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THEORY REVISITED

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ABSTRACT: A recursion equation has been derived for the multistep direct emission probability. The solution of the equation resembles the one given by Feshbach, Kerman and Koonin except for an overall multiplicative factor which depends on the number of steps considered, and the compound nucleus transmission coefficients for the grazing partial waves. An estimate of this factor is given and the consequences of its presence are discussed for the reaction $^{120}Sn(p,n)$ at 45 MeV.

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With increasing bombarding energies, nuclear direct reactions become more complex as multistep processes become important. Examples of these are preequilibrium reactions and heavy ion deep inelastic collisions (DIC). In either case many complex channels are involved. Exact, quantal many-coupled-channels calculations of these processes are certainly not viable. Recently several alternative theories have been developed having in common the statistical treatment of the couplings among the channels. In particular, we mention the theory of Agassi, Ko and Weidenmüller[1] for DIC, and the statistical multistep direct emission (SMDE) theory of Feshbach, Kerman, Koonin[2]. One distinctive feature of the quantal SMDE developed in Ref. (2) is its simplicity. Several calculations have been made[3] and the overall SMDE account of the data on preequilibrium spectra at different angles were found to be good.

Recently, it has been observed[4] that the individual one-step cross-sections that appear in the expression for the multistep cross section of SMDE of Ref. (2) are not of the DWBA type. The point raised in Ref. (4) refers to the fact that the matrix elements for successive transitions are of the type $\langle \tilde{\chi}_\nu^{(+)} | v | \chi_{\nu+1}^{(+)} \rangle$ and therefore not of the DWBA form $\langle \tilde{\chi}_\nu^{(-)} | v | \chi_{\nu+1}^{(+)} \rangle$.

Clearly one may calculate the former matrix element from the DWBA by inserting, in the partial wave expansion of the latter, appropriate partial wave elastic S-matrix $e^{-2i\delta_\ell^{\nu+1}}$, where $\delta_\ell^{\nu+1}$ is complex[5]. An immediate consequence is that

$$\left| \langle \tilde{\chi}_\nu^{(+)} | v | \chi_{\nu+1}^{(+)} \rangle \right|^2 \geq \left| \langle \tilde{\chi}_\nu^{(-)} | v | \chi_{\nu+1}^{(+)} \rangle \right|^2$$

since the modulus of $e^{-2i\delta_\ell^{\nu+1}}$ i.e $e^{+2\eta_\ell^{\nu+1}}$ is larger than unity due to the presence of absorption (compound nucleus, other open channels) in the system.

The above implies that the extracted values of the residual interaction strength, V_o , found in Ref. (3) should be smaller than reported.

In the present note, we demonstrate that by a proper treatment of the distorted waves and, further, by recognizing the non-unitary nature of the underlying average S-matrix, the above question discussed in Ref. (4) can be fully, albeit approximately, accounted for by the introduction, into the SMDE expression of Ref. (2), of an overall multiplicative factor that depends on the number of steps included, and the incident channel compound nucleus transmission coefficients for the grazing partial waves.

The transition matrix element is, as given in Ref. (2)

$$T_{fi} = t_{fi} + \langle \tilde{\varphi}_f^{(-)} | \mathcal{V} \mathcal{G}_{opt}^{(+)}(E) \mathcal{V} | \varphi_i^{(+)} \rangle \tag{1}$$

where t_{fi} is the usual DWBA amplitude, $|\tilde{\varphi}_f^{(-)}\rangle$ and $|\varphi_i^{(+)}\rangle$ are the final and initial channel distorted waves respectively. The coupling potential (that couples among the different channels) is indicated by $\mathcal{V}$. Finally $\mathcal{G}_{opt}^{(+)}(E)$ is the optical Green's function given by

$$\mathcal{G}_{opt}^{(+)}(E) = \left(E^{(+)} - H_{opt}\right)^{-1} \tag{2}$$

where H_{opt} is the usual optical Hamiltonian with an imaginary part that simulates absorption due to compound nuclear processes.

The multistep processes are describable by the second term in Eq. (1). In Ref. (2), the optical Green's function was expanded in terms of dressed channel Green's functions that were eventually replaced by their on-energy-shell parts. The contributions of the principal parts of these Green's functions average out to zero[2]. We shall adapt the same procedure later.

We now write Eq.(1) as

$$T_{fi} = t_{fi} + \sum\!\!\!\!\!\!\int_{\nu} d\varepsilon_\nu \, \langle \tilde{\varphi}_f^{(-)} | v \frac{|\psi_\nu^{(+)}\rangle\langle\tilde{\psi}_\nu^{(+)}|}{E^{(+)} - \varepsilon_\nu} v | \varphi_i^{(+)} \rangle \tag{3}$$

where

$$(E - H_{opt}) \, |\psi_\nu^{(+)}\rangle = 0 \tag{4}$$

Notice that ε_ν contains a kinetic energy part and an intrinsic excitation energy related to the excited target nucleus in stage ν. We now make the on-energy-shell approximation for $\mathcal{G}_{opt}^{(+)}(E)$, anticipating that the off-shell part will generate terms in the series (3) that eventually average to zero upon taking the modulus squared and averaging Eq. (3). Thus

$$T_{fi} = t_{fi} - \pi i \sum_\nu \langle \tilde{\varphi}_f^{(-)} | v | \psi_\nu^{(+)} \rangle \langle \psi_\nu^{(+)} | v | \varphi_i^{(+)} \rangle \tag{5a}$$

$$= t_{fi} - \pi i \sum_\nu T_{f\nu} \, \langle \tilde{\psi}_\nu^{(+)} | v | \varphi_i^{(+)} \rangle \tag{5b}$$

The amplitude $\langle \tilde{\psi}_\nu^{(+)} | v | \varphi_i^{(+)} \rangle$ can be further reduced to a manageable expression as follows. First we notice that $\langle \tilde{\psi}_\nu^{(+)} |$ may be formally related to $\langle \tilde{\psi}_\mu^{(-)} |$ through the relation

$$\langle \tilde{\psi}_\mu^{(-)} | = \sum_{\mu'} S_{\mu\mu'} \langle \tilde{\psi}_{\mu'}^{(+)} | \tag{6}$$

where $S_{\mu\mu'}$ is the $\mu\mu'$-element of the average (optical) S-matrix. Upon multiplying Eq. (6) from the left by $S_{\nu\mu}^{*}$ and summing over μ we obtain

$$\sum_{\mu} S^{*}_{\nu\mu} \langle \tilde{\Psi}^{(-)}_{\mu} | = \sum_{\mu'} (S^{\dagger}S)_{\nu\mu'} \langle \tilde{\Psi}^{(+)}_{\mu'} | \tag{7}$$

From unitarity, we know that

$$(S^{\dagger}S)_{\nu\mu'} = \delta_{\nu\mu'} - P_{\nu\mu'} \tag{8}$$

where $\underset{\sim}{P}$, is Satchler's penetration matrix(5). Accordingly we can recast Eq. (7) as

$$\sum_{\mu} S^{*}_{\nu\mu} \langle \tilde{\Psi}^{(-)}_{\mu} | = \langle \tilde{\Psi}^{(+)}_{\nu} | - \sum_{\mu'} P_{\nu\mu'} \langle \tilde{\Psi}^{(+)}_{\mu'} | \tag{9}$$

At this point, we employ the approximation

$$P_{\nu\mu'} \approx \delta_{\nu\mu'} P_{\nu\nu} \equiv \delta_{\nu\mu'} P_{\nu} \tag{10}$$

which states that the effects of directly coupled open channels on the compound nucleus processes are small. Thus

$$\langle \tilde{\Psi}^{(+)}_{\nu} | \cong \frac{1}{1 - P_{\nu}} \sum_{\mu} S^{*}_{\nu\mu} \langle \tilde{\Psi}^{(-)}_{\mu} | \tag{11}$$

where we understand P_{ν} above as the transmission coefficients for the grazing partial waves(6). Using the familiar relation between $\underset{\sim}{S}$ and $\underset{\sim}{T}$, we can rewrite Eq. (11) in the following simple form

$$\langle \tilde{\Psi}^{(+)}_{\nu} | \cong \frac{1}{1 - P_{\nu}} \left[\langle \tilde{\Psi}^{(-)}_{\nu} | + 2\pi i \sum_{\mu} T^{*}_{\nu\mu} \langle \tilde{\Psi}^{(-)}_{\mu} | \right] \tag{12}$$

which, upon insertion into Eq. (5b), gives our fundamental "on-shell nuclear Low-equation"

$$T_{fi} = t_{fi} - \pi i \sum_{\nu} T_{f\nu} \frac{1}{1-P_{\nu}} T_{\nu i} + 2\pi^2 \sum_{\nu} T_{f\nu} \frac{1}{1-P_{\nu}} \left(T^{\dagger}T\right)_{\nu i} \tag{13}$$

To calculate the inclusive preequilibrium cross section we have to take modulus squared of Eq. (13) and sum over the relevant final excitation energy interval. In doing this we shall, in keeping with the basic statistical hypothesis underlying the theory, retain only manifestly positive definite terms, thus

$$|T_{fi}|^2 = |t_{fi}|^2 + \pi^2 \sum_{\nu} |T_{f\nu}|^2 \frac{1}{(1-P_{\nu})^2} |T_{\nu i}|^2 + 4\pi^4 \sum_{\nu} |T_{f\nu}|^2 \frac{1}{(1-P_{\nu})^2} \left[\left(T^{\dagger}T\right)_{\nu i}\right]^2 \tag{14}$$

which is the recursion equation we seek. An important point which is to be recognized is that the overall ν^{th}-step delta function would guarantee, upon taking the modulus squared of Eq. (13), the appearence of the correct momentum phase space factors that are needed, in Eq. (14), to define the individual one-step DWBA cross--sections. We have not indicated this explicitly in Eq. (14) in order to simplify the notation. Eq. (14) exhibits the statistical multistep nature of the theory, as exemplified in the second and third terms. One may easily generate the SMDE expression of Ref.(2) by repeated application of the recursion equation above and by assuming the chaining hypothesis, namely $\nu = i+1$, etc.. The only major difference is the presence of the factor $(1-P_{\nu})^{-2}$, which is generated in a successive way.

In Table I we list the first three multiplicative corrections to the results of Ref. (2), namely for two-step, three-step, and four-step processes. As one clearly sees, these corrections depends on two quantities: the number of steps involved, and the incident channel, compound nucleus, transmission cofficients P_i . One further approximation was employed to obtain these correction factors, namely

$$P_\nu \approx P_i \tag{15}$$

This, we believe, is reasonable in view of the fact that the energy losses encountered in each step is rather small, and the optical potential one uses in the distorted waves defining the basic DWBA corss section, $|t_{\nu,\nu+1}|^2$, is usually taken to be the same as the one that generates the elastic channel distorted wave, $|\varphi_i^{(+)}\rangle$.

We have calculated the correction factors shown in Table I, for the recently analysed reaction $^{120}Sn(p,n)$ at $E_p =$ $= 45$ MeV. For this system the value of P_i , for the grazing partial wave, namely $\ell_g \cong 11$, is about 0.1 and therefore the correction factors are not very large. As a matter of fact, by introducing these factors into the calculations reported in Ref. (3), (see figure 1), we obtain an overall 35% reduction of the extracted value of the strength of the residual interaction, V_0 . This result, we believe, is reasonable as it goes in the direction of closing the gap between the values of V_0 extracted from multistep compound processes[7] and multistep direct processes reported in Ref. (3). An important feature of our fundamental Eq. (14), is the appearence of compound nucleus quantities in the cross section for direct processes. This is clearly a manifestation of unitarity. Notice that when one increases the incident energy, the grazing ℓ_g increases, and accordingly the correction factor becomes less important as it should, since multistep direct emission dominates over compound emission. Inversely when the energy is reduced, P_i becomes larger but $|T_{\nu i}|^2$ etc. become smaller since they are roughly proportional to $(1 - P_i)$. Thus the second term in Eq. (14) becomes less important (being proportional to $(1- P_i)$). The third term in Eq. (14) also becomes less significant since the fundamental DWBA cross sections that appear implicitly in it become small (since the average $\underset{\sim}{S}$ is reduced in magnitude). This is certainly what one would expect.

In conclusion, we have derived a simple correction to the expression for the statistical multistep direct emission cross section of Ref. (2) that restores approximately the constraints imposed by unitarity. In this way, the interplay between multistep direct and compound processes becomes quite clear.

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6) Notice that $\underset{\sim}{P}$ is not the usual, partial wave transmission matrix, as it depends on the angle formed by the momentum vectors $\vec{k}_\nu$ and $\vec{k}_\mu$. However, as a consequence of the surface nature of the multistep direct process, i.e. its being forward peaked, we expect that of all partial waves contributing to $P_{\nu\mu}$, only the grazing one to be important. Throughout the paper we shall therefore consider $P_{\nu\mu}$ or P_ν to be approximately given by P_{l_g}, where $l_g \simeq kR$, with R being the radius of the system.

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TABLE CAPTIONS

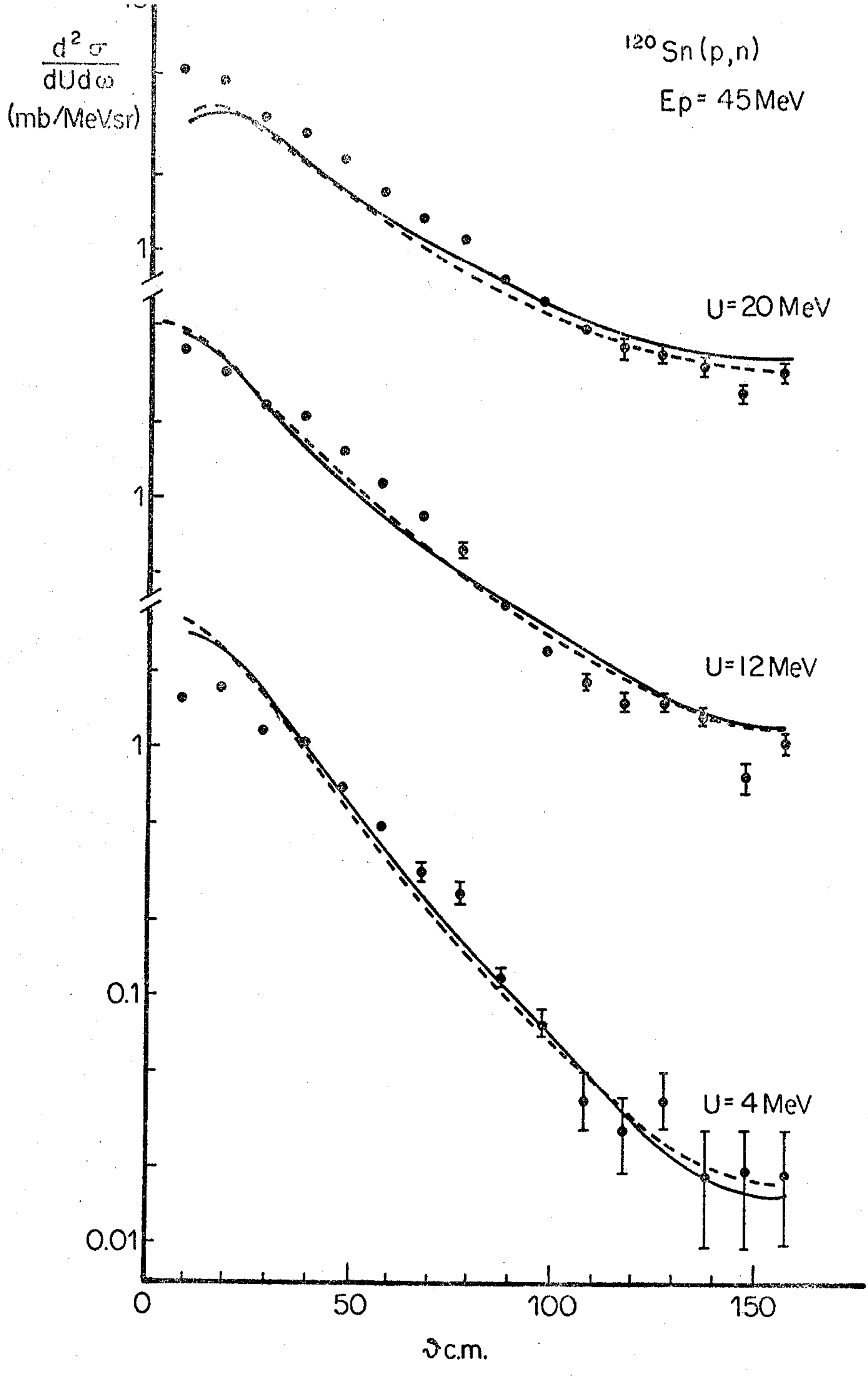
Table 1 - The correction factors listed for three different numbers, n , of the steps in the SMDE reaction. The symbol χ stands for $(1-\rho_i)^{-2}$ (see text for details). Note that for each value of n , the corresponding power of V_o in the cross section is 2n . Thus V_o is scaled by $(A_n)^{\frac{1}{2(n-1)}}$.

FIGURE CAPTION

FIGURE 1 - Calculated neutron angular distributions for the $^{120}Sn(p,n)$; Ep = 45 MeV at three final neutron energies. Dashed curves are from Ref. (3) (see this reference for details). Full lines include the correction factors of table 1. The data points are from Galonsky et al., Phys.Rev. C14 (1976) 748.

n	Correction Factor; A_n	A_n for $^{120}Sn(p,n)$ at $E_p = 45$ MeV
2	X	1.235
3	$2(2X + X^2)$	7.98
4	$20X^2 + X^3$	32.388

TABLE 1



$\frac{d^2\sigma}{dUd\omega}$
(mb/MeV.sr)
^{120}Sn (p,n)
Ep= 45 MeV
U=20 MeV
U=12 MeV
U=4 MeV
1
1
1
0.1
0.01
0
50
100
150
ϑ c.m.

FIGURE 1