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ABSTRACT

We propose a unified description of the following classes of nuclear collective states in terms of an interacting sp-boson model: (i) Low-lying collective states in the light nuclei, both odd-odd and even-even; (ii) Giant multipole resonances *(GMR)*, and (iii) pairing collective motions.

The interacting boson model (IBM) has been very successful in describing collective states in nuclei[1]. In this model one may describe the low-energy collective motion in the even-even nuclei as excitation of an interacting-boson system which consists mainly of s-bosons and d-bosons. It is the purpose of this letter to communicate the possibility of extending this elegant model to include isospin and p-bosons and thus giving an unified description of giant multipole resonances (GMR)[2], isovector pairing vibrations[3], and the low-lying collective states in light nuclei, both odd-odd and even-even.

In IBM, the s-boson and d-boson can be considered as typifying the correlated nucleon pairs coupled to angular momentum zero and two as a result of the pairwise interactions among the valence nucleons outside of a core. For example, the interaction between the nucleons in the s-boson can be thought of as pairing type. In view of the two existing components of pairing correlations in nuclei, namely, T=1 pairing and T=0 pairing, we are proposing a possible interpretation of both odd-odd and even-even nuclei as a mixture of isovector bosons, with angular momentum L=0 and with L=2, and isoscalar bosons with L=1. If, for simplicity, one excludes the L=2 components of isovector boson, a simple interacting sp-boson model would emerge with a group scheme U(4) × U(4). Several interesting features would therefore arise from the associated group symmetries. Among others, the most remarkable one is that experimental data seem to give evidences, as we will show, that **giant multipole** resonances and isovector pairing vibrations possess symmetry of the rotational group in four dimensions R(4). For example, we are able to reproduce reasonably well the general trend of the excitation energies of the giant multipole resonances. In view of the R(4) symmetry with the Hydrogen atom, one seems to be able to draw the parallel between the $1/r_o$ dependence of the Hydrogenic energy and the $A^{-1/3}$ dependence of the excitation energy of GMR. As to the isovector pairing vibrations, the rotational symmetry is with respect to the isospin space. It is remarkably interesting to find that the double pseudo-spins which characterizes the R(4) states may now be given physical meanings: they are identified with the number of removal pairing phonon and that of addition phonon in the Bohr classification scheme[3]. If we insist that the double pseudo- spins which characterize the GMR has the same physical meaning as the pairing-vibration, then the isoscalar GMR could be thought of as the isoscalar pairing vibrations.

We will also consider the SU(3) limit of the model which proves useful for many low-lying collective states in the light nuclei, both odd-odd and even-even. The results for $^{18}_{9}F$ and $^{20}_{10}Ne$ calculated in the SU(3) limit seem to reproduce roughly well the experimental trends of the data. All of the above mentioned interesting features demonstrate in an encouraging way that the model might be useful for a wide range of collective states in nuclei.

To begin with, we claim that a number of nuclear states can be generated as states of a system of N bosons occupying two levels, one level with angular momentum $\ell=0$, isospin $\tau=1$, and the other level with $\ell=1$, $\tau=0$. For convenience, we would call the former, isovector s bosons and the latter, isoscalar p bosons. If we regard the boson states $\phi_{\ell m \tau m_\tau}$ as providing us a basis for the representation of the $U(4) \times U(4)$, then N-boson states can be classified according to the irreducible product representations of this group and its subgroups. The irreducible representations of U(4) are fully characterized by the Young diagram which contains at most 4 rows. In order that the total wave function be totally symmetric, we require the Young diagram for both U(4) and U(4) to have the same shape $[f]$. It follows that $[f]$ contains at most 4 rows, namely $[f]=[f_1 f_2 f_3 f_4]$.

The energy splitting $\varepsilon=\varepsilon_s-\varepsilon_p$ and the boson-boson interaction would give rise to a definite spectrum for all the states belonging to $[f]$. The spectrum is then defined by ε and by the two body matrix elements in the model Hamiltonian for the system. In general this Hamiltonian can be written as a linear combination of operators that are diagonal for states either with the U(3)-symmetry, or R(4)-symmetry or with both. Thus in the spirit of IBM, the nucleus of interest characterized by a set of parameters and the energy levels can be found by diagonalizing the model Hamiltonian in either U(3) basis or R(4) basis.

The group theoretical methods which we use in the present work are the standard ones [4]. In the construction of a many-boson state according to U(3)-symmetry, the branching rules $U(4) \supset U(3)$ and $U(3) \supset R(3)$ are needed. The decomposition of the representation $[f]$ of U(4) into the representation $[\lambda]=[\lambda_1\lambda_2\lambda_3]$ of U(3) is given by: $f_1 \geq \lambda_1 \geq f_2 \geq \lambda_2 \geq f_3 \geq \lambda_3 \geq f_4 \geq 0$. Together with the well known decomposition rule[4] for U(3) $\supset$ R(3), we now can label the U(3) wave functions by $|N[f][\lambda]KLM;K_T TM_T\rangle$. As to the classification of the R(4) states, one needs to recall the fact that the R(4) group is locally isomorphic to the direct product of two three-dimensional rotation groups. The representation of R(4) are then characterized by that of R(3) $\times$ R(3), namely, (j_1,j_2), where either j_1 and j_2 are integers or both are semi-integers. The decomposition R(4) $\supset$ R(3) follows the well known coupling rule for two angular momenta $|j_1-j_2| \leq L \leq j_1+j_2$ or $|j_1-j_2| \leq T \leq j_1+j_2$. For this obvious reason j_1,j_2 are sometimes called double pseudo-spins in the literature. To obtain the allowed values (j_1,j_2) in a given representation of U(4), one may use the branching rule U(4) $\supset$ R(4) as stated in Racah[5]. Thus in the R(4) basis the wave functions are classified by $|N[f](j_1j_2)LM;(j'_1j'_2)TM_T\rangle$. Table I gives the classification of states in U(3) basis and in R(4) basis for a system of four bosons.

We now consider the case in which the energy ε is much larger than all interaction terms in the model Hamiltonian. In that case, U(4) $\supset$ U(3) $\times$ U(1). The states are now characterized by the number of isoscalar p-bosons, N_p and a U(3) symmetry. If we further restric ourselves, by setting $N_p=N$, to the low-lying states of self-conjugate nuclei, then the Hamiltonian can be written in terms of the generators of U(3) alone and analytic solutions to the eigenvalue problem for H can be found as in Racah[6]. The energy expression for the states of the symmetry $[f]=[\lambda_1\lambda_2\lambda_3 0]$ is given as:

$$E=\frac{N(N-1)}{2}a+\frac{b}{2}\{\lambda_1(\lambda_1-1)+\lambda_2(\lambda_2-3)+\lambda_3(\lambda_3-5)\}$$
$$+c\{\frac{1}{2}L(L+1)-N)\} \qquad (1)$$

Preliminary results of the calculations of the spectra (Eq.1) for chosen values of a, b and c for ^{18}F and ^{20}Ne have been communicated elsewhere[7]. Here we take the ground state of ^4He as the vacuum, and add 7 p-bosons and 8 p-bosons to reach the nuclei of interest. The model reproduces roughly well the experimental trends[8] of the energy levels. Out of 44 levels up to 11 MeV in ^{18}F, the model roughly covers 33 of them. The K=1 band built on the observed 1.7-MeV 1+ state is known to be dominated by configurations outside s-d shell, predominantly of a four-particle-two-hole (4p-2h) nature. Within the model this band can be assigned with the permutation symmetry [61]. We also assign certain symmetries for the negative-parity bands in ^{18}F which are again beyond the $(2s,1d)^2$ descriptions. For example, the symmetry [511] has been assigned to the band: 0^-(1.08 MeV), 2^-(2.1 MeV), 4^-(4.4 MeV). The results for ^{20}Ne are also encouraging. We assign the symmetries [44] and [422] to the bands of an eight-particle-four-hole nature. Although, within the model, the band with [422] - symmetry is cut off at 2^+ level, the 10.8-MeV 4^+ state is still reproduced well by the 10.85 MeV 4^+ member of the [44] - band.

It is also interesting to note that the two sets of parameters a, b, and c for the two nuclei are quite close to each other. For ^{18}F, a=0.37 MeV, b=-0.28 MeV, c=0.36 MeV and for ^{20}Ne, a=0.45 MeV, b=-0.33 MeV, c=0.43 MeV.

These results can be further improved by introducing symmetry breaking. A perturbation treatment for the totally symmetric states leads to the expression for the energy correction in the form: $a_1(N-L)(N+L+1)+a_2(N-L)^2(N+L+1)^2 + \ldots$ This should account for the observed softening effects to the axial rotors.

To test the wave functions, we have calculated the B(E2) values within the ground-state band of ^{20}Ne and within the K=1 band of ^{18}F. For ^{20}Ne, it reproduces quite well the experimental trend that B(E2) strengths increase from $2\to0$ to $4\to2$, and then turn around to decrease monotonically. The Bohr-Mottelson geometrical model[9] gives, on the other hand, monotonically increasing B(E2) strengths as one proceeds upward through the $2\to0$, $4\to2$, $6\to4$ and $8\to6$ transitions. It remains to be seen whether similar effects of the cutoff factor due to the finite particle number could be observed for ^{18}F experimentally.

We now turn to the $\hat{R}(4)$ limit of our model, switching our attention to high-lying collective states. We first noted the parallel between the $A^{-1/3}$ or $\frac{1}{R}$ dependence of the excitation energy of giant multipole resonance and $\frac{1}{r_o}$ dependence of the energy of the Hydrogen atom. Since R(4) is the well-known group of the H-atom, the above pa -

rallel seems to hint that GMR might possess R(4) symmetry. Using the group theoretical treatment of H-atom as a guide, the model Hamiltonian that preserves the R(4) symmetry and gives a possible description to GMR is found to be of the following form:

$$H=a\{4\hat{J}_1^2+1\}^{1/2}+b\{4\hat{J}_2^2+1\}^{1/2}+c\hat{L}^2+d\hat{T}^2 \qquad (2)$$

where $\hat{J}_1^2$ and $\hat{J}_2^2$ are the Casmir operators for the product group R(3) × R(3) that makes R(4) and the energy spectrum is given by:

$$E=(2j_1+1)a+(2j_2+1)b+cL(L+1)+dT(T+1) \qquad (3)$$

If we fix the free parameters in (2) to be a=b=19 $A^{-1/3}$ MeV, c=-2.5 $A^{-1/3}$ MeV and d=1.5 $A^{-1/3}$, with the spectrum given by (3) we are able to reproduce some of the known excitation energies[2] of giant multipole resonances. The results are given in Table II together with the classification of these resonance states according to R(4) symmetry.

To verify that the states listed in Table II are indeed those of GMR within the present framework, one needs to consider the multipole transition strengths between the ground state and other R(4) states. The multipole operator is generally taken from the unit tensor operators[4] that form the full set of generators of U(4) group. Now by the seniority consideration, these operators do not connect the ground state $|j_1=0, j_2=0\ L=0\ T\rangle$ and the excited states $|j_1>1, j_2>1, LT'\rangle$.

The seniority of state (j_1, j_2) is given by $v=j_1+j_2$. Therefore the states $|j_1=1, j_2=1, L=0,T\rangle$, $|j_1=1, j_2=1, L=1,T+1\rangle$ and $|j_1=1, j_2=1, L=2,T\rangle$ can be considered as the states of mo - nopole, dipole and quadrupole resonances. As to the isovector quadrupole resonances, one should remember that its connection to the ground state is achieved by the tensor product of the unit tensor operator of rank 2 and that of rank 1. Since this transition operator carries the seniority 4, we expect the GQR state to have $j_1=2$, $j_2=2$, L=2 T+1. This proves to be true from the above energy consideration.

Using the standard group theoretical technique, we have obtained various expressions for the multipole transitions among the R(4) states. There exists one interesting general feature that the states that belong to the yrast band are always very collective. The measure of this collectivity is somehow connected with the magnitude of the Runge-Lenz vector which is now a constant of motion. This should remind us of the fact that the orbits of the Hydrogen atom in the yrast states are circles or with the zero eccentricity.

If we exchange the role of angular momentum and isospin in the Hamiltonian (2), the energy spectrum (3) remains the same except the re-interpretation of j_1 and j_2 as double pseudo-isospins coupled to T. If we further restrict ourselves to the 0^+ states and apply (3) to ^{56}Ni, we find ourselves reproducing exactly the pairing vibrational spectrum[2)] if we identify j_1, j_2 with the number of the removal phonon and that of addition phonon in the Bohr classification scheme[2)]. 2a and 2b are then the one phonon energies. To verify the R(4) symmetry with the pairing vibrational states near ^{56}Ni, we have performed a calculation of ground state intensities $^{48}Ca \rightarrow ^{50}Ti \rightarrow ^{52}Cr \rightarrow ^{54}Fe \rightarrow ^{56}Ni$ assuming that these states are those of 4, 5, 6, 7 and 8 bosons and have R(4) symmetry. The model predicts the above sequence in the ratio 1.2:1.7:1.1:1. The observed ratio[3)] is 1:1.2:1.4:1.

Turning back to GMR, an interesting question would arise: Can we adopt the same interpretation of double pseudo-spins as the number of some kind of removal phonons and that of addition phonons. If yes, it seems possible to interpret the isoscalar giant multipole resonances as the isoscalar pairing vibrations with the phonon energy 38 $A^{-1/3}$ which is nearly the energy spacing between the major shells, $\hbar\omega \sim 41\ A^{-1/3}$. Each of these phonons carries $L^{\pi}=1^{+}$, T=0 and $\alpha=\pm 2$, where α is the nucleon transfer number. If this physical picture for isoscalar GMR is indeed valid, then it would be interesting to identify these one phonon states in the neighbouring odd-odd nuclei. In the case of ^{12}C, for example, one should look for 1^{+} state with the required symmetry and energy in $^{14}_{7}N$ and $^{10}_{5}B$.

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TABLE I

Classification of states in L space for a system of four sp-bosons

U(4)	U(3) $\supset$ R(3)		R(4) $\supset$ R(3)	
[f]	[λ]	L	($j_1 j_2$)	L
[4]	[4]	0,2,4	(22)	0,1,2,3,4
	[3]	1,3	(11)	0,1,2
	[2]	0,2	(00)	0
	[1]	1		
	[0]	0		
[31]	[31]	1,2,3	(21)	1,2,3
	[3]	1,3	(12)	1,2,3
	[21]	1,2	(11)	0,1,2
	[2]	0,2	(10)	1
	[11]	1	(01)	1
	[1]	1		
[22]	[22]	0,2	(20)	2
	[21]	1,2	(02)	2
	[2]	0,2	(11)	0,1,2
			(00)	0
[211]	[211]	1	(20)	0,1,2
	[21]	1,2	(10)	1
	[111]	0	(01)	1
	[11]	1		
[1111]	[111]	0	(00)	0

TABLE II

Excitation energies of the giant multipole resonances (in $A^{-1/3}$MeV)

Multipolarity	T	E_{exp} a)	E_{th} b)	R(4) Classification c) $(j_1j_2)L;\ T$
2	1	130	140	(22)2; $T'+1$
2	0	64.7	61	(11)2; T'
1	1	78	74	(11)1; $T'+1$
3	0	116	122	(22)3; T'
0	0	80	76	(11)0; T'

a) See Ref. 2

b) According to Eq.(3) and the fixed parameters shown in the text

c) The R(4) classification for the ground state is (00)0;T'. All the states are totally symmetric in L or T, namely, with the shape [N]