INSTITUTO DE FÍSICA

preprint

IFUSP/P-276

MAGNETIC MONOPOLES, DUALITY AND COSMOLOGICAL PHASE TRANSITIONS

by

C.O. Escobar , A.A. Natale
Instituto de Física Teórica, São Paulo, Brazil

and

G.C. Marques
Instituto de Física da Universidade de São Paulo,
São Paulo, Brazil

UNIVERSIDADE DE SÃO PAULO
INSTITUTO DE FÍSICA
Caixa Postal - 20.516
Cidade Universitária
São Paulo - BRASIL

MAGNETIC MONOPOLES, DUALITY AND COSMOLOGICAL PHASE TRANSITIONS

C.O. Escobar[1,2], A.A. Natale[3]

Instituto de Física Teórica, São Paulo, Brazil

and

G.C. Marques[2]

Instituto de Física da Universidade de São Paulo, Brazil

ABSTRACT

We show that duality for magnetic monopoles, as proposed by Montonen and Olive, does not hold in quantum field theory at finite temperatures. Furthermore, the evolution picture of the Universe looks different when analyzed in the original "electric" theory or in its dual "magnetic" counterpart.

1. Address after 1st. of August, 1981: Instituto de Física da Universidade de São Paulo, Brazil.

2. Work supported in part by CNPq.

3. Supported by FAPESP.

JUNE/1981

It is believed that below a critical temperature (T_c) the Universe underwent a phase transition to a new phase in which the order parameter (the vacuum expectation value, v.e.v., of the Higgs field - $<\phi>$) is different from zero. Below T_c, $<\phi>$ depends on the temperature(1). Since at zero temperature the theory exhibits, at the classical level, monopole-like solutions(2), whose mass is, in the Bogomolny-Prasad-Sommerfield (B.P.S.) limit(3), given by $M_M = g<\phi(T=0)>$, where g is the magnetic charge of the monopole, it has been argued that monopoles with mass

$$M_M(T) = g<\phi(T)> , \tag{1}$$

should be produced in such phase transition. Early estimates(4) of the cosmological monopole production, indicated an exceedingly large amount of such topologically stable particles, thus creating a puzzle when the evolution of the Universe is studied on the light of Grand Unified Theories. The current thinking about the suppression of monopoles, is to invoke a strongly first order phase transition for the symmetry breaking of the Grand Unified Theory(5).

In case we had a better description of the quantum dynamics of such magnetic monopoles, we could hope to get an improved understanding of the monopole suppression. A nice possibility for dealing with a quantum field theory of magnetic monopoles is to employ the duality scheme proposed by Montonen and Olive(6). These authors, inspired by the sine-Gordon-Thirring model relationship(2), by which the Thirring model is the quantum field theory of the solitons of the sine-Gordon model, conjectured that when quantized the magnetic monopoles would form, together with the photon, a multiplet of gauge particles of a dual theory, described by a gauge-invariant Lagrangian similar to the original one, provided that the magnetic charge replaces the electric charge.

This paper grew out of an attempt of applying the duality concept as a tool for understanding the cosmological production of monopoles, in its dynamical aspects. As a matter of fact expression (1) would arise naturally in the dual theory where the magnetic monopoles are the gauge bosons which acquire their masses via the Higgs-Kibble mechanism.

It is our intention in this paper to show that for a field theory at finite temperatures, the duality concept is untenable. In order to show this, we analyze the effective potential at finite temperatures(1). For simplicity, we only consider the Georgi-Glashow model(7), based on the gauge group SO(3) ‡1.

At the one loop level, the effective potential for small values of λ (the region in which we will be interested later on) is given by ‡2

$$V[\phi,T] = -\frac{\mu^2\phi^2}{2} + \frac{\lambda\phi^4}{4}\left(1 - \frac{25}{16\pi^2}\frac{e^4}{\lambda}\right) + \frac{3}{32\pi^2}e^4\phi^4 \ln\frac{\phi^2}{M^2} -$$

$$- \frac{\pi^2}{10\beta^4} + \frac{1}{24\beta^2}(6e^2\phi^2 - 3\mu^2 + 5\lambda\phi^2) \qquad (2)$$

($\beta = \frac{1}{T}$, k = 1). The renormalization conditions are,

$$\left.\frac{\partial^2 V}{\partial\phi^2}\right|_{\phi^2=0} = -\mu^2 \quad , \quad \frac{1}{6}\left.\frac{\partial^4 V}{\partial\phi^4}\right|_{\phi^2=M^2} = \lambda \qquad (3)$$

The critical temperature is defined by(9),

$$\frac{\partial V}{\partial\phi}(T_c,\phi_c) = 0$$

$$V(\phi(T_c),T_c) = V(0,T_c) \quad . \qquad (4)$$

It is now opportune to comment on the B.P.S. limit, where μ^2 and λ vanish, with their ratio kept fixed. This limit plays an important role in the arguments of Montonen and Olive, since only then the mass spectrum of the "electric" and "magnetic" theories would be consistent. Furthermore, it is only in this limit that one of their supporting arguments for the conjecture is valid: the two independent calculations of the force between monopoles, the one by Manton(10) and the other one considering the monopoles as gauge particles, only agree in the B.P.S. limit, where there is a long-range scalar field. We would like to keep with this limit, at the tree level, which is relevant to the classical monopole solutions.

Using eqs. (2) and (4), we obtain, in the B.P.S. limit, the following values for ϕ_c and T_c,

$$\langle\phi_c^2\rangle = M^2 \exp\left(\frac{19}{6}\right)$$

$$T_c^2 = \frac{3}{8\pi^2} e^2 M^2 \exp\left(\frac{19}{6}\right) \qquad (5)$$

We can verify that in the B.P.S. limit the phase transition is typically of first order(11), since

$$\langle\phi^2\rangle \equiv \langle\phi^2(T=0)\rangle \simeq \langle\phi_c^2\rangle \quad . \qquad (6)$$

It is this feature which enables us to disregard the explicit temperature dependence in the masses of theory.

The masses of $W^{\pm}$ bosons are given by,

$$M_W^2 = e^2 <\phi^2> = e^2 M^2 \exp \frac{11}{3} \tag{7}$$

and the critical temperature, now in terms of the W mass is,

$$T_c^2 = \frac{3}{8\pi^2} M_W^2 \exp\left(-\frac{1}{2}\right) \quad . \tag{8}$$

The Higgs boson mass is defined by $M_\phi^2 = \left.\frac{\partial^2 V}{\partial \phi^2}\right|_{\phi^2=<\phi^2>}$, leading to

$$M_\phi^2 = \frac{3}{4\pi^2} e^2 M_W^2 \quad . \tag{9}$$

At this point we can compare the pictures obtained in the "electric" theory and in its dual, "magnetic" theory. We can do this either by comparing the mass spectrum or the characteristic of the phase transitions in each theory.

Simple inspection of eq. (9) shows that the Higgs boson would acquire very different masses in the two theories. Calling $\tilde{M}_\phi$ the mass of Higgs particle in the dual theory, we obtain[‡3]

$$\frac{M_\phi^2}{\tilde{M}_\phi^2} = \frac{e^4}{g^4} \tag{10}$$

We can instead impose the same critical temperature for both theories, so as to obtain a similar cosmological picture for them, in which case we would arrive at the following expressions,

$$\begin{aligned} e^2 M^2 &= g^2 \tilde{M}^2 \\ g^2 M_\phi^2 &= e^2 \tilde{M}_\phi^2 \\ M_W^2 &= M_M^2 \end{aligned} \tag{11}$$

where M_M is the magnetic monopole mass and, as before, we indicate by $\tilde{A}$ the quantity A, computed in the dual theory. M, is the renormalization point as defined in eq. (3).

We have thus come to the conclusion that it is impossible to obtain a consistent picture, either for the spectrum or for the critical temperature, in both theories. Furthermore, it seems to be impossible, even on qualitative grounds, to reconcile duality with cosmology, witness the very different domain structure of the Universe obtained in each theory. This happens so because, following Kibble[(12)], the domains are characterized by the correlation lenghts ξ and η given by[‡4]

$$\xi^{-1} \approx \frac{1}{M_\phi} \quad , \quad \eta^{-1} = \frac{1}{M_V} \tag{12}$$

where M_V is the mass of the vector boson of the corresponding gauge theory ($M_V = M_W$ or M_M). Since the particle masses are so different in the two theories, we can see that we have a different domain structure for the Universe.

If the phase transition is strongly first order, the monopoles will appear at the temperature T^*, of the reheated Universe[(5)]. We should require $T^* = \tilde{T}^*$, and this would imply very restrictive conditions on the effective potential of the two theories which seem to us very difficult to be implemented by a simple replacement $e \leftrightarrow g$. Even admiting $T^* = \tilde{T}^*$, there is another problem in the context of monopole production since the monopoles will appear only at the temperature T^*, while the W's exist already at temperatures above T^*. The opposite situation would occur in the dual theory, in the sense of Montonen and Olive, since now the W's are the solitons of this dual theory, which will then appear at the temperature T^*!

We conclude that the duality concept, as formulated by Montonen and Olive, is untenable for finite temperatures. It seems to us that at the root of the problem is the strict sense in which duality is assumed to take place, i.e., the magnetic monopoles are taken to be the gauge bosons of a similar theory, described by the same Lagrangian as the original one, simply by replacing e by g. Remember that is not the way in which duality occurs in the sine-Gordon-Thirring model relationship, which are described by completely different fields and there is no identification of the soliton sector of the Thirring model with the elementary sector of the sine-Gordon theory. In this respect we would agree with Nahm[13], who pointed out that duality would be a much more interesting concept if the monopoles have instead spin zero. Alternatively, as remarked by Montonen and Olive, we should only apply the duality concept if we have magnetic monopoles present, to start with, in which case we conclude that the duality conjecture is only true at zero temperature.

FOOTNOTES

#1 - For the extension of the duality concept to other gauge groups, see ref. (8).

#2 - The effective potential has been calculated up to the second order in an expansion of its temperature dependent part.

#3 - We have used the quantization condition $eg = 4\pi$ and assumed the same $\langle\phi^2\rangle$ for both theories, i.e., $\langle\phi^2\rangle = \langle\tilde{\phi}^2\rangle$. In this case we have different critical temperatures, $T_c^2/\tilde{T}_c^2 = e^2/g^2$.

#4 - The correlation length ξ determines the scale of fluctuations in ϕ, i.e., the size of the radial oscillations, while η is the penetration depth, in analogy with the Meissner effect in superconductors.

REFERENCES

(1) For recent reviews see: A.D. Linde, Rep. Prog. Phys. 42 (1979) 389.

T.W.B. Kibble, Phys. Rep. 67 (1980) 183.

(2) See the review: P. Goddard and D. Olive, Rep. Prog. Phys. 41 (1978) 1357.

(3) M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. 35 (1975) 760.

E.B. Bogomolny, Sov. J. Nucl. Phys. 24 (1976) 449.

(4) Ya.B. Zeldovich and M.Yu. Khlopov, Phys. Lett. 79B (1978) 329.

J.P. Preskill, Phys. Rev. Lett. 43 (1979) 1365.

(5) A.H. Guth and S.-H.H. Tye, Phys. Rev. Lett. 44 (1980) 631.

(6) C. Montonen and D. Olive, Phys. Lett. 72B (1977) 117.

(7) H. Georgi and S.L. Glashow, Phys. Rev. Lett. 28 (1972) 1494.

(8) F.A. Bais, Phys. Rev. D18 (1978) 1206.

(9) J. Iliopoulos and N. Papanicolau, Nucl. Phys. B111 (1976) 195.

(10) N.S. Manton, Nucl. Phys. B126 (1977) 525.

(11) A.D. Linde, Phys. Lett. 99B (1981) 391.

(12) T.W.B. Kibble, J. Phys. A9 (1976) 1387.

(13) W. Nahm, Phys. Lett. 90B (1980) 412.