Heterogeneous Methods for Calculating Reactors: Survey of Results and Comparison with Experiment

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When calculating the critical sizes of a reactor with a heterogeneous core and the spatial distribution of the neutron flux the homogenization rule is usually applied. In such calculations the heterogeneous lattice is substituted by a homogeneous multiplying medium possessing the same mean neutron parameters as a heterogeneous lattice of infinite dimensions.

Even in the case of a "large" reactor the need arises for a proof of the homogenization principle and for an evaluation of its possible error. In the case of a reactor composed of a comparatively small number of uranium slugs the use of the homogenization principle is, generally speaking, not justified.

In this connection direct methods of calculation of heterogeneous reactors were developed. The latter make possible the calculation of a reactor composed only of a few uranium slugs as well as the evaluation of errors introduced by the homogenization method.

It must be additionally stressed that the heterogeneous method enables us to study the neutron field near the active zone boundary in the vicinity of a separate absorbing rod introduced into the heterogeneous lattice and in composite lattices, that is, in lattices composed of slugs with different properties placed in an order of regular periodic alternation.

Thus the problem of the heterogeneous method is not limited to the proof of the homogenization method; in some cases it is the only possible and indispensable means of investigation of the neutron field.

1. THE PRINCIPLES OF THE HETEROGENEOUS METHOD

In the heterogeneous method the reactor is considered as a system of uranium slugs placed in a neutron-moderating medium. The uranium slugs are sources of fast neutrons and sinks for thermal and resonance neutrons.

The heterogeneous method is based on the following assumptions:

1. The thermal neutron field near the slugs possesses spheric or axial symmetry so that the sinks may be considered as of point or line configuration. This assumption is valid if the distance between the slugs is much greater than their dimensions. It should be noted that symmetrical structure of the reactor lattice favors fulfillment of the symmetry conditions in the neutron field in the vicinity of the slugs. Thus, the theory is applicable in the case of quite large slugs, if the lattice is sufficiently symmetrical.

2. An elementary diffusion equation may be used in describing thermal neutron diffusion between the slugs. This is justified when assuming that the absorption of thermal neutrons in the moderator is low and that the diffusion length is small in comparison with the distances between the slugs.

3. The value of the logarithmic derivative of a flux of thermal neutrons on the surface of the slug is a characteristic of the absorbing properties of the uranium slugs. When absorption of thermal neutrons inside the slug is great then conditions in the vicinity of the surface calling for the diffusion approximation are violated; in this case the logarithmic derivative should be given for the effective surface of the slug removed from its real surface to a distance of $\lambda_s/2$ (λ_s is the scattering mean free path in the moderator). A simple relation may be established between the intensity of the source and the sink. Let i_k be the intensity of the thermal neutron sink, i.e., the number of thermal neutrons absorbed by this sink in unit time, and let q_k be the intensity of the same source of fast neutrons, i.e., the number of fast neutrons emitted in unit time by the source. Then

$$q_k = \eta_k i_k \tag{1.1}$$

where η_k is the number of fission neutrons generated in the slug as a result of the absorption of one thermal neutron.

Besides the sources-sinks, the system may also contain outer sources of fast neutrons and sinks of thermal neutrons. Each source of fast neutrons generates a field of moderated neutrons, the spacial distribution of which are described by the following function

$$q_k j(E_1,E_0,|r - r_k|) \tag{2.1}$$

where $j(E_1,E_0,r)$ is the flux of the moderating neutrons, E_1 is the energy of the flux generated by a point (or line) source of fast neutrons of unit intensity and E_0 is the primary energy of the neutrons. The function j is determined by the values E_1, E_0 and the properties of the slowing-down medium. (In Equation 2.1 r is the radius-vector of the observation point, r_k the radius-vector of the position of the source.) Assuming that $E_1 = kT$ in Equation 2.1 and denoting $j(kT,E_0, r) = j_T{}^{E_0}(r)$, we get for k of that source the following

$$q_k j_T{}^{E_0}(|r - r_k|) \tag{3.1}$$

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Knowing the thermal neutron generating field, it is not difficult to define their spatial distribution ($\Phi = nv$), still disregarding the neutron absorption in the sinks (i.e., considering the moderator as free from point sinks for thermal neutrons). To find the distribution of the thermal neutrons it is sufficient to integrate (throughout the volume of the moderator) the product of the generation density of the thermal neutrons by the function of the influence of the point source. In diffusion approximation this function is expressed as follows:

For a point source of thermal neutrons, letting L be the diffusion length for thermal neutrons and Σ_c the macroscopic thermal neutron absorption cross section,

$$f(r) = e^{-r/L}/4\pi L^2\Sigma_c r \qquad (4.1a)$$

and for a line source

$$f(r) = K_0(r/L)/2\pi L^2\Sigma_c \qquad (4.1b)$$

where $K_0(x)$ is the modified Bessel function of the second kind. As a result of integration we obtain the distribution of the thermal neutron flux, generated in the moderating medium free from point sinks, by a single fast-neutron source.

This spatial distribution we denote as F and call it the Green function

$$F(|r - r_k|) = \int_\infty j_T^{E_0}(|r' - r_k|) f(|r - r'|)\, dr' \qquad (5.1)$$

Evidently the Green function of the kth source may be written as:

$$q_k F(|r - r_k|) \qquad (6.1)$$

Due to additivity properties of neutron sources placed in a moderator the field of thermal neutrons in a medium where there is no sink is described by

$$\Phi(r) = \sum_k q_k F(|r - r_k|) \qquad (7.1)$$

where the sum is taken over all the fast-neutron sources in the reactor.

On the other hand, the influence of each kth point neutron sink decreases the thermal neutron flux by

$$i_k f(|r - r_k|) \qquad (8.1)$$

Therefore, the spatial distribution of the thermal neutron flux in the reactor is:

$$\Phi(r) = \sum_k [q_k F(|r - r_k|) - i_k f(|r - r_k|)] \qquad (9.1)$$

By employing the connection between q_k and i_k expressed in Equation 1.1 we find:

$$\Phi(r) = \sum_k i_k[\eta_k F(|r - r_k|) - f(|r - r_k|)] \qquad (10.1)$$

As it may be assumed that the values of $F(r)$, $f(r)$, η_k are known, it is necessary to know i_k, the intensity of the thermal neutron sinks, to calculate the thermal neutron flux $\Phi(r)$.

If the distance between the slugs, the diffusion length and slowing-down length are much greater than the slugs themselves, then the thermal neutron flux in the vicinity of the nth slug may be designated approximately as

$$\Phi(|r - r_n|) \approx \sum_k{}' i_k[\eta_k F(d_{nk}) - f(d_{nk})]$$

$$+ i_n[\eta_n F(0) - f(\rho)] \equiv \sum_k i_k[\eta_k F(d_{nk}) - f(d_{nk})] \qquad (11.1)$$

$$\rho = |r - r_n|; \quad d_{nk} = |r_n - r_k| \quad \text{when} \quad n \neq k$$
$$d_{nk} = \rho \quad \text{when} \quad n = k \qquad (12.1)$$

In Equation 11.1 the sum $\sum_k{}'$ is extended over all the slugs except the nth. As the equations determining the thermal neutron flux are lineal the intensity of absorption of neutrons in the nth sink is proportional to the thermal neutron flux on the effective surface of the slug (taking $\rho = R_n$). The coefficient of proportionality γ_n between i_n and $\Phi(R_n)$ determined the absorbing properties of the slug

$$i_n = \frac{1}{\gamma_n}\Phi(R_n) \qquad (13.1)$$

The value of γ_n may be determined either by experiment or by a non-diffusional calculation of the thermal neutron field in the vicinity of the slug. In a number of cases γ may be defined almost precisely from the elementary diffusion equation. From this and from Equation 11.1 we obtain:

$$\gamma_n i_n = \sum_k i_k[\eta_k F(d_{nk}) - f(d_{nk})] \qquad (14.1)$$

Evidently, $d_{nn} = R_n$. Thus, we have a system of homogeneous line equations for the definition of i_k; this system may be briefly designated as:

$$\gamma_n i_n = \sum_{k=0}^{N-1} i_k \alpha_{kn} \qquad (15.1)$$

where

$$\alpha_{kn} = \eta_k F(d_{kn}) - f(d_{kn}) \qquad (16.1)$$

This system of equations may be solved when and only when its determinant equals zero; in this way we obtain the following conditions for the critical size of a reactor:

$$\begin{vmatrix} (\alpha_{00} - \gamma_0) & \alpha_{01} & \alpha_{02} \cdots & \alpha_{0,N-1} \\ \alpha_{10} & (\alpha_{11} - \gamma_1) & \alpha_{12} \cdots & \alpha_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{N-1,0} & \alpha_{N-1,1} & \alpha_{N-1,2} \cdots & (\alpha_{N-1,N-1} - \gamma_{N-1}) \end{vmatrix} = 0 \qquad (17.1)$$

Equation 17.1 contains the values of η_i, γ_i, R_i, L, τ.* Each of these parameters may be the proper value of the problem.

Thus the calculation of a heterogeneous reactor composed of N uranium slugs is reduced to the solu-

* $6\tau = 4\pi \int_0^\infty r^4 j_T^{E_0}(r)\, dr$.

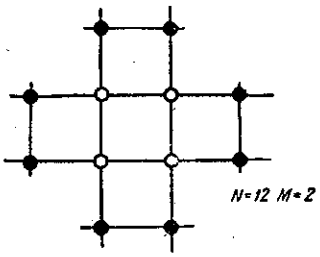

Figure 1. Scheme of an active zone consisting of N = 12 slugs; the number of unknown quantity M = 2

tion of a system of N uniform line equations. When the reactor is symmetrical and contains M groups of identically placed slugs, the number of equations decreases and becomes equal to $M < N$.

For instance, the reactor in Fig. 1 consists of $N = 12$ slugs; the number of groups of symmetrical slugs is $M = 2$.

Another case: an infinite lattice consisting of identical slugs; in this case $N = \infty$ and $M = 1$. If symmetry is taken into account, Equation 15.1 has the form:

$$\gamma_n i_n = \sum_{s=1}^{s=M} i_s \sum_{k'} \alpha_{nk'} \qquad (18.1)$$

where the index k' is extended over the values of the given group s (symmetrically placed rods).

For instance, the critical equation for an infinite lattice composed of identical slugs is:

$$\gamma_0 = \eta_0 \sum^{\infty} F(|r_k|) - \sum^{\infty\prime} f(|r_k|) - f(R_0) \qquad (19.1)$$

It should be noted that the function $f(R)$ when $R \to 0$ has a singularity:

For point slugs: $f(R) \to 1/4\pi\Sigma_c L^2 R$
For line slugs:

$$f(R) \to -\left(0.577 + \ln\frac{R}{2L}\right)/2\pi L^2\Sigma_c \qquad \text{when } R \to 0 \qquad (20.1)$$

We have not as yet considered the resonance absorption in the slugs, occurring in the course of slowing down of neutrons. Resonance absorption may be calculated for one or several rather narrow levels—so narrow that the width of the "dangerous" zone is small in comparison with the average loss of energy in each collision.† This assumption made the resonance absorption (for a single level) equivalent to a monochromatic "negative" source, i.e., a neutron sink with the energy E_r equal the energy of the resonance level.

Thus the field of the slowing down neutrons (with $E < E_r$) appears to be generated by two sources: The actual fast neutrons source and the "negative" source, the energy of which is E_r. So when $E < E_r$ we have instead of Equation 3.1:

$$q_k j_T^{E_0}(|r - r_k|) - \psi_k j_T^{E_r}(|r - r_k|) \qquad (21.1)$$

† See I. I. Gurevich, I. Y. Pomeranchouk, P/649 "The Theory of Resonance Absorption in Heterogeneous Systems," this Session.

where ψ_k is the intensity of the resonance neutron sink. The value of ψ_k may be calculated by the following formula:

$$\psi_k = A_k \sum_p q_p j_{E_r}^{E_0}(|r_p - r_k|) \qquad (22.1)$$

where A_k is a constant which determines the absorption of resonance neutrons by the kth slug.

The scheme of heterogeneous calculation described above does not change if resonance absorption is considered. However, instead of Equation 5.1 the following expression, in which resonance absorption is taken into account, should be used:

$$q_k \int_\infty f(|r' - r|) j_T^{E_0}(|r' - r_k|)\, dr' - A_k \sum_p q_p j_{E_r}^{E_0}(|r_p - r_k|) \int_\infty f(|r' - r|) j_T^{E_r}(|r' - r_k|)\, dr'$$

Substituting the latter into Equation 14.1 after several transformations we have:

$$\gamma_n i_n = \sum_k^{N-1} \eta_k i_k F^*(r_n, r_k) - \sum_k^{N-1} i_k f(d_{nk}) \qquad (14.1^*)$$

where

$$F^*(r_n, r_k) = F(d_{nk}) - \sum_m^{N-1} F_T^{E_r}(d_{nm}) j_{E_r}^{E_0}(d_{km}) A_m$$

$$F_T^{E_r}(d_{nm}) = \int_\infty f(|r' - r_n|) j_T^{E_r}(|r' - r_m|)\, dr' \qquad (5.1^*)$$

Let us note that in distinction from $F(|r_n - r_k|)$ the function $F^*(r_n, r_k)$ is, generally speaking, dependent not only on the distance between points r_n and r_k.

Above, we have given a method for calculating a system of infinite cylindrical slugs, placed in an infinite neutron moderator. However, for a lattice, consisting of finite cylindrical slugs it is possible to evaluate the influence of the finiteness of the lattice.

For a reactor with line slugs placed parallel to the axis z, it is possible to solve the heterogeneous equation by separating the variables:

$$\Phi(r,z) = \Phi(r) \cos \kappa_z z \qquad (23.1)$$

The value κ_z is determined by this formula:

$$\kappa_z = \frac{\pi}{H + 2\Delta_z} \qquad (24.1)$$

In Equation 24.1 H is the height of the core, Δ_z is the reflector savings.

At the same time it is supposed that even in the presence of a reflector the density distribution of the neutrons along the z axis remains proportional to $\cos \kappa_z z$; thus, all the above-mentioned formulas turn out to be valid if we substitute for $1/L^2$,

$$\frac{1}{L^2} + \frac{\pi^2}{(H + 2\Delta_z)^2} = \frac{1}{L^2} + \kappa_z^2 \qquad (25.1)$$

and for η,

$$\eta e^{-\kappa_z^2 \tau}$$

Further we shall give a short survey of the results obtained by different authors from the heterogeneous method of solving certain reactor problems. It should be noted that this survey is not complete in consequence of space limitations; the author includes the most important works, from this point of view.

2. AN INFINITE LATTICE CONSISTING OF SIMILAR POINT OR LINE SLUGS

I. Y. Pomeranchouk and A. I. Akhiezer have investigated the problem of a heterogeneous lattice consisting of point (spherical) and line (cylindrical) slugs, placed periodically in an infinite moderator. The lattice was assumed to consist of identical slugs. Then, in agreement with Equations 19.1 and 20.1, the critical equation has the form:

$$\gamma = \eta \sum_k^\infty F(r_k) - \sum_k^{\infty\prime} f(r_k) - f(R) \qquad (19.1^*)$$

The problem consists of computing the sums in Equation 19.1*.

For the slowing down flux $j(E,r)$ from a point source the authors adopted an expression obtained from an age theory approximation for a monochromatic source of fast neutrons:

$$j(E_1,E_0,r) = \frac{e^{-r^2/4\tau(E_1,E_0)}}{(2\sqrt{\pi\tau(E_1,E_0)})^3} \qquad (1.2)$$

It is notable that the authors of all subsequent works used in this survey also proceeded from this expression, although it is not essential.

Substituting Equation 1.2 into Equation 5.1 and summing them up, we obtain the following expression (for point sources):

$$\sum_k^\infty F(r_k) = \sum_k^\infty \int \frac{e^{-r^2/4\tau}}{(2\sqrt{\pi\tau})^3} \frac{e^{-|r-r_k|/L}\,dr}{4\pi L^2\Sigma_c|r-r_k|} \qquad (2.2)$$

where $r_k = ak_1 + bk_2 + ck_3$, the k_i are integers, and a, b, c are basic lattice vectors.

By means of the Fourier transformation it is possible to show that:

$$\sum_k^\infty F(r_k) \approx \frac{1}{V\Sigma_c} \qquad (2.2^*)$$

if $e^{-4\pi^2\tau/a^2} \ll 1$. Here V is the volume of a cell in a periodic lattice, a being the largest period.

For the sum of the functions

$$\sum_k^{\infty\prime} f(r_k)$$

we have:

$$\sum_k^{\infty\prime} f(r_k) = \sum_k^{\infty\prime} e^{-r_k/L}/4\pi L^2\Sigma_c r_k \qquad (3.2)$$

If the resonance absorption of neutrons is taken into consideration and assumed to be due to one resonance level, the following expression may be obtained:

$$\sum_k^\infty F^*(r_k) = \sum_k^\infty \int_\infty \frac{e^{-r^2/4\tau}}{(2\sqrt{\pi\tau})^3} \frac{e^{-|r-r_k|/L}}{4\pi L^2\Sigma_c|r-r_k|}\,dr$$

$$- \frac{A}{V\Sigma_c} \sum_k^\infty \int_\infty \frac{e^{-r^2/4\tau_r}}{(2\sqrt{\pi\tau_r})^3} \cdot \frac{e^{-|r-r_k|/L}}{4\pi L^2\Sigma_c|r-r_k|}\,dr \qquad (4.2)$$

where $\tau \equiv \tau(E_0,kT)$, $\tau_r \equiv \tau(E_r,kT)$.

In the case of an infinite rectangular lattice consisting of line slugs in accordance with Equations 19.1* and 20.1, the critical condition is:

$$\gamma = \eta \sum_k^\infty F(r_k) - \sum_k^{\infty\prime} f(r_k) + \frac{0.577 + \ln\frac{R}{2L}}{2\pi L^2\Sigma_c} \qquad (5.2)$$

where

$$\sum_k^\infty F(r_k) = \sum_k^\infty \int_\infty \frac{e^{-\frac{r^2}{4\tau}}}{4\pi\tau} \frac{K_0\left(\frac{|r-r_k|}{L}\right)}{2\pi L^2\Sigma_c}\,dr \qquad (6.2)$$

$$\sum_k^{\infty\prime} f(r_k) = \sum_k^\infty \frac{K_0\left(\frac{r_k}{L}\right)}{2\pi L^2\Sigma_c} \qquad (7.2)$$

Using the Fourier transformation in the case of line slugs we get the following expressions:

$$2\pi L^2\Sigma_c F(r) = \int_\infty \frac{e^{-r_1^2/4\tau}}{4\pi\tau} K_0\left(\frac{|r-r_1|}{L}\right) dr_1$$

$$= \int_0^\infty \frac{xe^{-x^2\tau} I_0(xr)}{x^2 + 1/L^2}\,dx \qquad (8.2)$$

To calculate infinite sums in the case of a square lattice consisting of identical slugs with a spacing a, the Poisson summation formula may be applied; after a set of intermediate computations the following simple result evolves:

$$2\pi L^2\Sigma_c \sum_k^\infty F(r_k) = \frac{2\pi L^2}{a^2} \sum_{\vec{\nu}}^\infty \frac{e^{-\frac{4\pi^2\tau\nu^2}{a^2}}}{1 + \frac{4\pi^2L^2}{a^2}\nu^2} \qquad (9.2)$$

If the condition $4\pi^2\tau/a^2 \gg 1$, is fulfilled, Equation 9.2 takes the form

$$2\pi L^2\Sigma_c \sum_k^\infty F(r_k) \approx \frac{2\pi L^2}{a^2}\left(1 + \frac{4e^{-4\pi^2\tau/a^2}}{1 + \frac{4\pi^2L^2}{a^2}}\right) \qquad (9.2^*)$$

Usually with sufficient precision,

$$\sum_k^\infty F(r_k) = \frac{1}{a^2\Sigma_c} \qquad (10.2)$$

It follows from this result that the slowing down density is almost constant if $e^{-4\pi^2\tau/a^2} \ll 1$. Furthermore, with this condition fulfilled, the slowing-down density does not depend on the configuration of the lattice cell.

A. D. Galanin‡ successfully replaced the slowly converging series $\sum_k^\infty f(r_k)$ by another with a more rapid convergence. This enabled him to estimate the accuracy of calculation of the thermal utilization θ by the method of equivalent cells and to find the dependence of θ on the form of the periodic lattice cell. Most important is the result of calculating $\sum_k^\infty f(r_k)$ for a lattice with a cell in the form of a parallelogram the sides of which are a_1 and a_2, the angle between them being ω.

When computing the sums of f the integral representation of $K_0(x)$ may be used.

$$2\pi L^2\Sigma_c f(r) = K_0\left(\frac{r}{L}\right) = \int_0^\infty e^{-\left(z+\frac{r^2}{4L^2z}\right)}\frac{dz}{2z} \quad (11.2)$$

Then, summing according to the Poisson rules gives§

$$\sum_k{}' K_0\left(\frac{|r-r_k|}{L}\right) = \frac{2\pi L^2}{\sigma}\sum_{\nu_1,\nu_2}^\infty \frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left[1+\frac{4\pi^2L^2}{\sin^2\omega}\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1a_2}\cos\omega\right)\right]} \quad (12.2)$$

If the conditions $4\pi L^2/a_1a_2\sin^2\omega \gg 1$ are fulfilled, then

$$\sum_k{}' K_0\left(\frac{|r-r_k|}{L}\right) = \frac{2\pi L^2}{\sigma}\left(1+\frac{\sin^2\omega}{4\pi^2L^2}\right)\sum_{\nu_1,\nu_2}{}' \frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1a_2}\cos\omega\right)} \quad (13.2)$$

σ is the area of the lattice-cell; a prime means that the terms with $\nu_1 = \nu_2 = 0$ must be dropped.

Thermal utilization θ of an infinite lattice where $K_\infty = 1$, is the absence of resonance absorption in the moderating process, is, by definition:

$$\theta = \frac{\sum_k^\infty i_k}{\sum_k^\infty q_k} = \frac{\sum_k^\infty i_k}{\eta\sum_k^\infty i_k} = \frac{1}{\eta} \quad \text{or} \quad \frac{1-\theta}{\theta} = \eta - 1 \quad (14.2)$$

‡ A. D. Galanin, P/666 "The Thermal Coefficient in a Heterogeneous Reactor," this Session.

§ It must be noted that the following series are not absolutely convergent. For greater details see the work of A. D. Galanin.

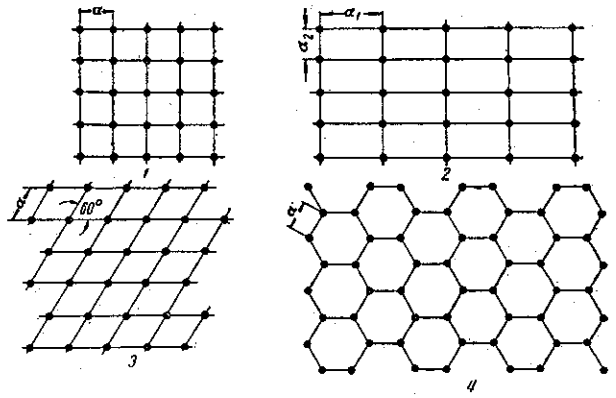

Figure 2. Scheme of lattices of the active zone of a reactor. 1. Square lattice with a spacing "a"; 2. Rectangular lattice with spacing "a" and "a_2"; 3. Rhomboid lattice with a spacing "a" and an angle $\omega = 60°$; 4. Hexagonal lattice with spacing "a"

From Equation 5.2, taking into account Equation 14.2, we get:

$$\frac{1-\theta}{\theta} = \eta - 1 = \frac{\gamma + \sum_k^\infty f(r_k)}{\sum_k^\infty F(r_k)} - 1 \quad (15.2)$$

In accordance with Equations 15.2 and 10.2 we further obtain:

$$\frac{1-\theta}{\theta} = \gamma a^2\Sigma_c + \left(a^2\Sigma_c\sum_k^\infty f(r_k) - 1\right) = \gamma a^2\Sigma_c + g \quad (16.2)$$

where g designates the value

$$g = a^2\Sigma_c\sum_k^\infty f(r_k) - 1 \quad (17.2)$$

Remembering that

$$\sum_k^\infty f(r_k) = \frac{1}{2\pi L^2\Sigma_c}\sum_k^\infty K_0\left(\frac{|\vec{\rho}-r_k|}{L}\right)$$

then

$$g = \frac{a^2}{2\pi L^2}\sum_k^\infty K_0\left(\frac{|r_k-\vec{\rho}|}{L}\right) - 1$$

In the case of the lattice under discussion we have for g:

$$g = \frac{\sin^2\omega}{4\pi^2L^2}\sum_{\nu_1,\nu_2}^\infty \frac{\cos\left[2\pi\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)\right]}{\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1a_2}\cos\omega} \quad (18.2)$$

For a square lattice $a_1 = a_2 = a$; $\omega = \pi/2$; calculations lead to:

$$g = \frac{a^2}{4\pi L^2}\left[\ln\frac{a^2}{\pi R^2} + \frac{\pi R^2}{a^2} - \ln 4\pi + \frac{\pi}{3}\right] \quad (19.2)$$

$$\ln 4\pi - \pi/3 = 1.48$$

Calculation of θ by the equivalent cell method gives the same formula with substitution of 1.48 by 1.5. This shows that the calculation of θ by the equivalent cell method is precise enough.

For a rectangular lattice when $\omega = \pi/2$ and $a_1 \neq a_2$ we get:

$$g = \frac{a_1 a_2}{4\pi L^2}\left[\ln\frac{a_1 a_2}{\pi R^2} + \frac{\pi R^2}{a_1 a_2} - \ln 4\pi + \frac{\pi}{6}\left(\frac{a_1}{a_2} + \frac{a_2}{a_1}\right)\right] \quad (20.2)$$

For a rhomboid lattice with the side of the cell a and the angle $\omega = 60$ degrees

$$g = \frac{\sigma}{4\pi L^2}\left[\ln\frac{\sigma}{\pi R^2} + \frac{\pi R^2}{\sigma} - \ln 4\pi + \frac{\pi}{3} + \ln 2 - \frac{\pi}{3}\left(1 - \frac{1}{\sqrt{3}}\right)\right] \quad (21.2)$$

And lastly, for a hexagonal lattice with a side a:

$$g = \frac{\sigma}{4\pi L^2}\left[\ln\frac{\sigma}{\pi R^2} - 1.61\right] \quad (22.2)$$

The lattice schemes are given in Fig. 2.

If we compare a rhomboid and hexagonal lattice with a square lattice having identical cell areas σ we find that the rhomboid lattice suffers a loss in $\delta g = 0.25\sigma/4\pi L^2$, the hexagonal lattice shows an increase of $\delta g = 0.13\sigma/4\pi L^2$ in comparison to the square lattice.

Composite Lattices (Double and Triple)

S. M. Feinberg, E. P. Kunyegin and B. D. Slutskaya, and also independently A. D. Galanin, applied the heterogeneous method in calculating "multilattices," consisting of regularly alternating cylindrical slugs, differing from each other in their absorbing (γ_k,R_k) and multiplying properties (η_k).

Such composite lattices may be made up of enriched uranium slugs and control rods or slugs intended for the production of radioactive isotopes.

The composite lattice calculations enable us to evaluate the efficiency of the control rods system used in the reactor, the redistribution of heat generation in the uranium slugs due to the control rods, and the part of neutrons absorbed in slugs of different types.

Figures 3 and 4 represent "double" and "triple" lattices. It is not difficult to observe that composite lattices may be considered as regular lattices, inserted one into the other. For instance, the double lattice in Fig. 3 splits into three regular lattices inserted into each other. Each of these regular lattices is composed of slugs in identical conditions. Thus, the number of unknown M values in the system of Equation 18.1 is equal to the number of the above-mentioned regular lattices. The equation system in the case of a double lattice (Fig. 3) is as follows:

$$\gamma_n i_n = \sum_{s=1}^{s=3} i_s \sum_k^{\infty} (\eta_s F_{nk} - f_{nk}); \qquad n = 1, 2, 3 \ldots$$
$$\gamma_3 = \gamma_2 \neq \gamma_1, \qquad \eta_3 = \eta_2, \qquad \eta_1 = 0 \quad (23.2)$$

The infinite sums were extended over all the identical rods of one of the three regular lattices.

The sum $\sum_k^{\infty} f_{nk}$ were computed by a method analogical to the above-mentioned one. They are equal with sufficient precision to $1/a^2\Sigma_c$. In more complicated cases the next terms of the series in Equation 9.2 may be computed.

In the work by S. M. Feinberg, E. P. Kunyegin and B. D. Slutskaya the sums $\sum_k^{\infty} f_{nk}$ were calculated numerically. In the work by A. D. Galanin, as has been mentioned above, the rapidly converging series of Equations 12.2 and 13.2 were used (see Fig. 4).

The results of several typical calculations for composite lattices are given in Tables Ia and Ib.

In one case of a composite lattice when the number of unknown values $M = 2$, a simple analytical expression may be obtained for K_∞ and L^2 in such a lattice.

A method for calculating composite lattices has also been elaborated for a case, when the moderator does not absorb thermal neutrons. In this case the method of heterogeneous calculation described in the first section is not applicable in consequence of the

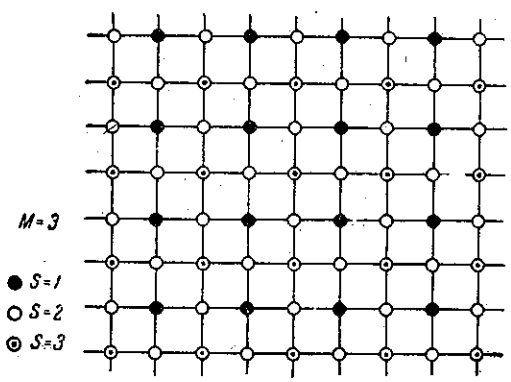

Figure 3. Scheme of a double lattice; the number of unknown quantities $M = 3$. 1. absorbing rods; 2, 3. multiplying slugs

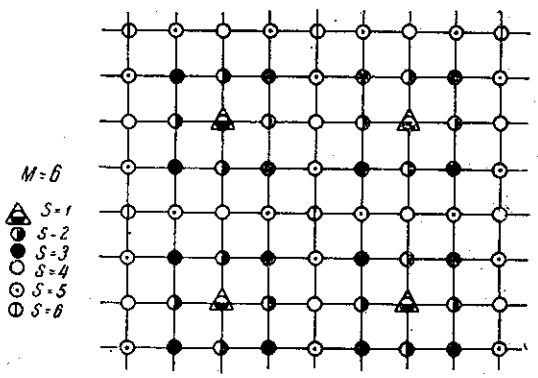

Figure 4. Scheme of triple lattice; the number of unknown quantities $M = 6$. 1. control rods; 2, 3, 5. multiplying slugs; 4, 6. slugs producing isotopes

Table Ia. For a "Double" Lattice, Fig. 3

$\frac{L^2}{\tau}$	$\frac{L}{a}$	$\frac{R}{a}$	$2\pi D\gamma_{2,3}$ $(D = \lambda_{tr}/3)$	$\gamma_{2,3}/\gamma_1$	$\eta_{2,3}$	η_1	Φ_1/Φ_2	Φ_2/Φ_3
10	2.74	0.107	4.75	0	1.17	0	1.41	1.03
10	2.74	0.107	4.75	0.5	1.36	0	1.17	1.01
10	2.74	0.107	4.75	1.0	1.49	0	1.00	1.00
10	2.74	0.107	4.75	1.5	1.60	0	0.873	0.992
10	2.74	0.107	4.75	2.0	1.68	0	0.775	0.985

divergence of $\sum_k^\infty f_{nk}$. Here use was made of the doubly-periodic solution of the Poisson equation $\Delta\Phi = \text{const.}$, possessing logarithmic singularities in the nodes of the lattice.

3. THE PROOF OF THE HOMOGENIZATION METHOD

A consequent derivation of formulas obtained by the homogenization method (i.e., the proof of this extensively applied rule) is of theoretical interest.

According to this method, as we know, at a distance from the boundary of the core of a large reactor the spatial distribution of a flux of neutrons Φ is described by superposition of the following expressions:

$$\Phi = e^{i\vec{\kappa r}} \tag{1.3}$$

where κ^2 is the real root of the characteristic equation

$$K_\infty = e^{\kappa^2\tau}(1 + \kappa^2 L_c^2) \tag{2.3}$$

it being known that

$$K_\infty = \eta\theta, \qquad L_c^2 = L^2(1 - \theta) \tag{3.3}$$

Here κ^2 is the buckling, characterizing the effective critical dimension of the core of the reactor, i.e. the active zone increased by the reflector savings; θ is the thermal utilization coefficient in an infinite heterogeneous lattice with $\kappa = 0$.

The problem consists in defining the conditions at which Equations 1.3 and 2.3 are consequences of "heterogeneous" equations.

A. D. Galanin and V. V. Vladimirski have shown that for the spatial distribution of a flux of neutrons Φ similar to Equation 1.3 a characteristic equation, approximately coincident with Equations 2.3 and 3.3, follows from the heterogeneous Equations 17.1.

We shall proceed from equations 15.1. Noting that α_{rk} is a function of $\alpha_{nk}(|r_n - r_k|)$ we may write Equations 15.1 as follows:

$$\begin{aligned} \gamma i_n &= \sum_k^\infty i_k \alpha(|r_n - r_k|) \\ \alpha_{nk} &= \eta F(|r_n - r_k|) - f(|r_n - r_k|) \end{aligned} \tag{4.3}$$

It is easy to discern that $i_n = e^{i\vec{\kappa r}_n}$ is the solution of a system of Equations 4.3. Actually, the sum $\sum_k^\infty \alpha(|r_n - r_k|)e^{i\vec{\kappa r}_k}$ may be transformed thus:

$$\sum_k^\infty \alpha(|r_n - r_k|)e^{i\vec{\kappa r}_k} = e^{i\vec{\kappa r}_n}\sum_k^\infty \alpha(|r_n - r_k|)e^{i\vec{\kappa}(r_k - r_n)} \tag{5.3}$$

It is quite clear that for an infinite lattice

$$\sum_k^\infty \alpha(|r_n - r_k|)e^{i\vec{\kappa}(r_n - r_k)} = \sum_k^\infty \alpha(|r_k|)e^{i\vec{\kappa r}_k} \tag{6.3}$$

And so, we have:

$$\gamma e^{i\vec{\kappa r}_n} = e^{i\vec{\kappa r}_n}\sum_k^\infty \alpha(|r_k|)e^{i\vec{\kappa r}_k} \tag{7.3}$$

From which

$$\gamma = \sum_k^\infty \alpha(|r_k|)e^{i\vec{\kappa r}_k} = \eta\sum_k^\infty F(|r_k|)e^{i\vec{\kappa r}_k} - \sum_k^\infty f(|r_k|)e^{i\vec{\kappa r}_k} \tag{8.3}$$

Table Ib. For a "Triple" Lattice, Fig. 4

$\frac{L^2}{\tau}$	$\frac{L}{a}$	$\frac{R}{a}$	$2\pi D\gamma_1$ $(D = \lambda_{tr}/3)$	$\gamma/\gamma_{4,6}$	$\gamma_{2,3,5}/\gamma_1$	$\eta_{2,3,5}$	η_2	η_3	Φ_2/Φ_1	Φ_3/Φ_1	Φ_4/Φ_1	Φ_5/Φ_1	Φ_6/Φ_1
10	2.74	0.107	2.0	0.6	0	1.27	0	0	0.504	0.478	0.587	0.473	0.578
10	2.74	0.107	2.0	1.0	0	1.33	0	0	0.496	0.470	0.460	0.455	0.451
10	2.74	0.107	2.0	1.4	0	1.36	0	0	0.490	0.464	0.378	0.444	0.370
10	2.74	0.107	2.0	0.6	1	1.35	0	0	1.018	1.019	1.276	1.039	1.281
10	2.74	0.107	2.0	1.0	1	1.42	0	0	1.0	1.0	1.0	1.0	1.0
10	2.74	0.107	2.0	1.4	1	1.46	0	0	0.988	0.988	0.822	0.975	0.820
10	2.74	0.107	2.0	0.6	3	1.38	0	0	2.046	2.101	2.654	2.171	2.688
10	2.74	0.107	2.0	1.0	3	1.44	0	0	2.010	2.061	2.080	2.090	2.098
10	2.74	0.107	2.0	1.4	3	1.49	0	0	1.984	2.035	1.711	2.037	1.720
10	2.74	0.107	4.75	0.6	0	1.331	0	0	0.707	0.685	0.769	0.681	0.760
10	2.74	0.107	4.75	1.0	0	1.405	0	0	0.697	0.674	0.665	0.661	0.651
10	2.74	0.107	4.75	1.4	0	1.47	0	0	0.690	0.665	0.588	0.645	0.576
10	2.74	0.107	4.75	0.6	3	1.48	0	0	1.629	1.679	1.925	1.731	1.958
10	2.74	0.107	4.75	1.0	3	1.56	0	0	1.605	1.651	1.668	1.679	1.688
10	2.74	0.107	4.75	1.4	3	1.63	0	0	1.587	1.631	1.471	1.639	1.484

The first of the sums, written on the right side of the equation obtained, may be reduced to:

$$2\pi L^2\Sigma_c \sum_k^{\infty} F(|r_k|)e^{i\kappa r_k}$$

$$= \frac{2\pi L^2}{\sigma} \sum_{\nu}^{\infty} \frac{e^{-\frac{4\pi^2\tau}{a^2}\left(\vec{\nu}+\frac{\vec{a\kappa}}{2\pi}\right)^2}}{1+4\pi^2\frac{L^2}{a^2}\left(\vec{\nu}+\frac{\vec{a\kappa}}{2\pi}\right)^2} \approx \frac{2\pi L^2}{\sigma}\frac{e^{-\kappa^2\tau}}{1+\kappa^2L^2} \tag{9.3}$$

The latter expression is valid when $4\pi^2\tau \gg a^2$; $\kappa a \ll 1$.

The second sum in Equation 8.2 is defined by this formula:

$$2\pi L^2\Sigma_c \sum_k^{\infty} f(|r_k|)e^{i\kappa r_k}$$

$$= \frac{2\pi L^2}{\sigma} \sum_{\nu_1,\nu_2}^{\infty} \frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{1+\frac{4\pi L^2}{\sin^2\omega}\left[\left(\frac{\nu_1}{a_1}+\frac{\kappa_1}{2\pi}\right)^2+\left(\frac{\nu_2}{a_2}+\frac{\kappa_2}{2\pi}\right)^2 - 2\left(\frac{\nu_1}{a_1}+\frac{\kappa_1}{2\pi}\right)\left(\frac{\nu_2}{a_2}+\frac{\kappa_2}{2\pi}\right)\cos\omega\right]} \tag{10.3}$$

If $\kappa a \ll 2\pi$, then

$$2\pi L^2\Sigma_c \sum_k^{\infty} f(|r_k|)e^{i\kappa r_k} \approx -\frac{2\pi L^2}{\sigma}\left(g+\frac{1}{1+\kappa^2L^2}\right) \tag{10.3*}$$

where g is given in Equation 17.2. And so, when conditions $4\pi^2L^2 \gg a^2$, $4\pi^2\tau \gg a^2$ and $\kappa a \ll 2\pi$ are fulfilled, Equation 8.3 may be written as:

$$\Sigma_c \cdot \gamma \approx \eta\frac{e^{-\kappa^2\tau}}{1+\kappa^2L^2}\frac{1}{\sigma} - \left(\frac{1}{1+\kappa^2L^2}+g\right)\frac{1}{\sigma} \tag{11.3}$$

where g does not depend on $|\kappa|$ and is determined by Equation 18.2. The last equation may be written as follows:

$$K_\infty = e^{\kappa^2\tau}(1+\kappa^2L_{lattice}^2) \tag{12.3}$$

where

$$K_\infty = \frac{\eta}{1+\gamma\Sigma_c\sigma+g}, \qquad L^2_{lattice} = L^2\frac{\gamma\sigma\Sigma_c+g}{1+\gamma\Sigma_c\sigma+g} \tag{13.3}$$

It remains to be pointed out that in the case of $\kappa = 0$, as has been shown in Section 2 (Formula 16.2) the following relation is true:

$$\gamma\Sigma_c\sigma + g = \frac{1-\theta}{\theta} \tag{14.3}$$

Consequently,

$$K_\infty = \eta\theta, \qquad L_{lattice}^2 = L^2(1-\theta) \tag{15.3}$$

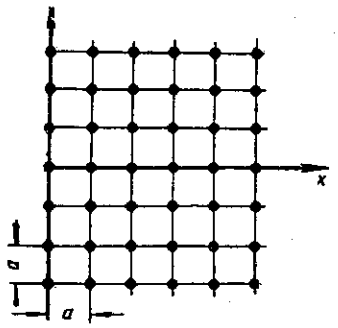

Figure 5. Scheme of a semi-infinite lattice with a spacing

It must be pointed out that if instead of a slowing down flux (j) of the form of Equation 1.2 another form of expression is used for "j", i.e., going over the limits imposed by the age approximation, then the function $e^{-\kappa^2\tau}$ in Equation 12.3 will be substituted by the Fourier-component j.

4. DEFINITION OF EFFECTIVE BOUNDARY CONDITIONS

In large multiplying systems near the core boundary or near the irregularities of the lattice (for instance, near the absorbing rod) a complicated irregular spatial distribution of the neutron flux is to be observed. On the contrary, far enough from the core boundary or the absorbing rod a regularly varying asymptotic spatial distribution of neutrons is established. To characterize the reactor as a whole this distribution is most important. If the variation of the neutron flux within one cell of the lattice be disregarded, the asymptotic dependence of i_n, when $|n|$ increases, may easily be found up to a certain quantity δ. This asymptotic dependence is not difficult to obtain from "homogeneous equations." With $k_\infty = 1$ the asymptotic dependence i on $|n|$ for a semi-infinite reactor has the form:

$$i_n \to \text{const}\ (an+\delta)$$

For an infinite reactor with spherical absorbing slugs:

$$i_n \to \text{const}\left(1-\frac{\delta}{r_n}\right)$$

The problem of determination of δ shall further on be designated as the problem of effective boundary conditions.

EFFECTIVE BOUNDARY CONDITIONS IN AN INFINITE REACTOR WITH A SEMI-INFINITE ACTIVE ZONE

A. M. Budker *et al.* considered a multiplying system in the form of an infinite moderator, in which a cubic lattice of point (spherical) slugs filling the half-space $x > 0$ (Fig. 5) was situated.

One shall proceed from the system of Equations 15.1, where $\gamma_n = \gamma = \text{const.}$:

$$\gamma i_n = \sum_{k=0}^{\infty} i_k\alpha_{nk}$$

Due to symmetry considerations for all slugs with fixed x the fluxes i_k are equal to each other and Equation 15.1 take the form:

$$\gamma i_n = \sum_{k=0}^{k=\infty} i_k \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} \alpha_{n(k,l,m)} = \sum_{k=0}^{k=\infty} b_{nk} i_k \quad (1.4)$$

where

$$b_{nk} = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} \alpha_{n(k,l,m)} \quad (2.4)$$

This equation system has an infinite number of unknown quantities.

Since for a semi-infinite reactor a stationary solution of Equation 1.4 is possible only when $\kappa = 0$, at a far enough distance from the boundary of the active zone, that is, when $x_n \gg a$ the solution may be written as follows:

$$i_n = \underset{n\to\infty}{\text{const}} (na + \delta) \quad (3.4)$$

where the const. and δ are both constant.

Let us recall that the problem of effective boundary conditions consists in determining the value of δ. Calculation of the value l_{nk} in the first approximation leads to the following result:

$$b_{nk} \approx \eta\pi \frac{L}{a} e^{\tau/L^2} \left\{ e^{(n-k)\frac{a}{L}} \left[1 - \operatorname{erf} \left(\frac{n-k}{2\frac{\sqrt{\tau}}{a}} + \frac{\sqrt{\tau}}{L} \right) \right] \cdot \right.$$

$$\left. + e^{-(n-k)\frac{a}{L}} \left[1 + \operatorname{erf} \left(\frac{n-k}{2\frac{\sqrt{\tau}}{a}} - \frac{\sqrt{\tau}}{L} \right) \right] \right\}$$

$$- 2\pi \frac{L}{a} e^{-(n-k)\frac{a}{L}} \quad (4.4)$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

It should be noted that Equation 4.4 is approximate, since the last term is obtained by substituting an integral for the sums of singular functions, which is, generally speaking, unjustified. We should also note that b_{nk} depends on the absolute value of the difference $(|n - k|)$.

$$b_{nk} = b(|n - k|) = b_s; \qquad s = |n - k| \quad (5.4)$$

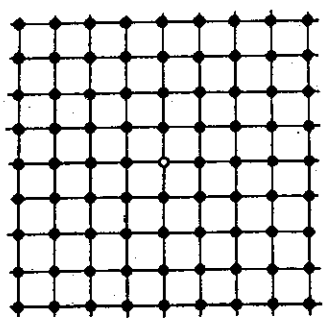

Figure 6. Scheme of an infinite lattice consisting of identical slugs; at the origin of the coordinates there is one slug with different properties

Then the developed system of Equation 1.4 has the form:

$$\begin{vmatrix} (b_{00} - \gamma) & b_{01} & b_{02} & \cdots \\ b_{10} & (b_{11} - \gamma) & b_{12} & \cdots \\ b_{20} & b_{21}, & (b_{22} - \gamma) & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

$$= \begin{vmatrix} Mb_1b_2 & \cdots \\ b_1Mb_1 & \cdots \\ b_2b_1M & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{vmatrix} = 0 \quad (6.4)$$

where

$$-M = \gamma - b_{nn} = \gamma - b_0 \quad (7.4)$$

To fulfill the stationary conditions for a semi-infinite reactor it is also necessary to fulfill them for an infinite reactor having a lattice identical to the first. This makes it possible to find the value of M. Indeed, for an infinite stationary lattice where $i_k = i_0 =$ const., Equations 15.1 take the form:

$$\gamma = \sum_{-\infty}^{+\infty} b_k = b_0 + 2\sum_{1}^{\infty} b_k \qquad \text{or} \qquad \gamma - b_0 = 2\sum_{1}^{\infty} b_k$$

Comparing with Equation 7.4 we get:

$$M = -2\sum_{1}^{\infty} b_k \quad (8.4)$$

Now the reflector savings δ may be evaluated. It is supposed that beginning from the mth slug there is an almost linear dependence of the neutron flux i_k on the number of the slug. Then for $k > m$

$$i_k = \frac{k + \delta}{m + \delta} i_m + \delta(i_k) \quad (9.4)$$

where $\delta(i_k)$ are presumably small.

Substituting Equation 9.4 into Equation 1.4 and using the above found value of M, we write:

$$i_n \left(2\sum_{1}^{\infty} b_k \right) = \sum_{k=1}^{k=m-1} i_k b_{n-k}$$

$$+ i_m \frac{1}{m + \delta} \sum_{k=m}^{\infty} (k + \delta) b_{n-k} + \sum_{k=m}^{\infty} \delta(i_k) b_{n-k} \quad (10.4)$$

The system of Equation 10.4 is "out of" at a certain value of m, i.e., it is assumed that the sum $\delta(i_k)$ may be disregarded. This gives us a homogeneous linear system of $m + 1$ equations. The eigenvalue of this system is the unknown quantity δ. The latter equals:

$$\frac{\delta}{a} = \frac{\begin{vmatrix} -\left(2\sum_1^\infty b_k\right) & b_1 & b_2 \cdots & \left(\sum_m^\infty k b_k\right) \\ b_1 & -\left(2\sum_1^\infty b_k\right) & b_1 \cdots & \left(\sum_m^\infty k b_{k-1}\right) \\ \cdots & \cdots & \cdots & \cdots \\ b_m, & b_{m-1} \cdots & \left(\sum_{m+1}^\infty k b_{k-m} - 2m\sum_1^\infty b_k\right) & \end{vmatrix}}{\begin{vmatrix} -\left(2\sum_1^\infty b_k\right) & b_1 & b_2 \cdots & \left(\sum_m^\infty b_k\right) \\ b_1 & -\left(2\sum_1^\infty b_k\right) & b_1 \cdots & \left(\sum_m^\infty b_{k-1}\right) \\ \cdots & \cdots & \cdots & \cdots \\ b_m & b_{m-1} \cdots & \left(\sum_{m+1}^\infty b_{k-m} - 2\sum_1^\infty b_k\right) & \end{vmatrix}} \tag{11.4}$$

Taking various values for m beginning from $m = 1$, we find, generally speaking, various values for δ. The extent of dispersion of the found values of δ characterizes the error made in computing this value.

As an example let us consider a case when $\tau/L^2 = 0.12$; $a/L = 0.4$; $\eta = 1.22$. Calculation in accordance with Equation 11.5 gives such a result:

$m = 1$	2	3
$\delta/a = 3.2$	3.15	3.14

Effective Boundary Conditions in an Infinite Reactor with an Absorbing Rod

For an infinite lattice, consisting of identical point slugs in which only one slug is substituted by a neutron absorbing rod (Fig. 6) with a different absorptive and emissive capacity ($\eta \neq \eta_0 = \eta_k$; $\gamma \neq \gamma_0 = \gamma_k$), the problem of effective boundary conditions consists in finding the value of δ in the asymptotic expression for the dependence of the neutron flux i_k absorbed by the kth slug on the index k

$$i_k \to \text{const}\left(1 - \frac{\delta}{|r_k|}\right), \quad |k| \to \infty \tag{12.4}$$

A method of calculation analogous to the one described in the last section may be used in solving this problem. Thus, Equation 14.1 for the problem under consideration may be written:

$$n \neq 0: \quad \gamma_0 i_n = \sum_{-\infty}^{+\infty}{}' \alpha\,(|r_k - r_n|)i_k - i_0 f(|r_n|)$$

$$\equiv \sum_{-\infty}^{+\infty} \alpha(|r_k - r_n|)i_k - i_0\eta F(|r_n|)$$

$$n = 0: \quad \gamma_\delta i_0 = \sum_{-\infty}^{+\infty}{}' \alpha(|r_k|)i_k - i_0 f(R_0)$$

$$\equiv \sum_{-\infty}^{+\infty} \alpha(|r_k|)i_k - i_0\eta F(0) \tag{13.4}$$

where γ_0 is the absorption coefficient for the slugs of the lattice; γ_δ is the same quantity for a slug placed in the origin of the coordinates (with a radius R_0); $r_n = an$, n is an integer vector, and a is the spacing of the cubic lattice.

It is possible, beginning with any n to substitute for the flux i_n its asymptotic expression Equation 12.4 and further, disregarding small terms, to obtain for calculating δ a system consisting of $\sim n^3$ "out-of" equations.

However, it is more interesting to describe another method used in solving this problem.

If $|n|$ in Equation 13.4 would be infinitely large, then the member $i_0\eta F(|r_n|) \to 0$ and therefore:

$$\gamma_0 i_n \to \sum_{-\infty}^{+\infty} \alpha(|r_k - r_n|)i_k \tag{14.4}$$

The solution of this asymptotic equation is: $i_n \to \text{const} = i_\infty$ i.e.,

$$\gamma_0 i_\infty = \sum_{-\infty}^{+\infty} \alpha(|r_k - r_n|)i_\infty \tag{15.4}$$

from which

$$\gamma_0 = \sum_{-\infty}^{+\infty} \alpha(|r_k|) \tag{16.4}$$

Subtracting separately from each term of Equation 15.4 the corresponding term of Equation 13.4 we find:

$$n \neq 0: \quad \gamma_0 i_n' = \sum_{-\infty}^{+\infty} \alpha(|r_k - r_n|)i_k' + i_0\eta F(|r_n|)$$

$$n = 0: \quad \gamma_0 i_0' = \sum_{-\infty}^{+\infty} \alpha(|r_k|)i_k' + i_0\eta F(0) + i_0\Delta\gamma \tag{17.4}$$

where $i_n = i_\infty - i_n$ and $\Delta\gamma = \gamma_\delta - \gamma_0$.

From Equation 14.4 it is easy to see that $i_n \to 0$ when $|n| \to \infty$. Then $i_n'| \sim 1/a|n||$ decreases, as we shall see later.

Let us multiply each of Equations 17.4 by $e^{i\vec{\alpha}\vec{\kappa}n}$ and summarize the results (κ is an arbitrary vector)

$$\gamma_0 \sum_{-\infty}^{+\infty} e^{i\vec{\alpha}\vec{\kappa}n} i_n' = \sum_{-\infty}^{+\infty} e^{i\vec{\alpha}\vec{\kappa}n} \sum_{-\infty}^{+\infty} \alpha(|r_k - r_n|) i_k' + i_0\eta \sum_{-\infty}^{+\infty} e^{i\vec{\alpha}\vec{\kappa}n} F(|r_n|) + i_0 \Delta\gamma \quad (18.4)$$

Changing the order of summation in the double sum, we write

$$\sum_{-\infty}^{+\infty}\sum_{-\infty}^{+\infty} = \sum_{-\infty}^{+\infty} i_k' \sum_{-\infty}^{+\infty} e^{i\vec{\alpha}\vec{\kappa}n} \alpha(|r_k - r_n|) = \sum_{-\infty}^{+\infty} i_k' e^{i\vec{\alpha}\vec{\kappa}k} \sum_{-\infty}^{+\infty} e^{i\vec{\alpha}\vec{\kappa}n} \alpha(|r_n|) \quad (19.4)$$

Now we introduce auxiliary periodic functions of the parameter $\vec{\kappa}$, defining them by the following Fourier series:

$$\tilde{i}(\vec{\kappa}) = \sum_{-\infty}^{+\infty} i_n' e^{i\vec{\alpha}\vec{\kappa}n} \quad \text{when} \quad |\vec{\kappa}| \neq 0,$$

$$\tilde{\alpha}(\vec{\kappa}) = \sum_{-\infty}^{+\infty} \alpha(|r_n|) e^{i\vec{\alpha}\vec{\kappa}n}, \quad (20.4)$$

$$\tilde{F}(\vec{\kappa}) = \sum_{-\infty}^{+\infty} F(|r_n|) e^{i\vec{\alpha}\vec{\kappa}n} \quad (21.4)$$

Note that the sum of Equation 20.4 converges at any $\kappa \neq 0$, but when $\kappa = 0$ it diverges. Using Equations 20.4, 21.4 and 19.4 Equation 18.4 may be written in the form

$$\kappa \neq 0: \quad \gamma_0 \tilde{i}(\vec{\kappa}) = \tilde{\alpha}(\vec{\kappa})\tilde{i}(\vec{\kappa}) + i_0(\eta\tilde{F}(\vec{\kappa}) + \Delta\gamma) \quad (22.4)$$

Noting from Equations 16.4 and 21.4 that $\gamma_0 = \tilde{\alpha}(0)$ we obtain

$$\vec{\kappa} \neq 0: \quad \tilde{i}(\vec{\kappa}) = i_0 \frac{\eta\tilde{F}(\vec{\kappa}) + \Delta\gamma}{\tilde{\alpha}(0) - \tilde{\alpha}(\vec{\kappa})} \quad (23.4)$$

Now i_n' may be computed from Equation 20.4 if the attention is paid to the fact that Equation 20.4 is an expression of the periodic function $\tilde{i}(\vec{\kappa})$ in the Fourier series. Thus the Fourier coefficient for this expansion is

$$i_n = \frac{i_0 a^3}{(2\pi)^3} \int_{\text{by the period}} \frac{\Delta\gamma + \eta\tilde{F}(\vec{\kappa})}{\tilde{\alpha}(0) - \tilde{\alpha}(\vec{\kappa})} e^{-i\vec{\alpha}\vec{\kappa}n} d\vec{\kappa} \quad (24.4)$$

Assuming that in Equation 24.4 n is equal to zero we obtain an equation connecting (i_∞/i_0) with γ_δ the coefficient of the absorbing slug:

$$\frac{i_\infty}{i_0} = 1 + \frac{a^3}{(2\pi)^3} \int_{\text{by the period}} \frac{\Delta\gamma + \eta\tilde{F}(\vec{\kappa})}{\tilde{\alpha}(0) - \tilde{\alpha}(\vec{\kappa})} d\vec{\kappa} \quad (25.4)$$

Thus the solution of a system of equations with an infinite number of unknown quantities is reduced to a quadrature.

To bring the solution to a numeric result it is necessary to calculate the integral on the right side of Equation 25.4 using the sums of Equation 21.4. The latter converge quite rapidly.

As has been pointed out in Part 2 this convergence may be made more rapid.

Now we return to the question of the behaviour of i_n' at infinity. When $|n| \to \infty$ in the integral of Equation 24.4 only the range $|\vec{\alpha}| \to 0$ is of essential value.

From Equation 21.4 it easily follows that

$$|\vec{\kappa}| \to 0: \quad F(\vec{\kappa}) \to \sum_{-\infty}^{+\infty} F(|r_k|) > 0$$

$$\tilde{\alpha}(0) - \tilde{\alpha}(\vec{\kappa}) \approx \frac{a^2\kappa^2}{2} \sum_{\infty}^{+\infty} k^2 \alpha(|r_k|) \cos^2(\vec{\kappa},\vec{k}) \quad (26.4)$$

Consequently, when $|n| \to \infty$

$$i_n' \to \frac{i_0}{|n|} \frac{\Delta\gamma + \eta \sum_{-\infty}^{\infty} F(|r_n|)}{\frac{1}{3} \sum_{-\infty}^{+\infty} k^2 \alpha(|r_n|)} \quad (27.4)$$

In the latter i_0 may be substituted by i_∞ from Equation 25.4. Comparing Equations 27.4 and 25.4 we find the connection existing between δ and i_0/i_∞ or the connection of δ and γ_δ

$$\delta = a \frac{3(\Delta\gamma + n \sum_k F_k) / \sum_k k^2 \alpha_k}{1 + \frac{a^3}{(2\pi)^3} \int \frac{\Delta\gamma + \eta\tilde{F}(\vec{\kappa})}{\tilde{\alpha}(0) - \tilde{\alpha}(\vec{\kappa})} d\vec{\kappa}} \quad (28.4)$$

This problem has been studied previously by B. T. Geilikman, by another method for cases with spherical and cylindrical slugs. This author obtained

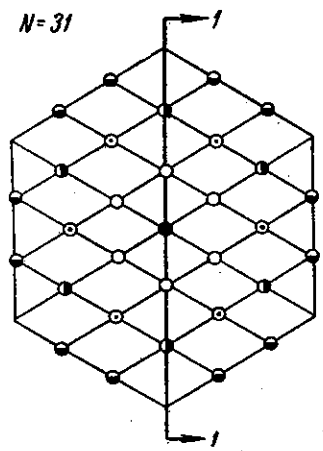

Figure 7. Scheme of a reactor with a small number of slugs (N = 31); the number of unknown quantities M = 5. Distribution of the neutron flux along the section 1-1 (see Fig. 9)

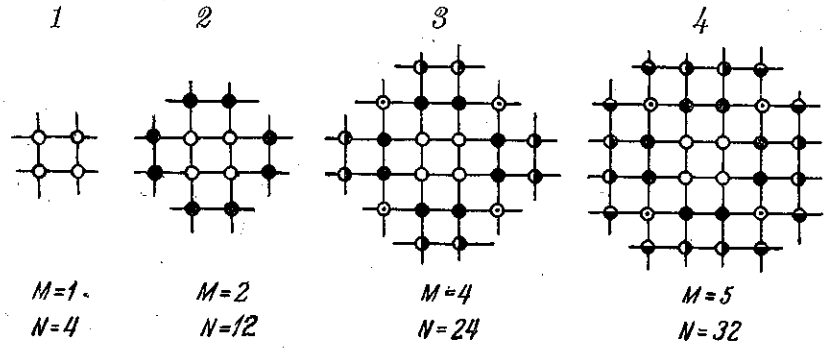

Figure 8. Scheme of a section with a small number of slugs. 1. Number of slugs N = 4; unknown quantities M = 1; 2. N = 12, M = 2; 3. N = 24, M = 2; 4. N = 32, M = 5

evaluations for δ assuming it possible to substitute integrals for infinite sums.

5. A REACTOR WITH A SMALL NUMBER OF SLUGS

For a reactor, the core of which consists of a small number of slugs, the application of the homogenization method is unjustified. The larger the number of slugs is the smaller is the difference between the calculations results for the heterogeneous and homogeneous methods. Comparison of the "homogeneous" and "heterogeneous" methods enables us to estimate the limits of validity and the error of the homogeneous method. The homogeneous method is especially unsatisfactory when applied to the calculation of the effect of an absorbing rod and the neutron field in a reactor in which the periodicity of the lattice has been violated.

The initial step for describing the stationary state of a reactor is again Equation 18.1.

$$\gamma_n i_n = \sum_{s=1}^{s=M} i_s \sum_{k'}^{N} (\eta_s F_{nk'} - f_{nk'}) \qquad (1.5)$$

For line slugs, in accordance with Equation 6.2

$$F_{kn} = F(d_{nk}) = \int_{\infty} (e^{-r^2/4\tau}/4\pi\tau) K_0\left(\frac{|\mathbf{r} - \mathbf{d}_{k,n}|}{L}\right) \frac{d\mathbf{r}}{2\pi L^2 \Sigma_c}$$

$$f_{kn} = \frac{1}{2\pi L^2 \Sigma_c} K_0\left(\frac{d_{k,n}}{L}\right) \quad \text{when} \quad k \neq n;$$

$$\text{when} \quad k = n \quad f_{n,n} = \frac{1}{2\pi L^2 \Sigma_c} K_0\left(\frac{R_n}{L}\right)$$

where $d_{kn} = |\mathbf{r}_k - \mathbf{r}_n|$ is the distance between slugs k and n. G. A. Bat used the numerical method of calculating $F(d)$, representing this function as:

$$F_{kn} = F(d_{kn}) \equiv F\left(\frac{L^2}{4\tau}, \frac{d_{kn}}{L}\right) \qquad (2.5)$$

where

$$F(\beta,x) = K_0(x) \int_0^x y I_0(y) e^{-\beta^2 y^2}\, dy + I_0(x) \int_0^x y K_0(y) e^{-\beta^2 y^2}\, dy$$

A. D. Galanin independently pointed out that when $\tau/L^2 \ll 1$ the function $F(d)$ may be represented as a rapidly converging series.

Keeping the first terms of the series, we may write:

$$F(d) = \frac{e^{\tau/L^2}}{2\pi L^2 \Sigma_c} \int_{\tau/L^2}^{\infty} e^{-\left(z + \frac{d^2}{4L^2 z}\right)} \frac{dz}{2z} \approx \frac{e^{\tau/L^2}}{2\pi L^2 \Sigma_c} \left[K_0\left(\frac{d}{L}\right) + \frac{\tau}{2L^2} e^{-d^2/4\tau} + \frac{1}{2}\left(1 + \frac{d^2}{4L^2}\right) Ei(-d^2/4\tau) \right] \qquad (3.5)$$

Assuming the determinant of the system of Equation 1.5 to be equal to zero, we obtain the critical conditions for the reactor.

Several reactors were calculated according to this method; their schemes are shown in Figs. 7 and 8.

Results of the "heterogeneous" calculations were compared with the results of the respective "homogeneous" calculations; the latter were done by a two-group method, the quantities K_∞, θ and L were computed by formulae which follow from the homogeneization rule mentioned in Part 3.

The results of calculations for a reactor, the scheme of which is given in Fig. 7, show that the difference between the critical masses was quite small when calculated by the heterogeneous and homogeneous methods respectively; it was equal to ~15 per cent.

Table III shows the results of calculations done by A. D. Galanin** for the schemes on Fig. 8.

The multiplication factors shown in Table II have been calculated by the known formulae:

$$K_{hetero} = \frac{\eta}{1 + Q + g}; \qquad Q = 2\pi D\gamma \frac{d^2}{2\pi L^2} \qquad (4.5)$$

$$g = \frac{a^2}{4\pi L^2}\left(\ln \frac{a^2}{\pi R^2} + \frac{\pi R^2}{a^2} - 1.48\right) \qquad (4.5)$$

$$R_{reactor} = \sqrt{\frac{Na^2}{\pi}}; \qquad \alpha^2 = -\frac{1}{2}\left(\frac{1}{L_{lattice}^2} + \frac{1}{\tau}\right) + \sqrt{\frac{1}{4}\left(\frac{1}{L_{lattice}^2} + \frac{1}{\tau}\right)^2 + \frac{K_\Pi - 1}{L_{lattice}^2 \tau}} = \frac{5{,}76}{R_{reactor}^2} \qquad (5.5)$$

$$K_{homo} = e^{\alpha^2\tau}(1 + \alpha^2 L_{lattice}^2) \qquad (6.5)$$

** For details see A. D. Galanin, P/663, "Critical Sizes of a Heterogeneous Reactor with a Small Number of Slugs," this Session.

Table II. For Fig. 4

$\frac{L^2}{\tau} = 10$; $\frac{L}{a} = 2.74$; $\frac{R}{a} = 0.107$; $\eta_1 = 0$; $\eta_{4,6} = 0$

$\frac{2\pi D\gamma_{2,3,5}}{D = \lambda_{tr}/3}$	$\gamma_{2,3,5}/\gamma_{4,6}$	$\gamma_{2,3,5}/\gamma_1$	$\eta_{2,3,5}$	Φ_2/Φ_1	Φ_3/Φ_1	Φ_4/Φ_1	Φ_5/Φ_1	Φ_6/Φ_1
2.0	0.6	0	1.27	0.504	0.478	0.587	0.473	0.578
2.0	1.0	0	1.33	0.496	0.470	0.460	0.455	0.451
2.0	1.4	0	1.36	0.490	0.464	0.378	0.444	0.370
2.0	0.6	1	1.35	1.018	1.019	1.276	1.039	1.281
2.0	1.0	1	1.42	1.0	1.0	1.0	1.0	1.0
2.0	1.4	1	1.46	0.988	0.988	0.822	0.975	0.820
2.0	0.6	3	1.38	2.046	2.101	2.654	2.171	2.688
2.0	1.0	3	1.44	2.010	2.061	2.080	2.090	2.098
2.0	1.4	3	1.49	1.984	2.035	1.711	2.037	1.720
4.75	0.6	0	1.331	0.707	0.685	0.769	0.681	0.760
4.75	1.0	0	1.405	0.697	0.674	0.665	0.661	0.651
4.75	1.4	0	1.47	0.690	0.665	0.588	0.645	0.576
4.75	0.6	3	1.48	1.629	1.679	1.925	1.731	1.958
4.75	1.0	3	1.56	1.605	1.651	1.668	1.679	1.688
4.75	1.4	3	1.63	1.587	1.631	1.471	1.639	1.484

Table III

Scheme No.	*Number of slugs N*	*R of reactor in unit a*	K_Π	K_{hom}	K_{het}	$\frac{K_{het} - K_\Pi}{K}$	$\frac{K_{het} - K_{hom}}{K}$
			1st variation slug radius $R = 0.1$				
			$a = \sqrt{\tau} = 1$; $L^2 = 49$; $Q = 0.015$; $g = 3.3 \times 10^{-3}$				
1	4	1.128	2.11	2.31	2.29	13.7%	−1.5%
2	12	1.954	1.48	1.51	1.53	9.8%	−3.4%
3	24	2.764	1.29	1.30	1.31	7.1%	+3.2%
4	32	3.192	1.23	1.24	1.25	6.4%	+3.2%
			2nd variation				
			$a = \sqrt{\tau} = 1$; $L^2 = 8$, $Q = 0.081$; $g = 0.020$				
1	4		2.41	2.86	2.97	28%	+5.6%
2	12		1.63	1.71	1.81	21.9%	+11.1%
3	24		1.38	1.41	1.47	18.5%	+12%
4	33		1.31	1.33	1.37	16.8%	+11.2%

For details see A. D. Galanin "Critical Sizes of a Heterogeneous Reactor with a Small Number of Slugs," this Session.

The results in Table III enable us to estimate the possibilities of "homogeneous" methods of calculating the critical K_∞.

It is interesting to compare the spatial distribution of the thermal neutron flux calculated by the heterogeneous and homogeneous methods. The distribution of the neutron flux in a reactor (scheme given in Fig. 7) is shown in Fig. 9.

The difference in the neutron flux in the reflector is fairly notable.

Of still greater interest are the results obtained by the heterogeneous method for altering the spatial distribution of the neutron flux when a neutron absorbing rod is inserted into the core.

Figure 10 shows the modification of the neutron field after insertion into the reactor of a central multiplying slug.

Figure 11 gives the modification of the neutron field and the values of η_{kp} of the slugs when a group of multiplying slugs is substituted by an absorbing group having other γ and R_0 (substitution of the group of slugs in Fig. 7).

Resonance Absorption in a Finite Lattice

In Part 1 of this survey we mentioned a method which enables us to estimate the resonance neutron capture in a heterogeneous system with an arbitrary number of slugs.

If the density of the neutrons above the resonance is constant in the system, or if it changes but slightly within the limits of one cell, then the resonance capture may be approximately computed by multiplying the quantity η by the factor φ†† which is defined by an empirical formula. This greatly simplifies the calculation, freeing us of the necessity of calculating double sums in Equation 14.1*.

However, for a system with a small number of slugs such a consideration is not valid; in this case a rigorous account of the influence of the resonance absorption is necessary (see Part 1). As has already been noted in Part 1, a strict account of the resonance absorption influence may be made if we assume that the energetic range of resonance absorption is sufficiently narrow.

†† The probability of resonance absorption of neutrons by uranium in an infinite lattice $= 1 - \varphi$.

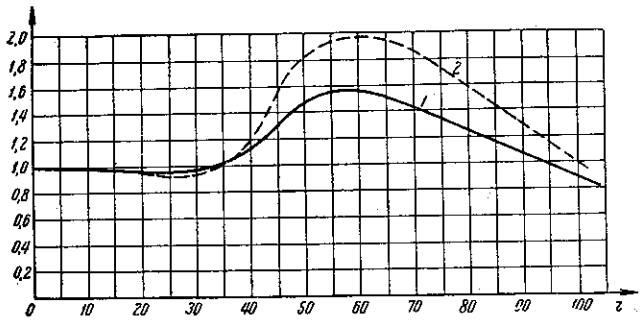

Figure 9. Spatial distribution of the thermal neutron flux in a reactor with a small number of slugs, the scheme of which is shown in Fig. 7. 1. $\Phi(r)$ was computed by the homogenization rule; 2. $\Phi(r)$ was calculated by the heterogeneous method; the intensity of neutrons in the core is averaged by the cell volume of the lattice

Actually two effects exist: (1) absorption at separate levels in the range of 6–100 ev; (2) a wide region of "unblocked absorption" extending to the energy fission neutron. The heterogeneous method of calculation is approximately valid only for estimation of the first effect. To estimate the influence of the position of the resonance level on the value of resonance absorption, S. M. Feinberg and B. D. Slutskaya calculated the heterogeneous system in Fig. 12. The calculation was made for $E_z \sim 10$ ev and $F_z \sim 400$; the computation results coincided within the range of calculation accuracy. This shows that the variation of the position of the resonance level in a wide range does not practically influence the calculation result. The dependence of the probability of resonance absorption on the number of slugs in a reactor may be clarified by the following example (see Fig. 12):

Figure 12 shows the scheme of successive increase of the number of slugs in the moderator.

The values of probability of resonance neutron capture depending on the number of slugs in the core are given in Table IV.

Thus resonance absorption in lattices consisting of a small number of slugs is essentially smaller than in infinite lattices with the same spacing and the same slugs. When the number of slugs is increased the resonance absorption quite rapidly approaches its value in infinite lattices.

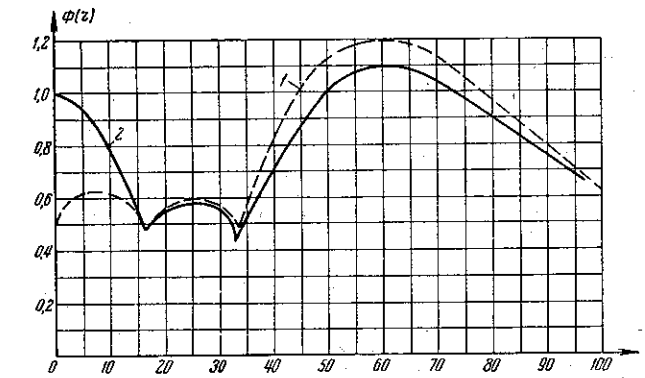

Figure 10. Change of the spatial distribution of the thermal neutron flux in a reactor (the scheme of which is given in Fig. 7) when the central slug is removed; 1. the reactor consists of 31 slugs; 2. the central slug is removed (the reactor consists of 30 slugs)

6. COMPARISON WITH EXPERIMENT

The correspondence of the results of heterogeneous calculations with experiment depends on the accuracy of the physical parameters applied in the calculation. To estimate the actual accuracy of the calculation and the dependence of the result on the variations of the physical parameters a special series of experiments has been made with a reactor‡‡ consisting of a small number of slugs.

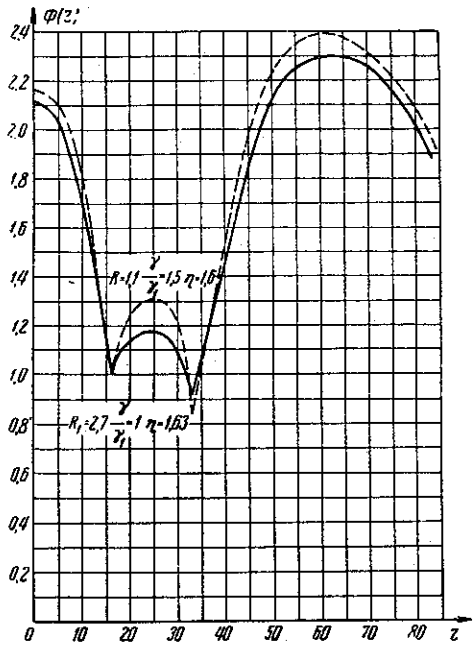

Figure 11. Change of the spatial distribution of the thermal neutron flux when γ is changed in the neutron absorbing rods (scheme shown in Fig. 7). R and γ correspond to the neutron absorbing slugs

As may be seen from Parts 1 and 5, the following physical parameters enter the calculation of a heterogeneous reactor with a small number of slugs:

$$\eta, L, \gamma, A, \tau, \tau_r, \Delta_z$$

If we build a number N of various critical systems containing identical slugs we get N connections between these seven parameters from the critical conditions. If $N \geq 7$ then the value of the parameters may be computed. However, if $N < 7$ then a part of the parameter must be given. Three critical reactors were consequently built in a large (practically infinite) graphite prism. Their schemes are shown in Fig. 13. The number of reactors is smaller than the number of values used in the calculation; consequently the values of four parameters must be picked beforehand.

(*a*) The reflector savings with respect to the height Δ_z may be estimated by means of the usual two or multi-group calculations. All the piles under consideration had a height considerably larger than the "diameter." Therefore the error in the given value $\Delta_z = 40$ cm may lead to a comparatively small error

‡‡ These experiments have been made on the suggestion of the author by a group of scientists at the Academy of Sciences of the USSR.

Table IV

Nos.	No. of slugs, N	Probability of resonance absorption $1 - \varphi\left(\frac{\eta_{A=0}}{\eta_A}\right)$
1	4	0.023
2	12	0.0374
3	16	0.0505
4	Infinite lattice	0.070

in computing the value of η. Thus, if $\delta\Delta_z = 10$ cm then this $\delta\eta_1 = 0.005$.

(*b*) A similar or even smaller error may be made in η if the position of the resonance level is wrong. Calculation shows that within reasonable limits, any choice of energy E_z (and therefore τ_z) leads to a change of not more than 0.005 in η. If we take $E_z = 15$ ev then $\tau_z = 100$ cm^2 and $\delta\eta_2 \sim 0.005$.

(*c*) A somewhat larger influence on the calculation results is exerted by the choice of the resonance absorption characteristic A. It was found by a semi-empirical formula and assumed to be equal to (40 ± 4) cm^2. The error contained in A is easy to define. Indeed in an infinite lattice with a spacing equal to 20 cm to the value of $A = (40 \pm 4)$ cm^2 corresponds the probability of the resonance absorption

$$1 - \exp\left(-\frac{40 \pm 4}{20^2}\right) = 0.1 \pm 0.01$$

In the piles under consideration the area of the moderator for one rod is on the average larger than in an infinite lattice with a spacing equal to 20 cm. For this reason the error in η is estimated as $\delta\eta_3 \sim 0.01$.

(*d*) An error in the given value of the age τ influences considerably the value of η. In accordance with the available experimental data τ is assumed to be

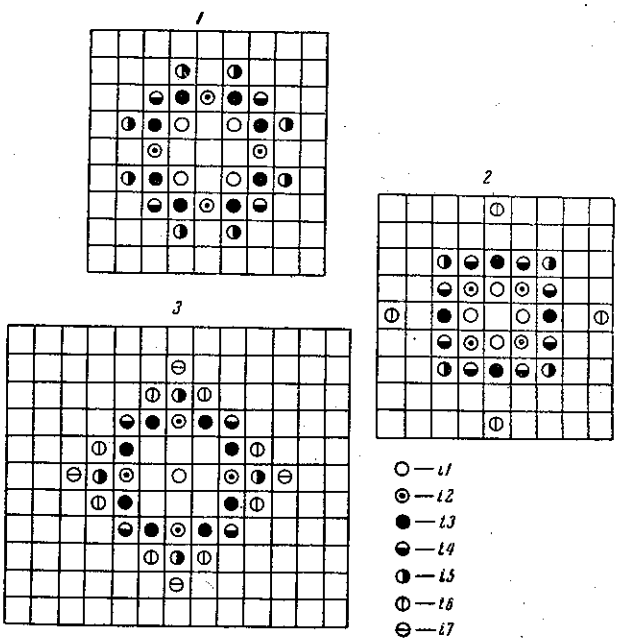

Figure 13. Scheme of reactors used in experiment. 1. Reactor consists of 28 rods; the number of unknown quantities 5. 2. Reactor consists of 28 rods; the number of unknown quantities 6. 3. Reactor consists of 33 rods: the number of unknown quantities 7

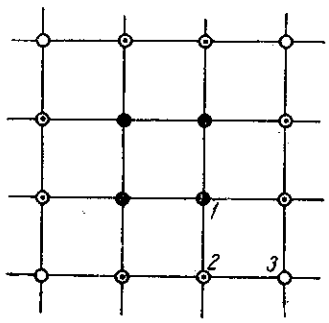

Figure 12. Schemes of lattices used for calculation of resonance absorption. 1. Lattice consists of four rods; 1 and 2. Lattice consists of 12 rods; 1, 2, 3. lattice consists of 16 rods

equal to (390 ± 10) cm^2 for the system under consideration; we may expect an error in $\delta\eta_4$ of about 0.015.

Thus the higher estimation of the error in η due to the uncertainty of the parameters Δ_z, E_r, A, τ, gives us the value $\delta\eta = \delta\eta_1 + \delta\eta_2 + \delta\eta_3 + \delta\eta_4 \leq 0.035$. It may be assumed that the actual error is not more than $\delta\eta = 0.02$ and does not exceed the error made when directly measuring η. Direct measurement made by B. G. Yerozolimsky using the method of measuring the effects from fissionable samples has given:

$$\eta = 1.647 \pm 0.017$$

The calculation results are given in Table V.§§

Calculation shows that there is no full correlation between the coincidences of η and the coincidences of i_k in calculation and experiment; however, the discrepancies are negligible and the precision attained is absolutely satisfactory from a practical point of view.

This is especially evident because in the accepted calculation scheme such factors as asymmetry in the neutron flux near the slug surface and also the dependence of the thermal neutron diffusion coefficient and the absorbing coefficient of the slug on the temperature of the neutron flux and several other factors are not accounted for.

The satisfactory agreement between the calculated and experimentally obtained values of the neutron flux distribution over slugs is quite notable. It confirms the assumption of the effectivity of the heterogeneous method especially for examining the spatial distribution of the neutron flux in the reactor.

Another point of interest is the application of the heterogeneous method for the calculation of the spatial distribution of a neutron field in a reactor with a rather large number of slugs and with considerable violations of the lattice.

A reactor employed for generation of radioactive isotopes had altogether about 240 channels. Part of them were loaded with uranium slugs, part with slugs for the generation of isotopes, and a part was used for the control system.

§§ See G. A. Bat, P/665, "Heterogeneous Calculations of Reactors with a Small Number of Slugs," this Session.

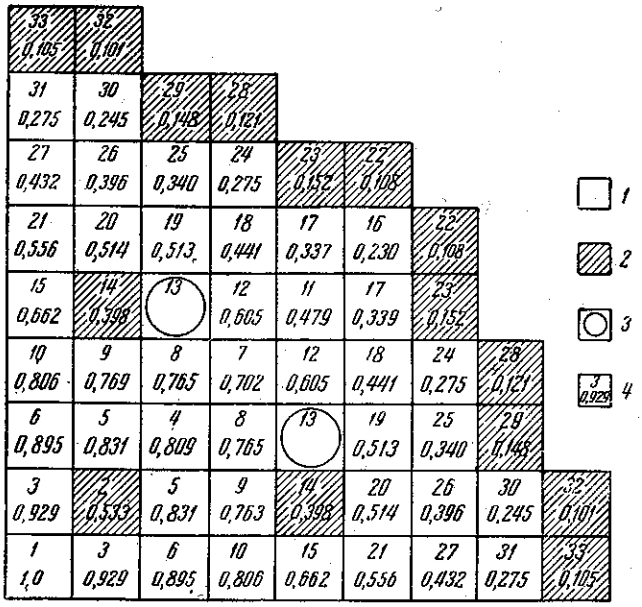

Figure 14. Cartogram of spatial distribution of a thermal neutron flux in a reactor. 1. Uranium slug cell; 2. cell of a slug producing isotopes; 3. cell of the control rod; 4. the upper number—number of the unknowns in the equations (18.1), the lower number—the neutron flux on the s-th slug (in relative units)

Owing to the symmetry in the position of the channels the problem has been reduced to solving the system of equations with 33 unknown quantities. This system of linear equations was calculated by means of computing machines. Various arrangements of the same rods producing isotopes and the control rods were considered. The use of rods producing isotopes with different degrees of "blackness" were also considered.

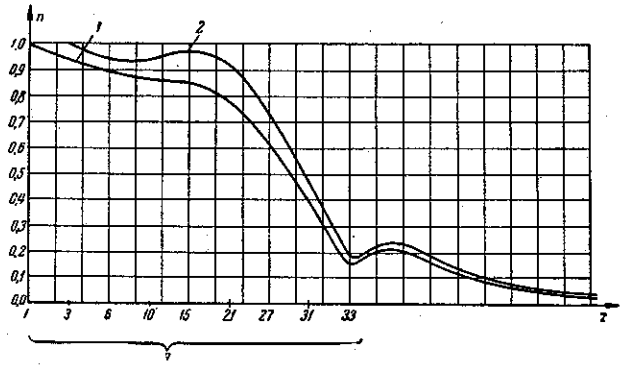

Figure 15. Radial distribution of the thermal neutron flux in a reactor. 1. Case of No. 2 position of slugs (see Table No. 6); 2. case of No. 3 position of slugs (see Table No. 6); 3. number of slugs "S"

A typical cartogram of the spatial distribution of the neutron flux over the rods of the lattice and a scheme of the loading of the reactor are shown in Fig. 14.

The purpose of the calculations was to define the optimal position and optimal absorbing capacity of the slugs producing isotopes for obtaining the maximum quantity of radioactive isotopes with the minimum power of the reactor and, consequently, a minimum consumption of enriched uranium.

Table V

$L = 52$ cm; $2\pi D_\gamma = 3.3$

Scheme		η	i_1	i_2	i_3	i_4	i_5	i_6	i_7	Average
1	Experiment	1.647 ± 0.017	1	0.96	0.84	0.80	0.79	—	—	0.86
	Calculation	1.63 ± 0.2	1	0.89	0.80	0.78	0.75	—	—	0.82
2	Experiment	1.647 ± 0.017	1	1.1	0.97	0.93	0.89	0.73	—	0.85
	Calculation	1.61 ± 0.02	1	1.1	0.98	0.93	0.87	0.74	—	0.85
3	Experiment	1.647 ± 0.017	1	0.70	0.66	0.59	0.54	0.51	0.49	0.59
	Calculation	1.64 ± 0.02	1	0.70	0.67	0.63	0.56	0.53	0.51	0.61

Table VI

No. of arrangement of different slugs in the active zone*	Number of cells with slugs producing isotopes†	Number of cells with control rods‡	Power by given i_{max}‡	Average density of neutrons along the radius $\sum_1^N i/Ni_{max}$	Absorption in the slugs producing isotopes	Absorption in the same slugs per power unit
2	2; 9	13	100	0.678	100	1
3	2; 9	1; 7	113	0.766	112	0.99
4	2; 9	12	103	0.697	102	0.99
5	2; 14	13	83.6	0.567	85.2	1.02
6	2; 14	1; 7	93.1	0.631	93.5	1
7	2; 20	13	89.7	0.608	93.1	1.04
8	2; 20	1; 7	96.6	0.655	98.8	1.02
9	2; 7; 9	13	82.4	0.572	82.4	1
10	2; 7; 14	13	71.4	0.496	72.6	1.02
11	2; 7; 20	13	72.8	0.505	75.4	1.04
12	2; 9; 12; 21	13	59.1	0.442	62.2	1.05
13	2; 9	13; 16; 24; 30	87.7	0.673	87.4	1

* The case of arrangement N 5 is given in Fig. 14.

†Total number of cells and their indications in all arrangements are the same as in Fig. 14; in all the arrangements the cells 22, 23, 28, 29, 32, 33 are occupied with slugs producing isotopes; all the other cells not mentioned here are occupied with uranium slugs.

‡ All the quantities are given in relative units.

The calculation results are given in Table VI.

The spatial distribution of the neutron flux is shown in Fig. 15. The calculation results were compared with experimental results obtained during the work of the reactor. The calculation prediction of the choice of an optimal loading of the reactor proved to be correct.

CONCLUSIONS

From the above-mentioned results the following conclusions may be made:

1. The application of the homogenization rule for the estimation of the critical size of a reactor is precise enough for practical purposes not only for a large reactor but also for a pile with a core consisting of a small ($\sim$20) number of slugs.

2. In order to determine the spatial distribution of the neutron flux within a reactor with a small number of slugs the heterogeneous method of calculation is necessary.

3. The heterogeneous method is necessary when examining neutron fields in composite lattices, in the vicinity of local violations of a regular lattice (the control rods, empty cells and so on) and near the boundary of the core.

4. Comparison of the results of calculating reactors by the heterogeneous method with experimental data showed a satisfactory agreement.