd-odd and even-even nuclei as a mixture of isovector bosons, with angular momentum L=0 and with L=2, and isoscalar bosons with L=1. It has been long established in the literature[2)] that isovector pairing is the dominant component in medium and heavy nuclei in comparison with isoscalar pairing and that the reverse is true for light nuclei. This seems to suggest that, for the light nuclei, one can deal mainly with the system of the isoscalar p-bosons and then perform the refinements by letting the system interact with the rest, namely, the isovector sd-bosons.

In the following, we will show that even in this simple interacting p-boson model we can identify the two limiting cases with the anharmonic vibrator and axial rotor limits of the classical geometrical description for the self-conjugate nuclei, both odd-odd and even-even. The transition between these two limits emerges naturally from the model. The results for ^{18}F and ^{20}Ne calculated in one of the limits seem to reproduce roughly well the experimental trends of the data. This demonstrates in an encouraging way that the model might be useful. An interesting feature of the present model is in the proposal that the entire variety of observed spectra would emerge from a complete calculation with a set of disperson-type equations.

To begin with, we claim that a number of states can be generated in self-conjugate nuclei as states of a system of N isoscalar p-bosons occupying M distinct levels in isospin space. The operator which creates such a boson in the ν-th level will be denoted as $P^{+}_{\mu\nu}$, where $\mu=-1,0,1$. If we regard the boson states, $\phi_{\mu\nu}=P^{+}_{\mu\nu}|0>$ as providing us a basis for the representation of the U(3)xU(M), then N-boson states can be classified according to the irreducible product representations of this group and its subgroups. The irreducible representations of U(3) are fully characterized by the Young diagrams which contain at most 3 rows while those of U(M) contain at most M rows. In order that the total wave function should be totally symmetric, we require the Young diagrams for both U(3) and U(M) to have the same shape $[\lambda]$. It follows that $[\lambda]$ contains at most 3 rows for $M\geq 3$ and has at most M rows for M<3.

We now consider first the strong coupling limit where the two-body interactions among bosons are much larger than the energy splittings. This is a standard group-theoretical problem. The method of solution is due to Racah[3].

If two p-bosons interact with each other, their interaction energy can assume only three values according to the three possible values of their angular momentum. Therefore by a convenient choice of three constants a,b, and c, it is always possible to express their interaction energy by the formula:

$$a + b\, P^{L}_{12} + c\, \vec{L}_1 \cdot \vec{L}_2 \qquad (1)$$

where P^{L}_{12} is the exchange operator acting upon the angular momentum coordinate only and $\vec{L_1} \cdot \vec{L_2}$ is the scalar product of the angular momenta of the two bosons. The energy of a state of a system of bosons in the strong coupling limit will then be:

$$E = \langle N;[\lambda]LM| \sum_{i<j} (a+bP^L_{ij}+c\vec{L}_i\vec{L}_j)|N;[\lambda]LM\rangle \qquad (2)$$

where

$$|N;[\lambda]LM\rangle = \sum_{\mu_1\mu_2\ldots\mu_N} \alpha_{[\lambda]}(\mu_1\mu_2\ldots\mu_N;LM)\prod_{k=1}^{N}\{\sum_{\nu=1}^{M}\phi_k(\nu)P^+_{\mu_k\nu}\}|0\rangle \qquad (3)$$

After the evaluation of the average value of $\sum_{i<j} P^L_{ij}$ in the state (3) with the SU_3 functions $\alpha_{[\lambda]}(\mu_1\mu_2\ldots\mu_N;\ LM)$, we finally arrive at a closed form for the energy:

$$E=\frac{N(N-1)}{2}a+\frac{b}{2}\{\lambda_1(\lambda_1-1)+\lambda_2(\lambda_2-3)+\lambda_3(\lambda_3-5)\}+c\{\frac{1}{2}L(L+1)-N)\} \quad (4)$$

The constants a,b and c here will be treated as our free parameters and are connected with the two boson interactions in the following way:

$$a=\frac{1}{2}\{\langle p^2D|v|p^2D\rangle+(p^2P|v|p^2P\rangle\}$$

$$b=\frac{1}{6}\langle p^2D|v|p^2D\rangle-\frac{1}{2}\langle p^2P|v|p^2P\rangle+\frac{1}{3}\langle p^2S|v|p^2S\rangle$$

$$c=\frac{1}{3}\{\langle p^2D|v|p^2D\rangle-\langle p^2S|v|p^2S\rangle\} \qquad (5)$$

The allowable values of L in (4) for $[\lambda]=[\lambda_1\lambda_2\lambda_3]$ are given by

$$L=K,K+1,K+2,\ldots K+\max(\lambda_1-\lambda_2,\lambda_2-\lambda_3) \qquad K\neq 0$$

$$=\max(\lambda_1-\lambda_2,\lambda_2-\lambda_3),\ \max(\lambda_1-\lambda_2,\lambda_2-\lambda_3)-2,\ldots 1,\text{ or } 0 \quad K=0 \qquad (6)$$

with the integer K taking values

$$K=\min(\lambda_1-\lambda_2,\lambda_2-\lambda_3),\min(\lambda_1-\lambda_2,\lambda_2-\lambda_3)-2,\ldots,1 \text{ or } 0 \tag{7}$$

The spectrum of Eq. (4) for chosen a,b, and c for ^{18}F and ^{20}Ne are shown inFig.1 and Fig.2. Here we take the ground state of ^{4}He as the vacuum, and add 7 p-bosons and 8 p-bosons to reach the nuclei of interest. The model reproduces roughly well the experimental trends of the energy levels. Out of 44 levels up to 11 MeV in ^{18}F, the model roughly covers 33 of them. The K=1 band builded on the observed 1.7-MeV 1+ state is known to be dominated by configurations outside s-d shell, predominantly of a four-particle-two-hole (4p-2h) nature. Within the model this band can be assigned with the permutation symmetry [61] and it reproduces well the results of a projected Hartee-Fock calculation based on a strong coupling model[4)]. We also assign certain symmetries for the negative-parity bands in ^{18}F which are again beyond the $(2s,1d)^2$ descriptions. Here we assume one of our p-bosons carrying negative parity. This can be thought of as due to the odd orbital angular momentum of the two nucleons which are supposed to form a p-boson. The results for ^{20}Ne are also encouraging. We assign the symmetries [44] and [422] to the bands of an eight-particle-four-hole nature. Although, within the model, the band with [422] - symmetry is cut off at 2^+ level, the 10.8-MeV 4^+ state is still reproduced well by the 10.85 MeV 4^+ member of the [44] - band. Furthermore, in order to extend the supposed [422] - band, there seems to be a need to incorporate f-bosons into the model. It is also interesting to note that the two sets of parameters a,b, and c for the two nuclei are quite close to each other.

Quadrupole transitions can be calculated by taking matrix elements of the quadrupole operator between the eigenstates (3). The quadrupole operator is a generator of U_3, u^k_q, with k=2, where u^k_q are given by

$$\mu^k_q = \frac{1}{\sqrt{3}} \sum_{\mu\mu'} \langle 1\mu 1\mu' | 11kq \rangle (-1)^{1-\mu'} p^+_{\mu\nu} p_{-\mu'\nu} \tag{8}$$

with k=0,1,2. Formulas for these matrix elements can be derived using the methods described by Elliott[5].

For the fully-symmetric states and the states with the [N-1 1] - symmetry, we have

$$B(E2;L+2\to 2)=X\frac{(L+K+2)(L+K+1)(L-K+2)(L-K+1)}{(L+1)(L+2)(2L+3)(2L+5)}f([\lambda],L) \tag{9}$$

where

$$f([\lambda],L)=\frac{\lambda_1-\lambda_2-L}{\lambda_1-\lambda_2+L+3}\left(\lambda_1-\lambda_2+\frac{\lambda_2-\lambda_3}{2}+L+3\right)^2;\ \lambda_1-\lambda_2-L=\text{even}$$

$$=\frac{\lambda_1-\lambda_2+1-L}{\lambda_1-\lambda_2+L+4}\left(\lambda_1-\lambda_2+\frac{\lambda_2-\lambda_3}{2}+L+3\right)^2;\ \lambda_1-\lambda_2-L=\text{odd} \tag{10}$$

Table 1 shows that within the ground-state band of ^{20}Ne, our B(E2)'s, according to (9), reproduce quite well the experimental trend that B(E2) strengths increase from 2→0 to 4→2, and then turn around to decrease monotonically. The Bohr-Mottelson geometrical model[6] gives, on the other hand, monotonically increasing B(E2) strengths as one proceeds upward through the 2→0, 4→2, 6→4 and 8→6 transitions. The E2 transitions between the members of the K=1 band of ^{18}F were also obtained and compared with the Bohr-Mottelson values. It remains to be seen whether similar effects of the cutoff factor due to the finite particle number could be observed experimentally.

So far we have seen that our model produces rotational-like spectra for a certain choice of the parameters such as boson energies and two-body matrix elements. We now show that, as these parameters change, the model should span the entire variety of observed spectra. This is done not by solving the Schrödinger equation for our model Hamiltonian directly. Instead, we would rather deal with a set of algebraic coupled equations.

In order to arrive at such equations, we should borrow some idea from the well known TDA treatment of the T=1 paring correlations. In that approximation, the equations that govern the T=1, J=0 boson states are given by:

$$\begin{cases} \dfrac{1}{g} = \sum\limits_{m}^{M} \dfrac{\Omega_m}{2\varepsilon_m - E_i} & i=1,2,3\ldots N \\ E = \sum\limits_{i=1}^{N} E_i & \end{cases} \tag{11}$$

where g, ε_j and Ω_j are the paring strength, single particle energies and pair degeneracies. If we introduce the anharmonic effects to better the results toward the exact solutions we will get a new set of equations instead of (11) as is shown in Ref. 8:

$$\begin{cases} \dfrac{1}{g} + \sum\limits_{j} \dfrac{k_{ij}}{E_j - E_i} = \sum\limits_{m=1}^{M} \dfrac{\Omega_m}{2\varepsilon_m - E_i}, \quad i=1,2,\ldots N \\ E \doteq \sum\limits_{i=1}^{N} E_i \\ k_{ij} = (N(N-3)+T(T+1))/N(N-1) \quad \text{for } [N] \end{cases} \tag{12}$$

The newly introduced constants k_{ij} play the role as the interaction energy between the i-th boson and j-th boson. When $k_{ij}=0$, Eqs. (12) reduce to (11), that of a system of independent TDA, T=1, J=0 bosons. It should be remarked that Eqs. (12) are derived from the exact solution of the Schrödinger equation for an isovector pairing system. The physical significance behind these equations is then clear. They provide a picture of a system of bosons interacting in a more complicated manner than that of IBM. The constants k_{ij} represent the average behavior of such interactions. It is interesting to remark in passing that the present interacting boson picture has an immediate link with the boson expansion approach[9].

The applications of these ideas to our isoscalar p-boson model are then straight forward, simply exchanging the role of angular momentum and isospin in the T=1, J=0 bosons.

Without touching the microscopic foundations of the present model, we propose the following set of equations similar to (12) to govern our boson states from which the entire variety of observed spectra would emerge:

$$\begin{cases} \dfrac{1}{\chi} + \sum\limits_{j} \dfrac{S_{ij}}{E_j - E_i} = \sum\limits_{\nu=1}^{M} \dfrac{D_\nu}{\eta_\nu - E_i}, \quad i=1,2,\ldots N \\ \\ E = \sum\limits_{i=1}^{N} E_i \end{cases} \tag{13}$$

where the constants χ, η_ν and D_ν are now treated as free parameters. We take the constants S_{ij} as the average of the calculated interaction energy between i-th boson and j-th boson from (4):

$$S_{ij} = a + \frac{b}{N(N-1)} \{\lambda_1(\lambda_1-1) + \lambda_2(\lambda_2-3) + \lambda_3(\lambda_3-5)\}$$

$$+ \quad \frac{c}{N(N-1)} \{L(L+1-2N)\} \tag{14}$$

For the states with $[N]$- symmetry and the choice of the parameters: a+b=c, $S_{ij}=c(N(N-3)+L(L+1))/N(N-1)$, the same form as k_{ij} in (12).

It is interesting to see that if we set $D_\nu=0$, $\eta_\nu=0$ and $\chi=1$, the energy E in (13) can be evaluated analytically and it reproduces exactly the rotational-like spectra (4).

It the other limit where the energy spacings between the levels are much larger than all interaction terms, one can expand E in (13) into a power series in S_{ij} by using the technique of Ref. 8:

$$E=\sum_{\nu} N_{\nu}\varepsilon_{\nu} + \sum_{\ell=1} b_{\ell}\,\{aN(N-1)+b[\lambda_1(\lambda_1-1)+\lambda_2(\lambda_2-3)+\lambda_3(\lambda_3-5)]$$

$$+c[L(L+1)-2N]\}^{\ell} \qquad (15)$$

where ε_{ν} are the boson energies and N_{ν} the occupation numbers at ν-th level. The coefficients b_{ℓ} can be expressed analytically in terms of N_{ν} and the other free parameters. Since the energy in (15) is in the form of $A+BL(L+1)+CL^2(L+1)^2+\ldots$, we are thus led to a description of our system in terms of a rotation-vibration picture.

In conclusion, we believe that a description of collective states for the self-conjugate nuclei in terms of a isoscalar p-boson model might be appropriate, especially in the above mentioned two limiting cases. For the other variety of observed spectra, a solution of the coupled equation (13) may be needed.

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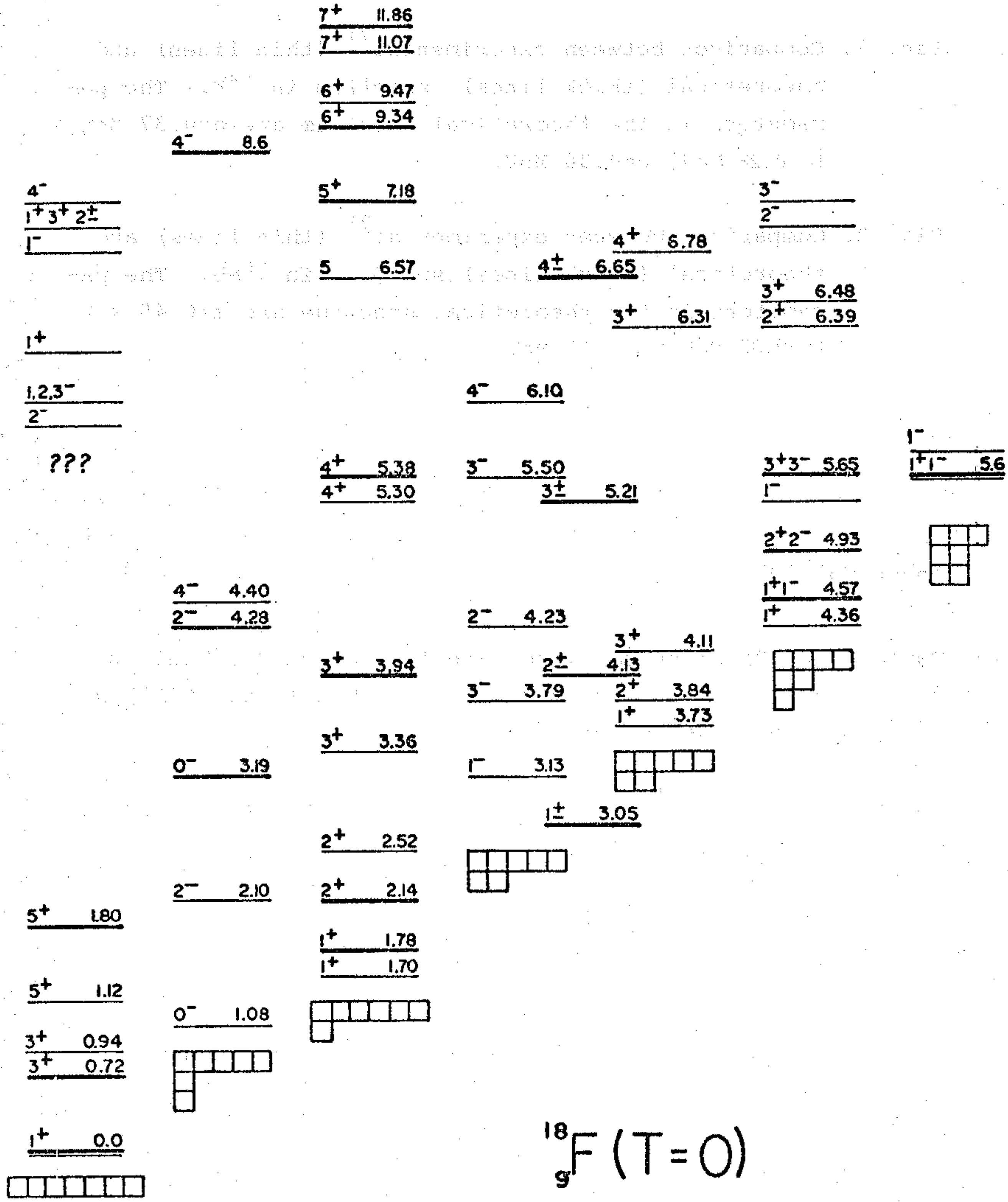
FIGURE CAPTIONS:

Fig. 1. Comparison between experimental[7] (thin lines) and theoretical (thick lines) spectrum in ^{18}F. The parameters in the theoretical spectrum are a=0.37 MeV, b=-0.28 MeV, c=0.36 MeV.

Fig. 2. Comparison between experimental[7] (thin lines) and theoretical (thick lines) spectrum in ^{20}Ne. The parameters in the theoretical spectrum are a=0.45 MeV, b=-0.33 MeV, c=0.43 MeV.

TABLE CAPTIONS:

Table 1. B(E2) strengths involving the lowest K=0 band in ^{20}Ne and the K=1 band in ^{18}F. The listed B(E2)'s are in Weisskopf unit.



4^-
$1^+ 3^+ 2^\pm$
1^-
1^+
$1,2,3^-$
2^-
???
5^+ 1.80
5^+ 1.12
3^+ 0.94
3^+ 0.72
1^+ 0.0
4^- 8.6
4^- 4.40
2^- 4.28
0^- 3.19
2^- 2.10
0^- 1.08
7^+ 11.86
7^+ 11.07
6^+ 9.47
6^+ 9.34
5^+ 7.18
5 6.57
4^+ 5.38
4^+ 5.30
3^+ 3.94
3^+ 3.36
2^+ 2.52
2^+ 2.14
1^+ 1.78
1^+ 1.70
4^- 6.10
3^- 5.50
2^- 4.23
3^- 3.79
1^- 3.13
4^+ 6.78
$4^\pm$ 6.65
3^+ 6.31
$3^\pm$ 5.21
3^+ 4.11
$2^\pm$ 4.13
2^+ 3.84
1^+ 3.73
$1^\pm$ 3.05
3^-
2^-
3^+ 6.48
2^+ 6.39
$3^+ 3^-$ 5.65
1^-
$2^+ 2^-$ 4.93
$1^+ 1^-$ 4.57
1^+ 4.36
1^-
$1^+ 1^-$ 5.6
$^{18}_{9}F$ (T=0)

Fig. 1

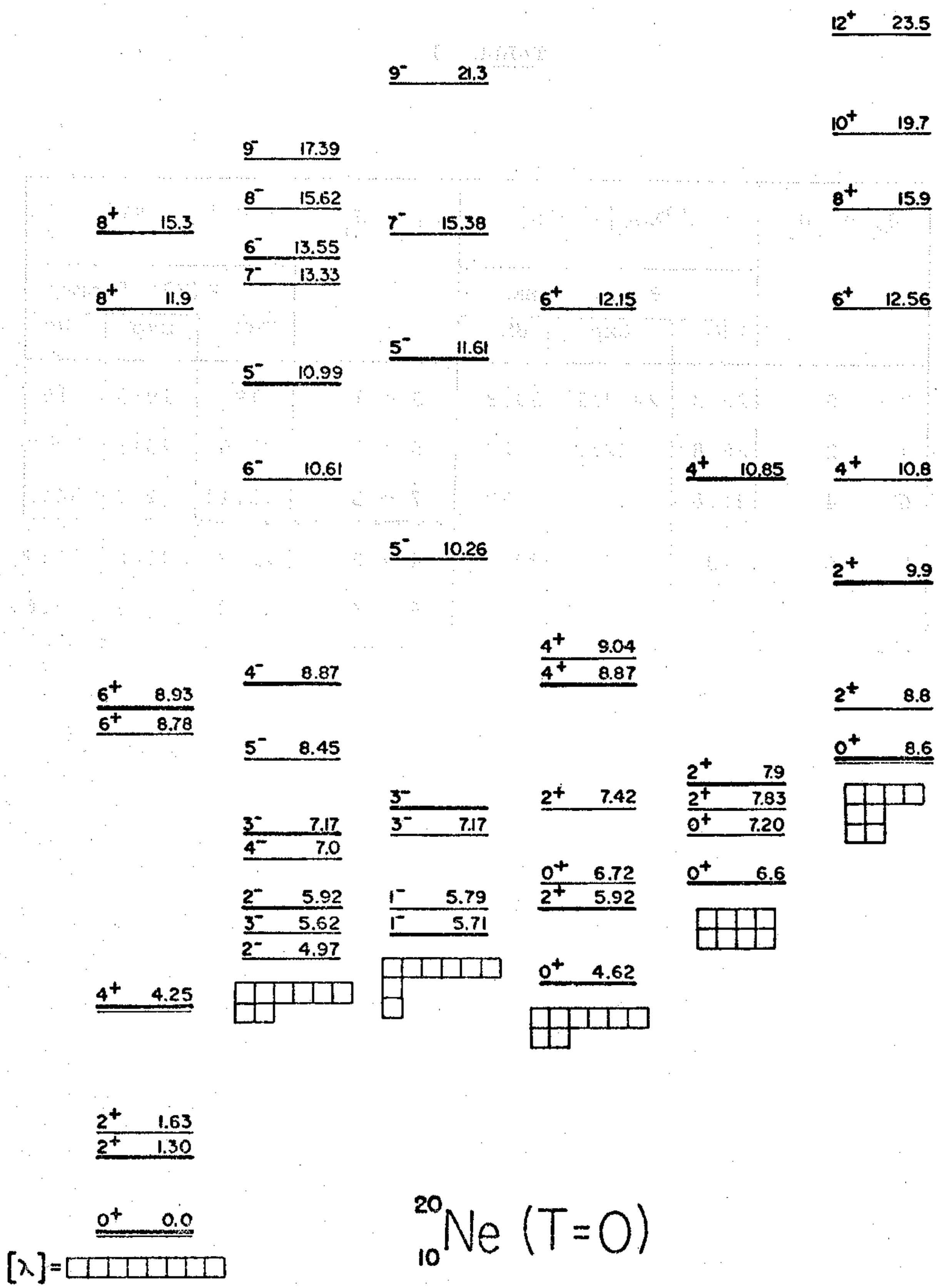

12+ 23.5
9- 21.3
10+ 19.7
9- 17.39
8- 15.62
8+ 15.9
8+ 15.3
7- 15.38
6- 13.55
7- 13.33
8+ 11.9
6+ 12.15
6+ 12.56
5- 11.61
5- 10.99
6- 10.61
4+ 10.85
4+ 10.8
5- 10.26
2+ 9.9
4+ 9.04
4+ 8.87
4- 8.87
6+ 8.93
6+ 8.78
2+ 8.8
5- 8.45
0+ 8.6
2+ 7.9
3-
2+ 7.42
2+ 7.83
3- 7.17
3- 7.17
0+ 7.20
4- 7.0
0+ 6.72
0+ 6.6
2- 5.92
1- 5.79
2+ 5.92
3- 5.62
1- 5.71
2- 4.97
0+ 4.62
4+ 4.25
2+ 1.63
2+ 1.30
0+ 0.0
[λ]=
$^{20}_{10}$Ne (T=0)

Fig. 2

TABLE 1

$J_i \to J_f$	^{20}Ne $[\lambda]=[8]$			$J_i \to J_f$	^{18}F $[\lambda]=[81]$		
	B(E2) from:				B(E2) from:		
	IBM	Exp	BM		IBM	Exp	BM
2 → 0	20.3	20.3±1	20.3	3 → 1	19	19±3	19
4 → 2	25.8	22±2	29	5 → 3	23.5	28±6	30.2
6 → 4	21.8	20±3	32	7 → 5	15.11	? ?	34.1
8 → 6	13	14±3	33.5	4 → 2	26.3	17±6	26.4
				6 → 4	25.3	? ?	32.6