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A B S T R A C T

I present evidence that suggests that the correct relationship between the optical transmission matrix, $\underset{\sim}{P}$, and the several correlation widths, Γ_n , found in multistep compound (preequilibrium) nuclear reactions, is $\sum_n 2\pi \Gamma_n / D_n = \mathrm{Tr}\, \underset{\sim}{P}$. A second sum rule is also derived within the shell model approach to nuclear reactions. Indications of the potential usefulness of the sum rules in preequilibrium studies are given.

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It is by now generally accepted that the correct formula for the correlation width extracted from autocorrelation studies of nuclear reactions is

$$\frac{2\pi\Gamma}{D} = Tr\, \underset{\sim}{P} \qquad (1)$$

where Γ is the correlation width, D the average spacing between compound nucleus levels and $\underset{\sim}{P}$ the transmission matrix, which is merely a coefficient in the absence of direct reactions, obtained from the optical (average) S-matrix. Equation (1) has been verified both numerically {1} and formally {2}, at least in the limit of large number of open channels.

In view of the recent upsurge of interest in preequilibrium studies, both experimentally {3}, and theoretically {2,4,5,6} one is tempted to generalize Eq. (1) to the case of multistep compound processes.

In the following I show that the correct generalization is

$$\sum_n \frac{2\pi\Gamma_n}{D_n} = Tr\, \underset{\sim}{P}_1 \qquad (2)$$

where Γ_n is the correlation width associated with the n-th stage of the preequilibrium reaction, D_n the corresponding average level spacing and $\underset{\sim}{P}_1$ is, just as in Eq. (1), the optical transmission matrix. The subscript 1 in $\underset{\sim}{P}_1$ will be made clear in what follows. The sum in Eq. (2) runs over all classes of overlapping resonances (doorways) that contribute to the reaction {5}.

I shall not present a rigorous proof of Eq. (2) but I will give plausible arguments. Let us first recalll some of the pertinent results of the recently developed Nested-Doorway (ND) model of multistep compound reactions {4,6}. In the ND model, the different classes of resonances are characterized by their dis-

tinctly different correlation widths, Γ_n, that are assumed to satisfy the inequality

$$\Gamma_1 \gg \Gamma_2 \gg \cdots\cdots \gg \Gamma_N \qquad (3)$$

where Γ_N corresponds to the equilibrated stage and may be estimated from the following empirically deduced formula {7}

$$\Gamma_N \approx 14 \exp\left[-4.69\sqrt{A/E_x}\right] [\text{Mev}] \qquad (4)$$

where A is the mass number of the compound nucleus, and E_x its excitation energy (in MeV).

Based on (3), the following expression for the average fluctuation cross section was obtained in {4} (we shall, in what follows, consider no direct reactions)

$$\sigma^{fl}_{cc'} = \sum_{n=1}^{N} \sigma^{fl}_{n,cc'} = \sum_{n=1}^{N} \frac{(P_{n,cc} - P_{n+1,cc})(P_{n,c'c'} - P_{n+1,c'c'})}{\mathrm{Tr}\,(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1})} \qquad (5)$$

with $\underset{\sim}{P}_{N+1} = 0$ and $\underset{\sim}{P}_1$ being the optical transmission matrix.

The corresponding expression for the generalized cross-section autocorrelation function was subsequently worked out in Ref. {6,8}

$$C_{cc'}(\varepsilon) = \left| \sum_{n=1}^{N} \frac{\sigma^{fl}_{n,cc'}}{1 + i\varepsilon/\Gamma_n} \right|^2 \qquad (6)$$

where $\sigma^{fl}_{n,cc'}$ is the same as that appearing in Eq.(5). The transmission matrices, $\underset{\sim}{P}_n$, were defined and discussed in Ref. {6} to which I refer the reader for fuller details.

Equations (5) and (6) should be contrasted with the conventional one-class expression, given by the Hanser-Feshbach formula for $\sigma_{cc'}^{fl}$

$$\sigma_{cc'}^{fl}(HF) = \frac{\rho_{N,cc}\ \rho_{N,c'c'}}{Tr\ \rho_N} \tag{7}$$

and Ericson is formula for $C_{cc'}(\varepsilon)$

$$C_{cc'}(\varepsilon) = \left| \frac{\sigma_{cc'}^{fl,(HF)}}{1 + i\varepsilon/\Gamma_N} \right|^2 \tag{8}$$

Recently, Agassi, Weidenmuller and Mantzouranis {2} have clearly demonstrated that, when Eq.(7) is valid the correlation width Γ_N that appears in (8) has the form of Eq.(2) namely

$$\Gamma_N = \frac{\mathcal{D}_N}{2\pi} Tr\ \underset{\sim}{P} \tag{9}$$

I therefore make the plausible assumption that the corresponding generalization for any n should be

$$\Gamma_n = \frac{\mathcal{D}_n}{2\pi} Tr\left(\underset{\sim}{P}_n - \underset{\sim}{P}_{n+1}\right) \tag{10}$$

where it is to be understood that condition (3) in obeyed. Upon multiplying (10) by $\frac{2\pi}{\mathcal{D}_n}$ and summing over n I obtain straightforwardly the sum rule, Eq. (2).

Another useful relation connecting the correlation widths, Γ_n, with the X_n-matrix introduced in {4}, follows directly from Eq. (9) of Ref.{ 4} , i.e.,

$$\sum_{n=1}^{N} \frac{2\pi\,\Gamma_n}{\mathcal{D}_n} = \left(Tr\ \underset{\sim}{X}_n\right)^2 \tag{11}$$

where it is to be understood that the condition for the validity

of Eq. (11) is attached to that of Eq.(5) in that the number of open channels is assumed to be very large, which, incidentally, is also the condition for the validity of Eq.(8) (see Ref. {2}).

In the shell model approach to multistep compound reactions {2}, the corresponding sum-rule involving the sum of the correlation widths $\sum_{n=1}^{N} \Gamma_n$, on the one hand, and the trace of the optical transmission matrix $Tr\ \underset{\sim}{P}_1$ on the other hand, comes out to be different from Eq.{2} and having the form {9}

$$\sum_{n=1}^{N} \Gamma_n = \sum_{n=1}^{N} \Gamma_n^{(0)\uparrow} + \sum_{n=1}^{N} \Gamma_n^{(0)\downarrow} \tag{12}$$

where $\Gamma_n^{(0)\downarrow}$ is the total spreading width of class n

$$\Gamma_n^{(0)\downarrow} = 2\pi \sum_m \overline{V_{nm}^2} \,/\, D_m \tag{13}$$

In (13), $\overline{V_{nm}^2}$ is the second moment of the assumed random matrix elements connecting states in different classes. Finally, $\Gamma_n^{(0)\uparrow}$ corresponds to the escape width of class n. The relation between $Tr\ \underset{\sim}{P}_1$, that appears in Eq.(2), and $\sum_n \Gamma_n^{(0)\uparrow}$ of Eq.(12), is most easily seen by introducing, as in Refs. {2} and {6}, model (not optical) transmission coefficients, $\underset{\sim}{T}_n$, for the different classes. Then

$$Tr\ \underset{\sim}{P}_1 = \sum_{n=1}^{N} Tr\ \underset{\sim}{T}_n \tag{14}$$

and

$$\sum_{n=1}^{N} \Gamma_n^{(0)\uparrow} = \sum_{n=1}^{N} \frac{D_n}{2\pi} Tr\ \underset{\sim}{T}_n \tag{15}$$

The approximation involved in obtaining Eq.(15) is that of small continuum shifts {2} .

Although the two sum rules, Eqs. {2} and {12}, are different, they should become equivalent in the limit of well nested doorways, namely $\Gamma_1 \gg \Gamma_2 \gg \cdots \gg \Gamma_N$. We expect that in this limit, the sum rule becomes quite useful since the Γ_n may be extracted from fluctuation analysis of the type reported in {3} and {10}, and the $Tr\, P_{n1} = \sum_c P_{1,c}$, with P_{n1}, being the optical transmission matrix, may be calculated, in principle, from an optical model analysis.

Aside from the above, we expect Eq.(2) to be useful insofar as it suggests a simple way of predicting the possible contribution of multistep compound processes to a given nuclear system. I suggest constructing a plot as shown schematically in figure 1 where the ratio

$$B \equiv \frac{2\pi \Gamma_N}{D_N} \Big/ Tr\, P_{n1} \qquad (16)$$

is shown as a function of incident energy. The quantity $2\pi \Gamma_N / D_N$ can be obtained from any Hanser-Feshbach code as it is nothing but the denominator, properly averaged over angular momentum, of the HF fluctuation cross section. As figure 1 indicates, I suggest that at low incident energies, the term $2\pi \Gamma_N / D_N$ exhausts the sum rule Eq.(2). Accordingly, B starts out with a value of unity. As the energy increases, B should then decrease indicating clearly the increasing contribution of multistep compound processes.

Another application I have in mind is in the problem of isospin mixing {11} . In this case the two correlation widths, $\Gamma_>$ and $\Gamma_<$, associated with the isobaric analog resonances ($T_>$) and the fine structure resonances ($T_<$), are not expected to be very different. For this problem I suggest using the second sum

rule Eq. (12) as an alternative way for obtaining information about the isospin mixing matrix element, $\sqrt{\overline{v_{12}^2}}$, contained in the second term of the right-hand side of Eq.(12), namely

$$\sum_{n=1,2} \Gamma_n^{(0)\downarrow} = 2\pi \overline{v_{12}^2} \left(D_>^{-1} + D_<^{-1} \right) \qquad (17)$$

In conclusion, I have derived two sum rules that should serve as the preequilibrium generalization of the usual one relating the correlation width Γ, with the trace of the optical transmission coefficient.

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9) See, e.g., W.A. Friedman, M.S. Hussein, K.W. McVoy and P.A. Mello, Phys. Lett. 92B, 253 (1980).

My result given in Eq. (12) is a multiclass generalization of the two-class case discussed in the above reference. In Ref. [2] the expression obtained for the fluctuations cross section has the structure $\sigma_{cc'}^{fl} = \sum_{m,n} T_m^c \, \Pi_{mn} \, T_n^{c'}$. Friedman et al al. cited above have shown that the correlation widths that should appear in the cross section auto correlation function are just the eigenvalues of the matrix $\Gamma_{mn} \equiv (2\pi)^{-1} \sqrt{D} \, \Pi^{-1} \sqrt{D}$ where $\sqrt{D}$ is a diagonal matrix with elements $\sqrt{D_i}$, the square root of the i^{th}-class average level spacing. Using the expressions for Π^{-1} given in Ref. [2] and calculating

$\mathrm{Tr}\,\Gamma_{\mu}$, we obtain Eq. (12).

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FIGURE CAPTION

FIGURE 1 - A schematic plot of the ratio $\beta \equiv \frac{2\pi \Gamma_N}{D_N \mathrm{Tr} P_{\sim 1}}$ (full line) vs. incident energy. The hatched area indicates the region where MCP become important.

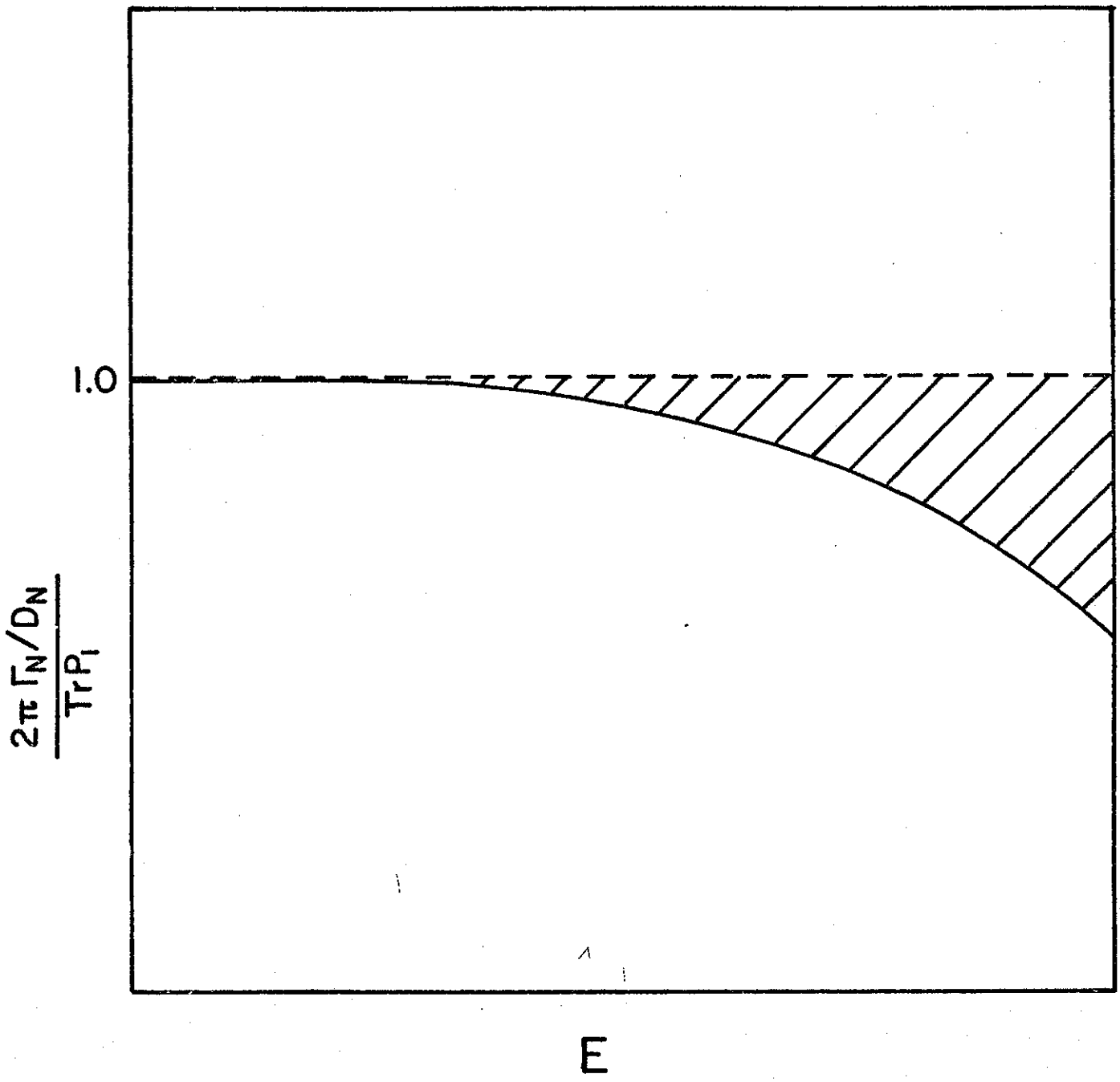

Figure 1