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COMPONENTS OF THE CHARGE-EXCHANGE VECTOR  
DIPOLE RESONANCE

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ABSTRACT

Recent experimental data for the energy splitting between the  $T = T_0$  and  $T = T_0 - 1$  components of the charge-exchange-vector-dipole resonance are discussed within the framework of a simple model.

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Sponsored by Fundo de Financiamento de Estudos de Projetos e Programas de Desenvolvimento (FINEP), Brasil.

November/1980

Charge-exchange reactions, such as  $(p,n)$ ,  $({}^3\text{He},t)$ ,  $(\pi^+, \pi^0)$ , etc., and the corresponding charge conjugate processes, are very interesting for various reasons. In the first place, they provide an excellent tool to probe the nucleus with respect to the isospin dependent residual interactions. Secondly, from the theoretical point of view, they have many features in common with the  $\beta$ -decay processes and the charge-exchange  $\gamma$ -decays. Thirdly, there is a great variety of charge-exchange vibrational fields which can be experimentally investigated in many nuclei. Fourthly, the nuclear photoresonances and the giant magnetic resonances can be seen in a wider perspective when they are recognized as a single component of an isovector mode, whose additional components represent the charge-exchange modes. Finally, they are clearly ultimately linked with many other aspects of isobaric symmetry in the nucleus and an understanding of the charge-exchange mechanism cannot but help to advance knowledge of the nuclear structure in general.

The isovector excitations are particularly rich in information when the isospin of the target  $T_0$  is non-zero ( $N > Z$ ). In such a case, a particle-hole operator may give rise to states with isospins  $T = T_0 + 1$ ,  $T = T_0$ ,  $T_0 + 1$  and  $T = T_0 - 1$ ,  $T_0$ ,  $T_0 + 1$  for  $\mu_\tau = 1$ ,  $\mu_\tau = 0$  and  $\mu_\tau = -1$ , respectively<sup>1-3</sup>. Furthermore, the Pauli principle causes a reduction in the number of proton hole-neutron particle excitations and a simultaneous increase of excitations of the type neutron hole-proton particle. As a consequence, both the energies and the transition strengths depend on the orientation of the corresponding states in isospace.

The charge-exchange collective fields can be classified in the same way as the corresponding inverse  $\beta$  transitions, i.e., as allowed, first forbidden, second forbidden, etc.. Furthermore, the allowed excitations ( $\kappa = 0$ ) may be of Fermi-type ( $\sigma = \lambda = 0$ ) and of Gamow-Teller type ( $\sigma = \lambda = 1$ ). There are four first-forbidden collective fields; one being of vector or non-spin-flip type ( $\sigma = 0$ ,  $\lambda = 1$ )

and three of axial-vector or spin-flip type ( $\sigma = 1$ ,  $\lambda = 0, 1, 2$ )<sup>2</sup>.

The Fermi resonance is known since the discovery of the well-defined isobaric multiplet structure by Anderson and Wong<sup>4</sup>. The gathering of the Gamow-Teller (GT) strength, in a nucleus with  $N > Z$ , was observed for the first time by Doering et al.<sup>5</sup> through the  $^{90}\text{Zr}(p,n)^{90}\text{Nb}$  reaction at incident proton energies of 35 and 45 MeV. This GT resonance was later confirmed by a subsequent  $^{90}\text{Zr}(^3\text{He},t)^{90}\text{Nb}$  experiment at 130 MeV in Jülich<sup>6</sup> and other one at 80 MeV in Grenoble<sup>7</sup>. More recently<sup>8</sup>, in a new study at  $E_p = 120$  MeV of the  $^{90}\text{Zr}(p,n)^{90}\text{Nb}$  reaction three resonances were found. They were interpreted as the  $T=4$  and  $T=5$  components of the GT strength and the  $T=4$  component of the giant vector dipole (GVD) state.

Quite recently, Sterreburg et al.<sup>9</sup> have measured neutron spectra resulting from bombardment with 45 MeV protons at small angle on 17 targets from  $^{90}\text{Zr}$  to  $^{208}\text{Pb}$ . Common features to the spectra were:

- i) a sharp peak representing the isobaric analog state or the Fermi resonance,
- ii) a broad peak (FWHM  $\approx 3-4$  MeV), interpreted as a GT state with isospin  $T = T_0 - 1$ , at a slightly higher but target dependent excitation energy, and
- iii) a similarly broad peak  $\sim 10$  MeV higher than the first bump, which was suggested to be the  $T = T_0 - 1$  component of the GVD resonance.

Based on the foregoing assumption, Sterrenburg et al.<sup>9</sup> were able to infer, from their experimental data, the energy splittings

$$D(T = T_0 - 1) = e(T = T_0, \mu_T = -1) - e(T = T_0 - 1, \mu_T = -1) \quad (1)$$

between the  $T=T_0$  and  $T=T_0-1$  components of the GVD states. The first energy was obtained by Coulomb displacement of the giant electric dipole (GED) resonance, while the mean energy of the observed neutron group above the GT was taken for the second. The resulting experimental values for  $D(T=T_0-1)$  are given in Table I.

Instead of discussing the experimental results individually we will compare them with simple, but rather general, theoretical estimates which were obtained in Ref.<sup>10</sup> at the expense of using a very schematic force of the form (see also Ref.<sup>11</sup>)

$$H = -\frac{1}{2} \chi \sum_{\mu_T} M^+(\tau=1, \mu_T) M(\tau=1, \mu_T) \quad , \quad (2)$$

where

$$M(\tau=1, \mu_T) = \sum_{i=1}^A r_i \vec{Y}_1(i)_{\tau \mu_T}(i) \quad , \quad (3)$$

and

$$\chi = \frac{\pi V_1}{A \langle r^2 \rangle} \quad , \quad (\langle r^2 \rangle = \frac{3}{5} (1.2A^{1/3})^2 \text{ fm}^2) \quad , \quad (4)$$

and  $V_1 = 130$  MeV is the symmetry potential. Furthermore, a degenerate model for the single-particle energies is assumed ( $\epsilon \equiv \hbar \omega_0 = 41 A^{-1/3}$  MeV) and the radial wave functions are approximated by those of an harmonic oscillator. The resulting expressions for the energy splitting  $D(T=T_0-1)$  are:

i) in the Tamm-Dancoff approximation (TDA)

$$D(T=T_0-1) = U \equiv U_0 \left( 1 - \frac{\langle r^2 \rangle_{n.\text{exc.}}}{2 \langle r^2 \rangle} \right) \quad (5)$$

where  $\langle r^2 \rangle_{n.exc.}$  is the mean square radius in the neutron excess region;

ii) in the Random Phase Approximation (RPA)

$$D(T=T_0-1) = U - \frac{2T_0 + 3}{2T_0 - 1} (K - K_0) \quad (6)$$

where

$$K_0 = \left[ (\epsilon + \chi S^{(0)}(\mu_T=0))^2 - (\chi S^{(0)}(\mu_T=0))^2 \right]^{1/2} \quad (7)$$

and

$$K = \left[ (\epsilon + \chi S^{(0)}(\mu_T=0))^2 - (1-\nu^2) (\chi S^{(0)}(\mu_T=0))^2 \right]^{1/2} \quad (8)$$

with

$$S^{(0)}(\mu_T=0) = \frac{3}{8\pi} A^{4/3} \text{ fm}^2, \quad (9)$$

being the total unperturbed transition strength for the giant electric dipole (GED) resonance.

The quantity

$$\nu = \frac{4T_0 \langle r^2 \rangle_{n.exc.}}{3 A^{4/3}} \quad (10)$$

expresses the neutron excess in units of the number of particles in a mayor shell of the harmonic oscillator. Explicitly,  $\nu = 22T_0/3A$  for all the nuclei considered here, except for the  $^{208}\text{Pb}$  for which  $\nu = 25/26$ .

When one does not work with a good isospin basis the effects of relative order  $T_0^{-1}$  are ignored and one has

$$D(T=T_0-1) = U - (K - K_0) \quad . \quad (11)$$

For the sake of completeness let us mention the theoretical results for the isospin splitting

$$D(T=T_0+1) = e(T=T_0+1, \mu_T=0) - e(T=T_0, \mu_T=0) \quad , \quad (12)$$

in the framework of the same approximations which were employed in obtaining the Eqs. (5), (6) and (11). The corresponding relations are

$$D(T=T_0+1) = \frac{T_0 + 1}{T_0} U \quad , \quad (5')$$

$$D(T=T_0+1) = \frac{T_0 + 1}{T_0} (U + K - K_0) \quad (6')$$

$$D(T=T_0+1) = U + (K - K_0) \quad (11')$$

In general the measured excitation energy of a giant resonance is that of a superposition of states with the same spin and parity but different isospins ( $T_0$  and  $T_0+1$  when  $\mu_T=0$  and  $T_0-1, T_0$  and  $T_0+1$  when  $\mu_T=1$ ). Then one might argue that it is really not necessary for the particle-hole excitations to be exact eigenstates of  $T$ . This would imply the direct use of expressions (11) and (11') instead of Eqs. (6) and (6') in comparing the experimental data with the theory. Clearly, such a procedure is not valid because it mixes up dynamical aspects of the process with kinematical ones

The effects of the ground state correlations on the excitation energies of the charge-exchange and charge conserving dipole states are measured by the last terms on the r.h.s. of Eqs. (7) and (8), respectively. The corresponding fractional changes in replacing the TDA by the RPA are of the order of  $(\chi S^{(0)}(\mu_T=0)/\epsilon')^2$  and  $(1-v^2)(\chi S^{(0)}(\mu_T=0)/\epsilon')^2$  where  $\epsilon' = \epsilon + \chi S^{(0)}(\mu_T=0)$ . This shows that both GVD states are less affected by the ground state correlations than the GED resonance in the ratio  $(1-v^2)$ . The origin of the term  $(1-v^2)$  becomes evident remembering that: i) the energy shift caused by the backward going graphs is proportional both to the collectivity of the state and to the amount of ground state correlations and ii) the collectivity of the state  $\mu_T = \pm 1$  is measured by the strength  $(1 \mp v) S^{(0)}(\mu_T=0)$  and its correlations by the quantity  $(1 \pm v) S^{(0)}(\mu_T=0)$ . This means, for example, that for  $^{208}\text{Pb}$ , where  $v \approx 1$ , the ground state correlations of the first forbidden  $\mu_T = -1$  modes (which are of  $\mu_T = +1$  type) die almost completely and we fall from RPA to TDA, while the corresponding modes with  $\mu_T = +1$  disappear themselves. It is now clear that the factor  $(K - K_0) > 0$  appearing in the expressions (6), (6') and (11') has its origin in the difference between charge-exchange and charge conserving correlations. This difference depends mainly on the neutron excess.

From the comparison of the experimental data for the splitting  $D(T=T_0-1)$  with the theory, shown in Table I, one sees that, while the estimate (5) based on the TDA reproduces reasonably well the experimental values, the RPA results calculated by means of Eq. (6) are systematically too small. One is then led to the conclusion that either the effects of the ground state correlations are overestimated in the present work or/and we are still missing some important physics.

It should be noted however, that the splittings  $D(T=T_0-1)$  were not measured directly but obtained as small differences of relatively large excitation energies of the GED and GVD



resonances, which were extracted from different experiments. Noting in addition, that i) both resonances are rather broad and ii) in the vicinity of the GVD resonance lie also the remaining three first forbidden axial-vector states<sup>2</sup>, it may be a little chance that the experimental values for the splittings were overestimated. There is evidently considerable scope for more experimental work and theoretical analysis of reactions leading to charge-exchange collective states.

Finally it should be also mentioned that, in the context of the approximations employed here, it is essential to include the effects of the ground state correlations in order to account for the experimental data for the isospin splittings  $D(T = T_0 + 1)$ <sup>12</sup>.

## REFERENCES

- 1 The notation is the same as in Ref.2. The quantum numbers  $\kappa, \sigma, \lambda, \tau$  and  $\mu_\tau$  stand for the orbital angular momentum, the spin, the total angular momentum ( $\vec{\lambda} = \vec{\kappa} + \vec{\sigma}$ ), the isospin and the third component of the isospin, respectively. Through this paper the unnecessary quantum numbers will be always omitted.
- 2 F. Krmpotiċ, K. Ebert and W. Wild, Nucl. Phys. A342, 497 (1980).
- 3 A. Bohr and B.R. Mottelson, Nuclear Structure (Benjamin, N.Y., 1975), Vol. II.
- 4 J.D. Anderson and C. Wong, Phys. Rev. Lett. 7, 250 (1961).
- 5 R.R. Doering, A. Galonsky, D.M. Patterson and G.F. Bertsch, Phys. Rev. Lett. 35 1961 (1975).
- 6 A. Galonsky, J.P. Didelez, A. Djalseis and W. Oelert, Phys. Lett. 74B 176 (1978).
- 7 D. Ovazza, A. Willis, M. Molert, N. Marty. P. Martin, P. de Saintignon and M. Buenerd, Phys. Rev. C19, 2438 (1978).
- 8 D.E. Bainum, J. Rapaport, C.D. Goodman, D.J. Horen, C.C. Foster, M.B. Greendfield and C.A. Gouding, Phys. Rev. Lett. 44, 1751 (1980).
- 9 W. Sterrenburg, S. Austin, R. De Vito and A. Galonsky, Proceedings of the International Conference on Nuclear Physics, Berkeley, California (1980), pag. 175.
- 10 F. Krmpotiċ, preprint IFUSP/P-225, Universidade de São Paulo, 1980 and Nucl. Phys., to be published.
- 11 F. Krmpotiċ and F. Osterfeld, Phys. Lett. 93B, 218 (1980).
- 12 F. Krmpotiċ, to be published.

TABLE I - Comparison of the experimental data for the isospin splitting  $D(T=T_0-1)$  with the theoretical estimates given by Eqs. (5) and (6) which correspond to the Tamm-Dancoff and Random Phase Approximation, respectively.

NUCLEUS	THEORY		EXPERIMENT
	TDA	RPA	
$^{90}_{\text{Zr}}$	2.1	1.1	2.2
$^{91}_{\text{Zr}}$	2.3	1.1	2.0
$^{92}_{\text{Zr}}$	2.5	1.2	1.7
$^{94}_{\text{Zr}}$	2.9	1.2	2.0
$^{96}_{\text{Zr}}$	3.3	1.3	2.1
$^{93}_{\text{Nb}}$	2.3	1.2	2.0
$^{94}_{\text{Mo}}$	2.1	1.1	1.7
$^{96}_{\text{Mo}}$	2.5	1.3	2.5
$^{97}_{\text{Mo}}$	2.7	1.3	2.2
$^{98}_{\text{Mo}}$	2.9	1.4	2.4
$^{100}_{\text{Mo}}$	3.3	1.5	-
$^{112}_{\text{Sn}}$	2.4	1.5	2.7
$^{116}_{\text{Sn}}$	3.1	1.8	2.5
$^{120}_{\text{Sn}}$	3.8	2.1	3.6
$^{122}_{\text{Sn}}$	4.2	2.2	4.3
$^{124}_{\text{Sn}}$	4.6	2.3	4.6
$^{208}_{\text{Pb}}$	4.6	1.5	4.5