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by

I. Kimel
Instituto de Física, Universidade de São Paulo
São Paulo, Brasil

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

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ABSTRACT

The proton-neutron mass difference is calculated in a Born-like approximation but taking into account off-mass-shell variations of the form factors. In this way, the right sign and, possibly, the right magnitude can be obtained.

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"The proton-neutron mass difference, probably the oldest puzzle in hadron physics, had challenged and frustrated generations of theorists. The solution is nowhere in sight". These are the opening sentences of the review by A. Zee[1] on the proton-neutron mass difference (Δ=Mp-Mn) problem. I have borrowed Zee's words to set the stage for the present work where a new attempt is made at solving the puzzle.

Since the time people started worrying about Δ (which is experimentally - 1.29 MeV) it was clear that the Coulomb interaction would make it positive. A possible way out was suggested by Feynman and Speisman (FS)[2] who pointed out that by taking the negative anomalous magnetic contribution into account, the right sign might emerge if the integrals are cut off at a suitable energy. Then, Cini, Ferrari and Gatto (CFG)[3] showed that the nucleon form factors made the artificial cut-offs of FS unnecessary, but, at the same time, lead to a Δ with the wrong sign.

All the above was done in a purely electromagnetic Born approximation. With the failures, however, other types of ideas started to come in: feedback mechanisms, tadpoles, Regge poles, fixed poles, relation with electroproduction (via Cottingham[4] formula) and other approaches described in Zee's review. More recent attempts invoke the unified electroweak (Salam-Weinberg-Glashow) theory. In most of these approaches the tendency is to shift the responsability for producing a negative Δ from the low to the high energy region.

In the present work I go back to FS and CFG with two differences in relation to the latter paper. First, I use for the nucleon electromagnetic current a parametrization naturally suited for the Sachs form factors which are taken to be of the usual dipole type when both nucleon legs are on-mass-shell. Second, and more important, the form factors are allowed to vary when one of the nucleons is off-mass-shell. With a plausible ansatz as to what the off-mass-shell behaviour of the form factors is, and after some tedious calculation, a Δ with the right sign and, possibly, with the right magnitude is obtained.

Let me start with the nucleon electromagnetic current which I chose to write as

$$(p_o p_o'/M) \; \langle p'|J_\mu(0)|p\rangle = \bar{u}(p')\Gamma_\mu(k)u(p)$$

$$= e\left(1-k^2/4M^2\right)^{-1}\bar{u}(p')\left[M^{-1}G_E(k^2)P_\mu - (2M)^{-2}G_M(k^2)(\gamma_\mu \not{P}\not{k} - \not{k}\not{P}\gamma_\mu)\right]u(p). \quad (1)$$

where $P_\mu = \frac{1}{2}(p_\mu + p_\mu')$, $k_\mu = p_\mu - p_\mu'$ and G_E and G_M are the electric and magnetic Sachs form factors. For the metric etc., I use the notation of Bjorken and Drell[5].

The parametrization (1) for the current, which I will use in the following, is, of course, not new[6] although it had not been much employed. One of its advantages is that it is specially suited for the Sachs form factors. Another, is that it provides convenient convergence factors for integral that will appear later.

In lowest order, the electromagnetic self-mass of a fermion can be written in terms of the forward Compton scattering amplitude with an off-mass-shell photon[3] $\left[T_{\mu\nu}(p,k)\right]$ as

$$\delta M = e^2 \int \frac{d^4k}{(2\pi)^4} \frac{(g^{\mu\nu} - k^\mu k^\nu/k^2)}{k^2 + i\epsilon} T_{\mu\nu}(p,k) \quad , \qquad (2)$$

with

$$\frac{M^2}{p_o}\,\bar{u}(p)T_{\mu\nu}(p,k)u(p) \quad =$$

$$= \int \frac{d^4x}{(2\pi)^3} \; e^{-ikx} \langle p|T\{J_\mu(x), J_\nu(0)\}|p\rangle \quad . \qquad (3)$$

In a standard fashion, we can arrive at a Low equation for

$T_{\mu\nu}$ which, in the Born approximation, is of the form

$$\bar{u}(p)T^{B}_{\mu\nu}u(p) = i\sum_{s'}\int \frac{d^3p'\,\delta^3(p'-p-k)\langle p|J_\mu(0)|p',s'\rangle\langle p',s'|J_\nu(0)|p\rangle\theta(k_o)}{p'_o-p_o-k_o+i\epsilon}$$

$$+ \begin{bmatrix} \mu \longleftrightarrow \nu \\ k \longrightarrow -k \end{bmatrix} \tag{4}$$

where p' is the momentum and s' the spin of the intermediate nucleon.

Using parametrization (1) for the matrix elements of the currents appearing in Eq. (4) and putting the $T_{\mu\nu}$ thus obtained in Eq. (2) leads us, after some γ matrices gymnastics, to the formula

$$\delta M^N = -i8e^2 \left\{ \int \frac{d^4k}{(2\pi)^4} \frac{\left[2M^2\,k^{-2}\,{}^N G_E^2(k^2) + {}^N G_M^2(k^2)\right]\left[M^2k^2-(pk)^2\right](2M-\not{k})}{(k^2+i\epsilon)(k^2-2pk+i\epsilon)(k^2-4M^2)^2} \right\}_{\not{p}=M}, \tag{5}$$

where N can be either p (proton) or n (neutron). We would, of course, have arrived at this same expression, had we written the formula for the lowest order Feynman diagram for the electromagnetic mass of an elmentary fermion and then changed the electromagnetic vertex according to

$$\bar{u}\,\gamma_\mu\,u \longrightarrow \bar{u}\,\Gamma_\mu\,u \quad , \tag{6}$$

with Γ_μ as in Eq. (1).

One disturbing feature of Eq.(5) is the double pole at the pair production threshold $k^2=4M^2$. The factor $\left[M^2k^2-(pk)^2\right]$ has a simple zero at that threshold which is not enough to kill the double pole. In order to properly assess what happens at $k^2=4M^2$ one should go beyond the Born approximation and take into account intermediate states with one nucleon plus one pair also. In this paper I will assume that the nucleon form factors provide sufficient damping to guarantee that whatever happens at $k^2\sim4M^2$ has a negligible effect on the final result. The integrals in Eq.(5) can be performed once a suitable $i\epsilon$ term is added to the (k^2-4M^2) factor.

G_E and G_M appearing in Eq.(5) will be taken to be of the standard dipole form

$$\frac{{}^{n}G_M(k^2)}{\mu_n} = \frac{{}^{p}G_E(k^2)}{\mu_p} = {}^{p}G_E(k^2) = G_D(k^2) = \left[(k^2/\beta M^2) - 1\right]^{-2} \quad , \tag{7}$$

where μ_p and μ_n are the proton and neutron magnetic moments and the mass $m^2 = \beta M^2$ has to be taken from experiment.

I calculated the proton and neutron mass shifts from Eq.(5) with the form factors (7) and obtained, as everybody else, the wrong answer: proton heavier than neutron. So, the question is: the simple Born approximation is not sufficient; how can we get something better?

In dispersion relation language, the higher intermediate states contribute to a sidewise dispersion relation of the form factors[7] (dispersion in the mass of one of the nucleon legs). In Feynman diagram language, the intermediate nucleon is off-mass-shell with an effective mass

$$p'^2 = W^2 = M^2 + k^2 - 2pk \quad . \tag{8}$$

In either way of looking at the problem the conclusion is the same: the form factors should be taken with one nucleon leg off-mass-shell, i.e., $G_D(k^2, W^2)$.

It would be interesting to study the off-mass-shell form factors in a sidewise dispersion in order to see their connection with the electroproduction amplitudes. In the meantime I propose the following ansatz.

$$G_D(k^2, W^2) = \left[(k^2/\beta W^2) - 1\right]^{-2} = \left[\frac{k^2}{\beta(M^2 + k^2 - 2pk)} - 1\right]^{-2} \quad . \tag{9}$$

Notice that in the Bjorken limit $(-k^2) \to \infty$, $(-pk) = \nu \to \infty$, with $x = (k^2/2pk)$, G_D scales as

$$G_D \to (1-x)^2 \left[1 + x(\beta^{-1} - 1)\right]^{-1} \quad . \tag{10}$$

Substituting in Eq.(5) the on-mass-shell form factors by form factors varying like (9) the resulting mass shift can be split according to

$$\delta M^N = \delta M^N_c + \delta M^N_V \quad , \tag{11}$$

where δM^N_c is the mass shift one would get from Eq.(5) if the form factors were constant, namely

$$M^N_c = -i8e^2 \int \frac{d^4k}{(2\pi)^4} \frac{\left[2M^2k^{-2}\, q_N^2 + \mu_N^2\right]\left[M^2k^2 - (pk)^2\right]}{k^2(k^2-2pk)(k^2-4M^2)^2} \quad , \tag{12}$$

where q_N is the nucleon charge in units of e. δM^N_V results from the variation of the form factors and is given by

$$\delta M^N_V = -i\,\frac{4}{3}\,e^2\,\frac{\partial^3}{\partial\beta^3}\,\frac{1}{\beta(1-\beta)}\int\frac{d^4k}{(2\pi)^4}\,\frac{\left[2M^2k^{-2}q_N^2+\mu_N^2\right]\left[M^2k^2-(pk)^2\right](2M-\not{k})}{(k^2-4M^2)^2(k^2-2pk)\left[k^2-\frac{\beta}{1-\beta}(M^2-2pk)\right]} \,. \tag{13}$$

δM^N_c can be obtained without much difficulty; it is

$$\delta M^N_c = \frac{\alpha M}{8\pi}\left[q_N^2(8\ell n\, 2+1) - \mu_N^2(7-4\,\ell n\, 2)\right] \quad . \tag{14}$$

This part alone contributes to the p-n mass difference with

$$\Delta_c = \delta M^p_c - \delta M^n_c = \frac{\alpha M}{8\pi}\left[8\,\ell n\, 2+1 - (\mu_p^2-\mu_n^2)(7-4\,\ell n\, 2)\right]$$
$$= -\ 2{,}99 \text{ MeV} \tag{15}$$

Unfortunately, δM^N_V is not that simple. It has the form

$$\delta M^N_V = -\,\frac{\alpha M}{2\pi}\,\beta^4\,\frac{\partial^3}{\partial\beta^3}\left[(\mu_N^2+\frac{1}{2}q_N^2)\,A - \frac{1}{2}\,q^2\,B\right] \quad , \tag{16}$$

where

$$A = \frac{1}{(4-\beta)}\left[\frac{1}{6} + \frac{\beta(\ell n\beta-1)}{12} + \frac{(7/9)+(4/3)\,\ell n 2}{\beta} + \frac{(1/9)-(1/8)(\beta-2\,\ell n\,\beta)}{(4-\beta)}\right.$$
$$\left. - \frac{8\,\ell n\, 2-(14/9)}{\beta(4-\beta)}\right] + \frac{1}{12}\int_0^1 dx\ \ell n\left[x^2+\beta(1-x)\right] \quad , \tag{17}$$

and

$$B = \frac{167}{288\beta} + \frac{2\,\ell n\, 2-(167/288)}{\beta(4-\beta)} + \frac{2-\ell n\,\beta}{(4-\beta)} - \frac{\ell n\,\beta}{12} - \frac{(4-\beta)\,\ell n\beta}{24} - \frac{(4-\beta)}{24}\int_0^1 \frac{dx}{x^2+\beta(1-x)} \,. \tag{18}$$

After taking the derivatives, I calculated everything analytically except for integrals like the one in Eq.(18) which were done numerically in a desk calculator. The result depends on $\beta=(m^2/M^2)$ in Eq.(7). Data up to about $(-k^2)\sim 5$ GeV^2 used to give $m^2=0.71$ GeV^2 [8] corresponding to a $\beta\approx 0.8$ which yields

$$\Delta_V = 2.81 \text{ MeV} \quad \rightarrow \quad \Delta = \Delta_C + \Delta_V = -0.18 \text{ MeV} \quad , \tag{19}$$

right sign but wrong magnitude. If one tries to fit the higher energy data with dipole-type form factors one needs $m^2=0.63$ GeV^2 with $\beta\approx 0.7$ leading to

$$\Delta_V = 1.68 \text{ MeV} \quad \rightarrow \quad \Delta = -1.31 \text{ MeV} \quad . \tag{20}$$

According to the point of view I have taken in this work, Δ is a low energy effect. To be coherent, the lower energy data for the form factors would have to be prefered over the higher energy data, which leaves us with the worse of the two values in (19) and (20). The off-shell form factors are not that complicated but lead to energy shifts which depend on a sensitive way on the parameter β.

In spite of the shortcomings just mentioned, I believe that by taking their off-mass-shell variations into account, the form factors cut off the integrals less abruptly allowing for the negative magnetic energy to overtake the positive Coulomb energy. The net result is a negative proton-neutron mass difference. If that is correct, the oldest puzzle in hadron physics would have ceased to be such.

REFERENCES

1. A. Zee, Phys. Lett. 3C (1972) 127.
2. R.P. Feynman and G. Speisman, Phys.Rev. 94 (1954) 500.
3. M.Cini, E.Ferrari and R.Gatto, Phys.Rev.Letters 2 (1959) 7.
4. W.N. Cottingham, Ann.Phys. (N.Y.) 25 (1963) 424.
5. J.D.Bjorken and D.Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).
6. P.Urban, Topics in Applied Quantum Electrodynamics (Springer,Wien,1970).
7. A.M. Bincer, Phys.Rev. 118 (1960) 855.
8. D.H. Coward et al., Phys.Rev.Letters 20 (1968) 292.
9. R.E.Taylor in Proceedings of the 1975 Symposium on Lepton and Photon Interactions at High Energy.