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S-MATRIX DESCRIPTION OF ANOMALOUS LARGE-ANGLE HEAVY-ION SCATTERING+

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ABSTRACT: We present a quantitative description of the well-known anomalous features observed in the large-angle scattering of n.α type heavy ions, in particular of the pronounced structures in the backangle excitation function for $^{16}O + {}^{28}Si$. Our treatment is based on the close connection between these anomalies and particular structural deviations of the partial-wave S-matrix from normal strong-absorption behaviour. The properties of these deviations are found to be rather well specified by the data: they are localized within a narrow "ℓ-window" centered at a critical angular momentum significantly smaller than the grazing value, and have a parity-dependent as well as a parity-independent part. These properties provide important clues as to the physical processes causing the large-angle enhancement.

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1. INTRODUCTION

The unusual structures observed in the large-angle scattering of certain heavy-ion systems have been the subject of intensive study and have attracted lively interest in recent years. Following the discovery[1)] of a strong, oscillatory enhancement of the angular distribution at large angles, and of pronounced regular structures in the backward-angle excitation function[2)] for the elastic and inelastic scattering of $^{16}O + ^{28}Si$ in the c.m. energy range E = 19-37 MeV, similar phenomena were found in many other heavy-ion systems: $^{12}C + ^{28}Si$, $^{12}C + ^{32}S$, $^{16}O + ^{24}Mg$, $^{12}C + ^{40}Ca$, $^{16}O + ^{40}Ca$. (For references see the recent review by Barrette and Kahana[3)]). It seems most significant that in all of these systems both the projectile and the target nucleus are of n.α type (i.e., they may be regarded as "consisting of" an integer number of α particles); the anomalous large-angle scattering (ALAS) disappears or is greatly reduced if one or both reaction partners are not of n.α type (see ref.[3)]).

Anomalous large-angle scattering with enhanced, oscillatory angular distributions had been observed earlier in the scattering of α particles by n.α targets (notably ^{40}Ca) and of lighter heavy ions, and it is an intriguing question whether or not all of these phenomena have the same or a similar dynamical origin.

Numerous attempts have been made to give a theoretical description and find a physical interpretation of the new structures (see the literature quoted in refs.[3,4)]), since it soon became clear that conventional theories appropriate for "normal" heavy-ion scattering fail to reproduce the data. The basic problem of what are the new features of the heavy-ion interaction responsible for the anomalous scattering, already brought into focus by Barrette and Kahana[3], can be stated as follows: are the new structures due to resonances or "resonance-like" processes associated with the specific composition of individual nuclei, or do they arise from a "universal" feature of the heavy-ion interaction which only manifests itself in a particularly pronounced way in n.α type nuclei?

We shall argue that there is overwhelming evidence for the second alternative, at least as far as the gross structure properties of ALAS in heavy ions are concerned.

With few exceptions[5,6], most theoretical attempts to account for ALAS as a "universal" feature of the average heavy-ion interaction are based on modifications of the optical potential: addition of a "Regge pole", surface transparency, ℓ-dependence (in particular parity-dependence) and energy-dependence, inclusion of elastic transfer, etc. Some of these modifications are based on dynamical or microscopic considerations, others are purely phenomenological. Although a few of these attempts have been successful in giving good fits to the data (see, especially, ref.[7]), no satisfactory description has as yet been given which reproduces all the experimental features on the basis of a convincing dynamical model.

In this situation, one of us[4] has proposed a different approach, based on investigating in general those properties of the elastic partial-wave S-matrix $S_\ell(E)$ that are capable of producing enhancement of the large-angle scattering cross section. Since normal heavy-ion scattering, with angular distributions falling rapidly to small values towards larger angles, is characterized by S-matrices with a "normal strong-absorption profile"[4], any backward-angle enhancement must be due to deviations from the normal structure of $S_\ell(E)$. This suggests an "inductive" method based on the hope that the anomalous features of the large-angle angular distributions, and especially of the backward-angle excitation functions, are related specifically enough to the corresponding structure of the S-matrix as to determine the angular momentum and energy dependence of its "deviating" part $\tilde{S}_\ell(E)$. This hope, in turn, is grounded on the much more direct connection between cross section and S-matrix than between cross section and potential because of the well-known ambiguities of potential parameters for heavy-ion scattering. Moreover, our approach employs closed-form expressions for the angular distributions and excitation functions[4], allowing us to study the relation between cross section and S-matrix in analytic detail.

If $\tilde{S}_\ell(E)$ can be successfully determined in this way, the final stage of our program would be to identify the dynamical origin of the large-angle enhancement, by constructing a model of the physical mechanism which generates a contribution to the total S-matrix having all the characteristic properties of $\tilde{S}_\ell(E)$ that are demanded by the experimental data.

In the present paper we show that the first stage of our "minimum assumption" approach is successful. From an analysis of the dominant features of the large-angle cross sections and the backward-angle excitation function we first determine the essential properties of $\tilde{S}_\ell(E)$, and then show that this together with a "background" S-matrix $\bar{S}_\ell(E)$ of normal strong-absorption profile gives an adequate description of the full angular distributions and the total 180° excitation function.

In this paper we do not address the final-stage problem of identifying a specific physical mechanism for the ALAS of heavy ions. Here we use simple phenomenological forms of the function $\tilde{S}_\ell(E)$ (as well as of $\bar{S}_\ell(E)$), but show that the salient features are independent of the detailed analytic form of its ℓ- and E-dependence. However, our analysis provides significant and highly suggestive clues as to the nature of the underlying dynamic process which will be taken up in a subsequent investigation. At the least our results indicate that a number of physical mechanisms currently under discussion are ruled out or are highly improbable.

We confine ourselves here to the first and best-known example of ALAS for heavy ions, the system $^{16}O + ^{28}Si$ [1,2]. Because of our conviction that we are dealing with a "universal" phenomenon, we suggest that very similar conclusions apply to other systems.

2. EXPERIMENTAL FEATURES

Before formulating our basic assumptions, let us recall and emphasize the essential features of the experimental data.

(i) The angular distribution $\sigma(\theta)/\sigma_R(\theta)$ (at a particular energy, E = 35 MeV, say) shows, at forward angles, the familiar damped Fresnel pattern characteristic of normal heavy-ion scattering. This is followed by a relatively smooth fall-off which, beyond the grazing angle θ_R, becomes more and more oscillatory. At intermediate angles ($\theta \approx 90$-150°) the cross section levels off with irregular structure, but at larger angles shows regular oscillations of increasing amplitude, and at 180° has a pronounced maximum of about one-hundreth of the Rutherford value. The period of these large-angle oscillations is determined by a critical angular momentum ℓ_c "close to" the grazing angular momentum ℓ_g. The significant features are the backward-angle enhancement and the irregular structure at intermediate angles. "Normal" strong-absorption scattering of heavy ions of similar mass would show a continued steep fall-off beyond θ_R, but the appearance of regular oscillations at backward angles, of the form $P_{\ell_c}(\cos\theta) \cong J_o\left[(\ell_c+\frac{1}{2})(\pi-\theta)\right]$, though normally of small amplitude, can be shown to be a universal (diffractive) phenomenon[4)] arising from the interference between the "near-side" and "far-side" branches of the scattering amplitude. Therefore, much more sensitive to the specific nature of the enhancement mechanism is the structure of

(ii) the backward-angle excitation function $\sigma(\pi)/\sigma_R(\pi)$ as a function of energy E. (Experimentally this is an average over a small angle interval of typically 5^o near 180^o). This shows a pronounced gross structure in the range E = 19-37 MeV, which at the higher energies becomes fairly regular, with oscillations of gradually increasing "period" and gradually decreasing amplitude. (These oscillations, which we shall call "E-oscillations", are often referred to as "resonance-like", yet the very nature of this gross structure is the cardinal point in question! In some systems (e.g. $^{12}C + ^{28}Si$) there is an additional fine structure in the excitation function, but this is not the subject of our present investigation.) The average cross section ratio decreases slowly with increasing energy indicating that the backangle enhancement phenomenon tends to disappear at high energies. Again this behaviour is "anomalous" mainly because of the magnitude of the effect: for "normal" backangle scattering of heavy ions the excitation function drops sharply with increasing energy to very small values, but the presence of small-amplitude E-oscillations with periods determined by the energy dependence of the grazing angular momentum $\ell_g(E)$ had been predicted long ago[8)]. Such weaker oscillatory behaviour has indeed been observed for systems in which one reaction partner is not of n.α type, e.g., $^{16}O + ^{29}Si$, $^{16}O + ^{30}Si$, $^{13}C + ^{28}Si$ and $^{18}O + ^{28}Si$ (see ref.[3]). Unlike the oscillations in the angular distribution, these E-oscillations are not of diffractive origin but arise from the interference between the leading terms (mostly $m = -1$ and $m = 0$) of the Poisson series representation of the backward-scattering amplitude[4]. Their omnipresence, albeit with widely different amplitudes, is the most significant indication of the universal nature of the gross structure in backangle excitation functions.

However, a very important and unexpected feature of the enhanced E-oscillations in $^{16}O + ^{28}Si$ (and other systems) is that their "periods" are twice as large as predicted for normal scattering. This indicates a parity-dependence of the interaction that produces the enhancement phenomenon: it favours either odd or even partial waves over those of the opposite parity. This is probably the most telling hint toward the nature of the underlying dynamical process.

On the other hand, it can be shown (see eqs.(3.11) and (3.12) below) that the very existence of pronounced oscillations in the enhanced part of the excitation function indicates that there must also be a parity-independent contribution to the enhancement-causing interaction: a purely parity-dependent deviation from the normal background S-matrix ("odd-even staggering") would render the enhanced part of $\sigma(\pi)$ essentially smooth, leaving only the tiny oscillations arising from interference with the small normal components of the backward scattering amplitude. In fact we shall see in the following section how the pronounced E-oscillations in $\sigma(\pi)$ come about by interference between the contributions arising from the parity-dependent and the parity-independent part of the S-matrix deviation $\tilde{S}_\ell(E)$.

3. BASIC ASSUMPTIONS

On the basis of these experimental features and the general theory of ref. [4], we are now in a position to specify the main properties of $\tilde{S}_\ell(E)$. First we recall that the total partial-wave S-matrix, regarded as a function of the continuous variable $\lambda = \ell+1/2$ (and omitting the dependence on E for simplicity) is written as

$$S_\ell = S_{\ell,N}\, e^{i2\sigma_\ell} \rightarrow S(\lambda) = \bar{S}(\lambda) + \tilde{S}(\lambda) = \left[\bar{S}_N(\lambda)+\tilde{S}_N(\lambda)\right] e^{i2\sigma(\lambda)} , \tag{3.1}$$

where the Rutherford (point-charge) phase shifts $\sigma_\ell \rightarrow \sigma(\lambda)$ have been factored out. By assumption, $\bar{S}(\lambda)$ has a "normal strong-absorption profile" as defined in ref. [4], while the "anomalous" part $\tilde{S}(\lambda)$ engenders the backangle enhancement.

It is well known from the Closed Formalism (CF) for heavy-ion collisions[9,10] that the "normal" scattering is determined by the Fourier transforms of the normal absorptive shape function $D_N(\lambda) = d\bar{S}_N(\lambda)/d\lambda$,

$$F_N[\bar{\Delta}x] = \int_{-\infty}^{\infty} d\lambda\, D_N(\lambda) e^{i\bar{\mu}\bar{\Delta}x} , \qquad \bar{\mu} = \frac{\lambda-\bar{\Lambda}}{\bar{\Delta}} , \qquad x = \theta_R \mp \theta , \tag{3.2}$$

where the parameters $\bar{\Lambda}$ and $\bar{\Delta}$ characterize the position and width, respectively, of the normal ℓ-space "window" defined by $|D_N(\lambda)|$, and where

$$\theta_R = 2\arctan(\eta/\bar{\Lambda}) \tag{3.3}$$

is the Rutherford grazing angle determined by the Sommerfeld parameter η and $\bar{\Lambda} = \ell_g + 1/2$. In an optical model description, scattering functions of the form $\bar{S}(\lambda)$ are generated by strongly absorbing potentials, e.g. of the type known as E18[11] which fits the forward-angle scattering cross section of $^{16}O + {}^{28}Si$ over a wide range of energies.

It has been shown in ref.[4] that in order to produce backward-angle enhancement, the deviation function $\tilde{S}(\lambda)$ must be localized within a narrow region of ℓ-space centred about a critical angular momentum $\tilde{\Lambda}$ not too far from $\bar{\Lambda}$, and that the width $\tilde{\Delta}$ of $|\tilde{S}_N(\lambda)|$ must satisfy the condition

$$\tilde{\Delta} \ll \bar{\Delta} \quad . \qquad (3.4)$$

In view of the experimental evidence we assume that $\tilde{S}_N(\lambda)$ has a parity-dependent and a parity-independent part, of the same form in ℓ-space, and we write

$$\tilde{S}_N(\lambda) = d\left[1+\gamma(-)^{\ell}\right]\omega(\lambda) \quad , \qquad (3.5)$$

where d is the overall strength of the deviation and γ is the parity parameter. The (complex) function $\omega(\lambda)$ defines the form of the "anomalous window" in ℓ-space, and may be considered a function of $\tilde{\mu} = (\lambda-\tilde{\Lambda})/\tilde{\Delta}$.

If the enhancement condition (3.4) is satisfied, the "normal" contribution to the backangle scattering cross section is negligibly small, and the 180^o excitation function, in the form of the cross section ratio $\rho(E) = \sigma(\pi)/\sigma_R(\pi)$, can be well approximated by the anomalous part. This is given by eq.(7.12) of ref. [4] as

$$\rho(E) \approx \frac{\tilde{\sigma}(\pi)}{\sigma_R(\pi)} = (2\tilde{d}\cot\frac{1}{2}\tilde{\theta}_R)^2$$

$$\{(H[\tilde{\Delta}(\tilde{\theta}_R-\pi)])^2+(H[\tilde{\Delta}(\tilde{\theta}_R+\pi)])^2-2H[\tilde{\Delta}(\tilde{\theta}_R-\pi)]H[\tilde{\Delta}(\tilde{\theta}_R+\pi)]\cos(2\pi\tilde{\Lambda})$$

$$+\gamma^2(H[\tilde{\Delta}\tilde{\theta}_R])^2+2\gamma H[\tilde{\Delta}\tilde{\theta}_R](H[\tilde{\Delta}(\tilde{\theta}_R-\pi)]+H[\tilde{\Delta}(\theta_R+\pi)])\sin(\pi\tilde{\Lambda})\}, \quad (3.6)$$

where

$$H[\tilde{\Delta}x] = \int_{-\infty}^{\infty} d\lambda\omega(\lambda)e^{i\tilde{\mu}\tilde{\Delta}x}, \qquad \tilde{\mu} = \frac{\lambda-\bar{\Lambda}}{\tilde{\Delta}}, \quad (3.7)$$

is the Fourier transform of the anomalous window function (assuming $H[\tilde{\Delta}x]$ to be real for simplicity), and

$$\tilde{\theta}_R = 2\arctan(\eta/\tilde{\Lambda}) \quad (3.8)$$

is the Rutherford scattering angle associated with $\tilde{\Lambda}$.

The "normal scattering" contributions to $\rho(E)$, neglected in eq.(3.6), contain the Fourier transforms $F_N[\bar{\Delta}(\theta_R-\pi)]$ and $F_N[\bar{\Delta}(\theta_R+\pi)]$ (see eq.(3.16) of ref.[4]), which because of the condition (3.4) are very much smaller than the Fourier transforms $H[\tilde{\Delta}x]$ in eq.(3.6). Thus the relative broadness of the function $H[\tilde{\Delta}x]$ compared with $F_N[\bar{\Delta}x]$ (a diffractive effect!) is the formal reason for the backangle enhancement.

Equation (3.6) contains two kinds of contributions: The terms in the first line within the curly brackets arise from the parity-independent part of eq.(3.5), those in the second line come from the parity-dependent part of $\tilde{S}_N(\lambda)$. Both parts have contributions, proportional to $\cos(2\pi\tilde{\Lambda})$ and $\sin(\pi\tilde{\Lambda})$, respectively, which are E-oscillations if $\tilde{\Lambda}$ depends on energy.

Their periods are given by[4)]

$$\tilde{P}(E) = (\frac{d\tilde{\Lambda}}{dE})^{-1}, \qquad \tilde{P}_\gamma(E) = 2\left(\frac{d\tilde{\Lambda}}{dE}\right)^{-1}, \tag{3.9}$$

respectively, the oscillations arising from the parity-dependent interaction having twice the period of those from the parity-independent contribution. Further, because of the relative magnitude of the Fourier transforms $H[\tilde{\Delta}x]$, the parity-dependent terms dominate over the others at intermediate energies. More specifically, under the condition

$$H\left[\tilde{\Delta}(\tilde{\theta}_R+\pi)\right] << H\left[\tilde{\Delta}(\tilde{\theta}_R-\pi)\right] , \tag{3.10}$$

eq.(3.6) reduces to

$$\rho(E) \simeq (2d\cot\frac{1}{2}\tilde{\theta}_R)^2$$

$$\{(H[\tilde{\Delta}(\tilde{\theta}_R-\pi)])^2+\gamma^2(H[\tilde{\Delta}\tilde{\theta}_R])^2+ 2\gamma H[\tilde{\Delta}\tilde{\theta}_R]H[\tilde{\Delta}(\tilde{\theta}_R-\pi)]\sin(\pi\tilde{\Lambda})\}, \tag{3.11}$$

containing only the parity-dependent E-oscillations of period $\tilde{P}_\gamma(E)$.

If $\tilde{S}_N(\lambda)$ had no parity-independent part (corresponding in (3.5) to the limit $\gamma\to\infty$ with finite $d\gamma$) eq.(3.11) would reduce to

$$\rho(E) = (2d\gamma\cot\frac{1}{2}\theta_R)^2 \left(H\left[\tilde{\Delta}\tilde{\theta}_R\right]\right)^2 , \tag{3.12}$$

and the enhanced part of the excitation function would be non-oscillatory.

Now, assuming that eq.(3.11) represents the dominant part of the gross structure in the 180^{o} excitation function for $^{16}O + ^{28}Si$, we can determine the energy dependence of $\tilde{\Lambda}(E)$ quite accurately from the spacing of the maxima and minima of the E-oscillations as

$$\tilde{\Lambda}(E) = \tilde{A}(E-\tilde{E})^{1/2}, \qquad \tilde{A} = 5.138\ \mathrm{MeV}^{-1/2}, \qquad \tilde{E} = 17.80\ \mathrm{MeV}\ . \tag{3.13}$$

Thus from the second eq.(3.9) we have the proportionality

$$\tilde{\Lambda}(E) = (\tfrac{1}{2}\tilde{A})^2\, \tilde{P}_{\gamma}(E) \quad . \tag{3.14}$$

On the other hand, $\tilde{\Lambda}$ also determines the period of the large-angle diffraction oscillations in the angular distributions, as $\pi/\tilde{\Lambda}$, and we have found that the values of $\tilde{\Lambda}$ derived from the angular periods for energies at which angular distributions have been measured are in close agreement with the relation (3.13).

It is highly significant to compare the result (3.13) with the energy variation of the grazing angular momentum $\bar{\Lambda}(E)$, determined from the forward-angle scattering by means of the "normal" part of the S-matrix $\bar{S}(\lambda)$ as

$$\bar{\Lambda}(E) = \bar{A}(E-\bar{E})^{1/2}, \qquad \bar{A} = 6.033\ \mathrm{MeV}^{-1/2}, \qquad \bar{E} = 17.58\ \mathrm{MeV}\ . \tag{3.15}$$

These energy dependences of $\bar{\Lambda}(E)$ and $\tilde{\Lambda}(E)$ are shown in fig.1. It is seen that at all energies the peak $\tilde{\Lambda}$ of the "anomalous ℓ-window" $|\omega(\lambda)|$ is several units below the grazing value $\bar{\Lambda}$. This finding is at variance with the popular notion that the structure causing the backangle enhancement is located "close to" the centre $\bar{\Lambda}$ of the "normal" window defined by $|D_N(\lambda)|$, corresponding to a process occurring near the strong absorption radius $\bar{R}$. Regarding (3.15) as a semiclassical relation between grazing angular momentum and $\bar{R}$, the latter can be calculated as

$$\bar{R} = \frac{\hbar}{(2\mu)^{1/2}} \bar{\Lambda} = 8.65 \text{ fm}, \qquad r_o = \bar{R}/(A_1^{1/3}+A_2^{1/3}) = 1.56 \text{ fm} \tag{3.16}$$

(where μ is the reduced mass), while the semiclassical radius associated with $\tilde{\Lambda}(E)$ is

$$\tilde{R} = \frac{\hbar}{(2\mu)^{1/2}} \tilde{\Lambda} = 7.36 \text{ fm} \quad , \tag{3.17}$$

about 1.3 fm inside the strong-absorption radius! Although we regard this difference as a further important clue as to the physical nature of ALAS, we note it here merely as a "semi-empirical" fact. Another noteworthy feature is that the difference between the "threshold energies" of $\tilde{\Lambda}$ and $\bar{\Lambda}$ is quite small, $\tilde{E}-\bar{E} = 0.22$ MeV. For later reference we calculate the Coulomb barrier at the strong absorption radius as

$$V_c(\bar{R}) = \frac{Z_1 Z_2 e^2}{\bar{R}} = 18.65 \text{ MeV} \ . \tag{3.18}$$

Having determined $\tilde{\Lambda}(E)$ rather uniquely from the periods of the dominant E-oscillations alone, one would hope to be able to specify the other characteristics of the window function $\omega(\lambda)$, its width $\tilde{\Delta}$ and its phase, in a similarly unique way from the amplitude of the E-oscillations and the energy variation of the average excitation function. This turns out to be a less unambiguous task than expected, if the anomalous S-matrix (3.5) and the corresponding features in excitation function and angular distribution are considered in isolation, i.e. without taking into account the interference with the normal part $\bar{S}(\lambda)$.

The main reason is that both the strength parameter d and the parity parameter γ in (3.5) are expected to vary (decrease) with energy: whatever physical process causes the backangle enhancement, it will at higher energies have to compete with the rapidly growing number of channels causing "normal" absorption, and thus diminish in relative importance. We expect further that the rate of decrease with energy is different for the parity-dependent and the parity-independent parts of the interaction. Without implying a specific dynamical model we simply assume the energy dependence of d and γ to be exponential,

$$d(E) = d_o e^{-bE}, \qquad \gamma(E) = \gamma_o e^{-cE}, \qquad (3.19)$$

with the understanding that this behaviour may be modified at lower energies where the "normal" part of the 180° excitation function, $\bar{\sigma}(\pi)$, becomes dominant.

Without a priori information about the constants b and c, the energy variations of d(E), γ(E) and of the width $\tilde{\Delta}(E)$ cannot all be determined unambiguously from $\tilde{\sigma}(\pi)$ alone. We therefore make the assumption

$$\tilde{\Delta} = \text{const.} \qquad (3.20)$$

merely for "parameter economy", and show that a consistent description of the data can be achieved this way. On physical grounds, however, we consider assumption (3.20) rather unrealistic for processes characterized by constant spatial parameters; these will give rise to ℓ-windows whose width tends to increase with energy, roughly as $\tilde{\Delta} \sim E^{1/2}$. Such an energy dependence must of course be expected for the width $\bar{\Delta}(E)$ of the normal S-matrix $\bar{S}(\lambda)$; this is semiclassically related to

$\bar{\Lambda}(E)$ by [8]

$$\bar{\Delta}(E) = \frac{a}{\bar{R}} \frac{1-\tilde{E}/2E}{1-\bar{E}/E} \bar{\Lambda}(E) \quad , \qquad (3.21)$$

where a is a surface diffuseness parameter. From an analysis of the forward-angle scattering of $^{16}O + {}^{28}Si$ we find the value

$$a = 0.47 \text{ fm} \quad . \qquad (3.22)$$

Finally, from eq.(3.6) or (3.11) it can be seen that the sign of the parity parameter γ determines the phase of the parity-dependent E-oscillations relative to the parity-independent ones. Conversely, the sign of γ can be determined unambiguously from the experimental excitation function.

4. PARAMETRIC MODELS

Now we choose as a convenient analytic form of $\omega(\lambda)$ the function

$$\omega(\lambda) = \frac{1}{2\tilde{\Delta}} \left[1 + \cosh(\tilde{\mu}+i\tilde{\alpha})\right]^{-1} , \quad \tilde{\mu} = \frac{\lambda-\tilde{\Lambda}}{\tilde{\Delta}} \quad . \qquad (4.1)$$

The form of (4.1) is that of the derivative of the Ericson parametrization[12] often used for the "normal" strong-absorption S-matrix,

$$\bar{S}_N(\lambda) = \left[1 + \exp(-\bar{\mu} - i\bar{\alpha})\right]^{-1}, \quad \bar{\mu} = \frac{\lambda-\bar{\Lambda}}{\bar{\Delta}} \quad . \qquad (4.2)$$

Writing $\omega(\lambda) = |\omega(\lambda)| \exp\left[i2\tilde{\delta}_N(\lambda)\right]$, we obtain

$$|\omega(\lambda)| = \frac{1}{2\tilde{\Delta}} \frac{1}{\cosh\tilde{\mu} + \cos\tilde{\alpha}} , \qquad (4.3)$$

$$2\tilde{\delta}_N(\lambda) = -\arctan\left(\frac{\sin\tilde{\alpha}\ \sinh\tilde{\mu}}{1 + \cos\tilde{\alpha}\ \cosh\tilde{\mu}}\right) , \qquad (4.4)$$

and from eq.(4.4) for the anomalous part of the nuclear deflection function

$$\tilde{\Theta}_N(\lambda) = \frac{d2\tilde{\delta}_N(\lambda)}{d\lambda} = -\frac{1}{2\tilde{\Delta}} \frac{\sin\tilde{\alpha}}{\cosh\tilde{\mu} + \cos\tilde{\alpha}} = -|\omega(\lambda)|\sin\tilde{\alpha} . \qquad (4.5)$$

The minimum of $\tilde{\Theta}_N(\lambda)$ is at $\tilde{\Lambda}$ and has the value

$$\tilde{\Theta}_N(\tilde{\Lambda}) = \frac{1}{\tilde{\Delta}} \tan\tfrac{1}{2}\tilde{\alpha} , \qquad (4.6)$$

which shows that the "dip" in the deflection function is the deeper the smaller $\tilde{\Delta}$. Since the phase parameter is restricted by $0 \leqq \tilde{\alpha} < \frac{1}{2}\pi$, the lowest possible value is $-\tilde{\Delta}^{-1}$.

The Fourier transform $H[\tilde{\Delta}x]$ of (4.1) is[4)]

$$H\left[\tilde{\Delta}x\right] = \frac{\pi\tilde{\Delta}x}{\sinh\pi\tilde{\Delta}x} e^{\tilde{\alpha}\tilde{\Delta}x} , \qquad (4.7)$$

thus a real function as assumed in eqs. (3.6) and (3.11). Using (4.7) in the latter equations allows a detailed analytic discussion of how the backangle excitation function depends on $\tilde{\Delta}$, $\tilde{\alpha}$ and $\tilde{\theta}_R(E)$, in addition to the more explicit dependences on $\tilde{\Lambda}(E)$, $d(E)$ and $\gamma(E)$.

If a satisfactory description of the data can be given with a window of the form (4.1), to what extend does this depend on the specific shape of this function? To answer this question we have also carried out analyses with a window of the following quite different analytic form,

$$\omega_r(\lambda) = \frac{1}{2\pi i} \frac{1}{\lambda - \tilde{\Lambda} + i\frac{1}{2}\tilde{\Delta}_r} , \qquad (4.8)$$

which may be regarded as arising from a "Regge pole" projected onto the real ℓ-axis. The Fourier transform of $\omega_r(\lambda)$ is a pure exponential,

$$H\left[\tilde{\Delta}_r x\right] = e^{-1/2\tilde{\Delta}_r x} . \qquad (4.9)$$

In this simple case the full Poisson series for the scattering amplitude (see ref.[4]) can be summed, and the following closed expression for (the anomalous part of) the 180° excitation function is obtained

$$\rho(E) = \frac{\tilde{\sigma}(\pi)}{\sigma_R(\pi)} = d^2(\cot\tfrac{1}{2}\tilde{\theta}_R)^2\, e^{-\tilde{\Delta}_r\tilde{\theta}_R}\, \frac{e^{-\pi\tilde{\Delta}_r} + \gamma^2 + 2\gamma e^{-\frac{1}{2}\pi\tilde{\Delta}_r} \sin(\pi\tilde{\Lambda})}{1 + e^{-2\pi\tilde{\Delta}_r} + 2e^{-\pi\tilde{\Delta}_r}\cos(2\pi\tilde{\Lambda})}$$

(4.10)

This shows that the parity-dependent E-oscillations are dominant under the condition

$$2e^{-\pi\tilde{\Delta}_r} \ll 1 , \qquad (4.11)$$

for which eq.(4.10) reduces to

$$\rho(E) \approx d^2(\cot\tfrac{1}{2}\tilde{\theta}_R)^2 e^{-\tilde{\Delta}_r\tilde{\theta}_R}\left[e^{-\pi\tilde{\Delta}_r} + \gamma^2 + 2\gamma e^{-\frac{1}{2}\pi\tilde{\Delta}_r}\sin(\pi\tilde{\Lambda})\right] . \qquad (4.12)$$

5. RESULTS AND DISCUSSION

We have fitted the 180° excitation function for $^{16}O + ^{28}Si$, and the angular distribution at E = 35.0 MeV, using, in the partial-wave sum, a total S-matrix of the form

$$S_N(\lambda) = \bar{S}_N(\lambda) + \tilde{S}_N(\lambda)$$

$$= \left[1 + \exp\left(\frac{\bar{\Lambda}-\lambda}{\bar{\Delta}} - i\bar{\alpha}\right)\right]^{-1} + \frac{de^{i\phi}}{2\tilde{\Delta}} \left[1 + \gamma(-)^{\ell}\right] \left[1 + \cosh\left(\frac{\tilde{\Lambda}-\lambda}{\tilde{\Delta}} - i\tilde{\alpha}\right)\right]^{-1} . \quad (5.1)$$

The parameters appearing in (5.1) have been discussed in Section 3. $\bar{\Lambda}$ and $\tilde{\Lambda}$ are given respectively in eqs.(3.13) and (3.15). The width $\bar{\Delta}$ is given by (3.21), with $a/\bar{R} = 0.054$, compared to $\tilde{\Delta} = 0.79$, and the phase parameters are $\bar{\alpha} = 1.5$ and $\tilde{\alpha} = 0.2$. The energy dependences of the strengths d and γ were indicated in eqs.(3.19) with $d_o = 3.1$, $b = 0.07$, $\gamma_o = -3.0$ and $c = 0.09$. The relative phase ϕ was found to be equal to π indicating clearly that the abnormal part $\tilde{S}(\lambda)$ corresponds to a dip in the total reflection function $|S(\lambda)|$. Notice that the parameters $\bar{\Lambda}$, $\bar{\Delta}$ and $\bar{\alpha}$ of the normal, strong-absorption, component of $S(\lambda)$ were chosen in such a way as to reproduce rather closely the results of optical model calculation with the "E18" potential[11).

Our results for the excitation function and the angular distribution at E = 35 MeV are shown in figures 2 and 3 , respectively. Considering the simplicity of our model, we consider our overall fit of the data quite satisfactory. It is important to note that although the abnormal part of $S(\lambda)$ gives by far the dominant contribution to the 180° excitation function at c.m. energies larger than about 20 MeV, the normal part of $S(\lambda)$ contributes equally importantly at $E \lesssim 20$ MeV and therefore results in an important interference effect in this energy region. Clearly at $E < V_c(\bar{R})$ (see eq.(3.18)), $\bar{S}(\lambda)$ dominates completely. The above interference effect between $\bar{S}(\lambda)$ and $\tilde{S}(\lambda)$ would certainly reflect itself in the angular distribution at these energies. Further, at energies below $V_c(\bar{R})$ the angular distribution at backward angles would be characterized by $\bar{\Lambda}$ in contrast to higher energies where $\tilde{\Lambda}$ determines the angular oscillations (see our discussion in Section 3). Lastly, to test our contention that the oscillations in the 180° excitation function are due to interference between the ℓ-independent and ℓ-dependent windows, we have changed the sign of γ. The resulting oscillations were found to be exactly 180° out of phase with the ones shown in figure 2.

One notices in figure 2 that at $E_{Lab} < 37$ MeV the calculation becomes qualitatively and quantitatively different from the data. This is expected owing to the assumption that the strengths d and γ increase exponentially with decreasing E. More realistic energy dependences of d and γ, showing maxima at an energy of about 35 MeV and a decrease with decreasing energy below $E_{Lab} \approx 35$ MeV, would certainly improve our fits in this energy region.

Our results for the angular distribution at E = 35 MeV may be easily understood along the lines of our discussion in Section 3. At extreme backward angles both the parity-independent and parity-dependent windows come into play, with the parity-dependent window giving an enhancement of both the magnitude of $\sigma/\sigma_R(\pi)$ (by a factor of ~ 10) and the amplitude of the E-oscillations.

At smaller angles ($\theta \sim 120^{o}$), where the peaking of the contribution of the parity-independent window occurs, the oscillations seen are due to this window alone. At angles in the range $50^{o} \lesssim \theta \lesssim 100^{o}$, one notices clearly another kind of oscillation arising from the interference between the contributions of the normal part of $S(\lambda)$ and the parity-independent window. At forward angles $\theta \lesssim \theta_R$, $\overline{S}(\lambda)$ dominates completely. Figure 3 shows that in the "interference region" at intermediate angles $50^{o} \lesssim \theta \lesssim 100^{o}$ our present, preliminary, calculation does not yet reproduce the data very well. However, in both the small-angle and large-angle regions the agreement is quite satisfactory.

6. CONCLUSIONS

The results of our analysis have shown that it is possible to obtain adequate fits to both the 180^{o} excitation function and the full angular distributions for a typical case of anomalous heavy-ion scattering, by assuming a certain structure in the total S-matrix superimposed on a background of "normal strong-absorption profile".

It turns out that this structure is quite well defined by the features of the experimental data, especially regarding its position $\tilde{\Lambda}$ (well below the grazing value), and as to its width, but the data are rather insensitive to the detailed shape of the "anomalous window".

Our main argument that the gross structure seen in the 180^{o} excitation functions for the scattering of $n.\alpha$ type heavy ions is (a strongly enhanced form of) a "universal" phenomenon, goes as follows. The presence of regular oscillations in the backward-angle excitation function, with an energy-dependent "period" $P(E) \sim (E-\overline{E})^{\frac{1}{2}}$, has been predicted[8)] for the scattering of all strongly absorbed nuclei. However, under normal strong-absorption conditions the magnitude of the 180° cross sections and the amplitude of the "E-oscillations" are extremely small. Only recently has the early prediction been confirmed for heavy-ion systems in which one of the reaction partners differs by one or two nucleons from an n.α composition, such as $^{16}O + ^{29}Si$, $^{16}O + ^{30}Si$, $^{13}C + ^{28}Si$ and $^{18}O + ^{28}Si$, for which the E-oscillations are visibly enhanced. The pronounced gross structure in the 180° excitation functions for systems in which both partners are of n.α type, has the same form (except for the doubling of the period due to the parity-dependent part of the interaction), the strong enhancement being presumably associated with the "α-cluster" composition of the nuclei. If the gross structure were due to resonances or "resonance-like" interactions at fixed angular momenta, it would imply regular sequences of resonant levels whose spacings and widths increase systematically with energy, for all "compound" systems formed by n.α type nuclei.

We now turn to the clues our analysis gives as to the dynamical origin of the enhancement phenomenon. Firstly, the parity dependence in the interaction suggests an exchange process. However, it seems highly unlikely that it is due to a contribution from a single-step elastic transfer, for the following reasons. In the case of $^{16}O + ^{28}Si$ this would involve the transfer of a ^{12}C cluster for which the spectroscopic factor is very small. Further, a single-step elastic transfer gives a refractive contribution to the S-matrix (see refs.[9,13]), while our analysis indicates a predominantly absorptive effect. Secondly, the very existence of E-oscillations in the 180° excitation function implies, according to the discussion of eq.(3.11), the presence of a parity-independent component in the anomalous part of the S-matrix. It has been shown in ref.[13] that such an (absorptive) component can arise from a two-step transfer coupling contribution to elastic scattering. Our third clue is the fact that the "anomalous window" in ℓ-space is centered at a value $\tilde{\Lambda}$ several units smaller than the grazing angular momentum $\bar{\Lambda}$. This suggests that the contribution causing the enhancement takes place at a distance where the overlap of the densities of the colliding nuclei is considerably larger than that at the strong absorption radius.

These considerations, together with the all-important fact that the ALAS phenomenon in heavy-ion scattering is confined to n.α type nuclei, lead us to the tentative suggestion that the enhancement is caused by multi-step α-transfer processes : a two-step α-transfer contribution giving rise to the parity-independent part, and a (much weaker) three-step successive transfer of α-clusters resulting in the parity-dependent part of the anomalous S-matrix. Such a picture would account qualitatively for the smallness of the parity parameter $\gamma(E)$ in the relevant energy range, and for the energy dependence of $d(E)$ and $\gamma(E)$: both types of transfer processes will become less likely at higher energies due to the shorter collision time; moreover, they will gradually lose out in the competition with the rapidly growing number of other channels causing "normal" absorption.

We shall investigate the validity of this picture in subsequent work, where we replace the present phenomenological form of the anomalous S-matrix $\tilde{S}(\lambda)$ by expressions derived by means of the coupled-channels S-matrix formalism described in ref.[13].

It is instructive to compare our results with those of the Minnesota group[7], who have obtained satisfactory fits to the $^{16}O + ^{28}Si$ data by means of an energy-dependent, surface-transparent and parity-dependent optical potential. It turns out that the S-matrix generated by this potential[14] shows very similar features to our "inductively" determined form, which indicates that these features are essential for reproducing the ALAS phenomena.

Finally, we mention that because of the close relationships between elastic scattering and quasi-elastic heavy-ion reactions, the enhancement-causing anomalous part of the elastic S-matrix engenders closely related anomalous structures in the angular distributions and excitation functions for quasi-elastic reactions of n.α type nuclei, as demonstrated explicitly by the closed formalism for inelastic scattering[15] and transfer reactions[16]. These processes will also be studied in our further work.

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FIGURE CAPTIONS

Figure 1. Energy dependence of the angular momentum parameters $\overline{\Lambda}$ and $\tilde{\Lambda}$.

Figure 2. Fit to the 180^{o} excitation function for the elastic scattering of $^{16}O + ^{28}Si$ obtained with the S-matrix given by eq.(5.1) and the parameter values indicated in section 5. The data points were taken from Ref.1.

Figure 3. Fit to the angular distribution for the elastic scattering of $^{16}O + ^{28}Si$ at E_{lab} = 55 MeV, obtained with the S-matrix given by eq.(5.1) and the parameter values indicated in section 5. The data points were taken from Refs.1 and 7.

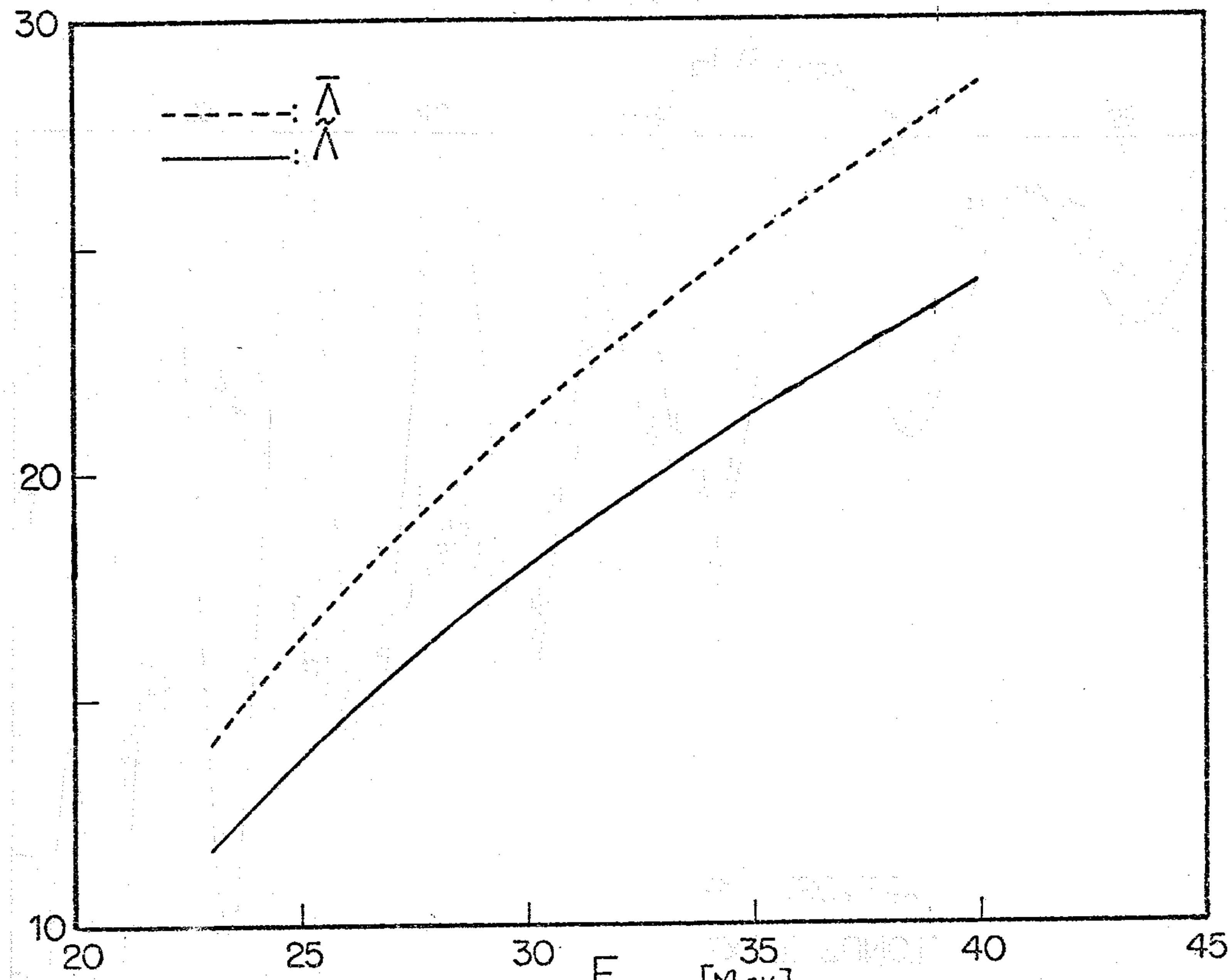
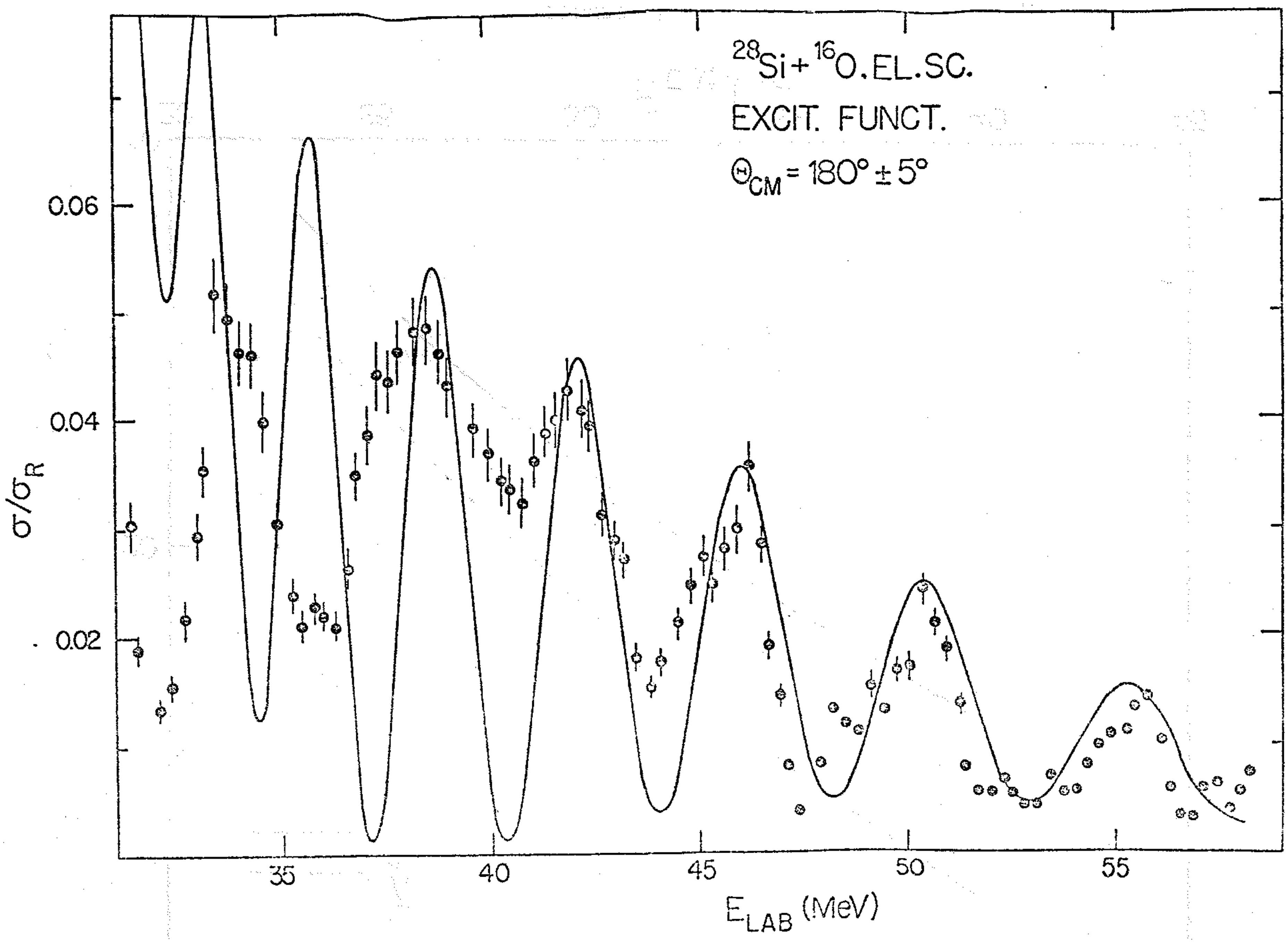
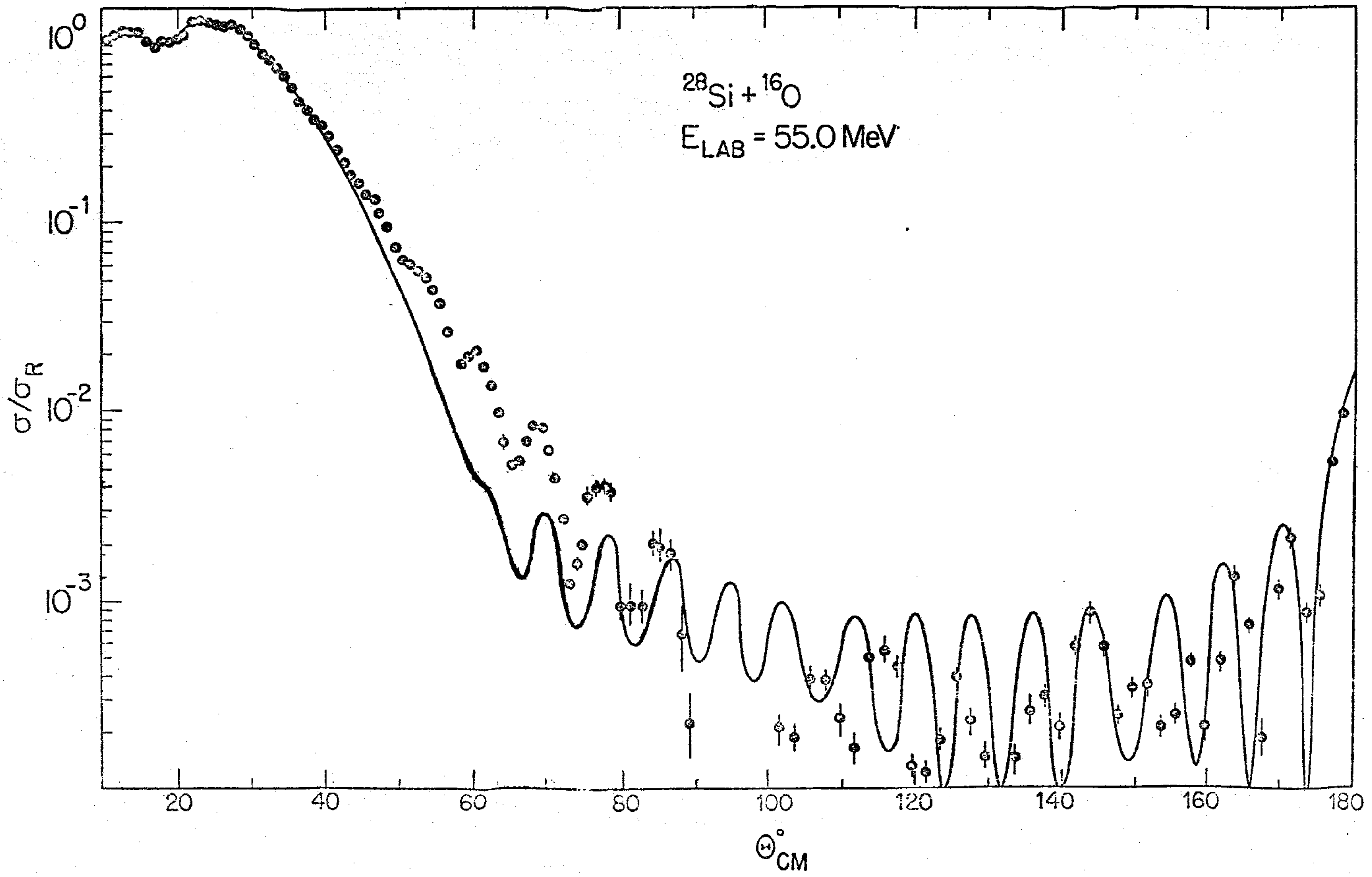


FIGURE 1



$^{28}Si + ^{16}O$. EL. SC.
EXCIT. FUNCT.
$\Theta_{CM} = 180° \pm 5°$
0.06
0.04
0.02
σ/σ_R
35
40
45
50
55
E_{LAB} (MeV)

FIGURE 2



$^{28}Si + ^{16}O$
$E_{LAB} = 55.0$ MeV
σ/σ_R
10^0
10^{-1}
10^{-2}
10^{-3}
20
40
60
80
100
120
140
160
180
Θ°_{CM}

FIGURE 3