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## EXTENDED DOMAIN OF VALIDITY OF THE HAUSER-FESHBACH FORMULA IN THE FRAMEWORK OF THE STATISTICAL THEORY

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ABSTRACT

It is shown that in the domain of overlapping resonances the Hauser-Feshbach formula can be derived in the framework of the statistical theory of nuclear reactions even when the energy dependence of the characteristic quantities (e.g. level density and widths) is not neglected.

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Using a random-matrix model for the coupling matrix elements connecting the channels with the nuclear resonance levels, Agassi et al. /1/ have derived the Hauser-Feshbach formula for the compound nucleus cross section in the limit of strongly overlapping resonances. The derivation given in Ref. /1/ is based on a number of assumptions which limit the domain of validity of the derivation. Among these, we mention an assumption explicitly or implicitly made in many theories /2/: the neglect of all terms involving derivatives of the characteristic parameters (level density, width) of the compound nucleus with respect to the excitation energy.

It is the purpose of this paper to point out that the Hauser-Feshbach formula can be derived, in the framework of the formalism of Ref. /1/, without using the last-mentioned assumption. As a result, we find for the compound-nucleus cross section pertaining to fixed angular momentum and parity, an expression of the form

$$\overline{|S_{ab}^{f\ell}|^2} = \frac{T_a T_b}{\sum_c T_c} (1 + \delta_{ab}) \ , \qquad (1)$$

where $S_{ab}^{f\ell}$ is the fluctuation part of the S matrix; a,b,... denote the channels; a bar the energy (or ensemble) average; and where the transmission coefficients $T_a$ are, in the absence of direct reactions ($\bar{S}_{ab} = 0$ for $a \neq b$), given by

$$T_a = 1 - |\bar{S}_{aa}|^2 \ . \qquad (2)$$

Equations (1) and (2) can be extended to include direct reactions in the usual way /1,2/. We note that the formal structure of (1), (2), including the value 2 for the elastic enhancement factor, is identical to that obtained /1/ under the neglect of all energy dependences. The difference arises from the explicit form of $\bar{S}_{aa}$, and of $T_a$. This point is irrelevant in practice since $T_a$ is anyway commonly calculated from a phenomenological optical model potential.* The main point of the present paper is, therefore, the statement that in the framework of the statistical theory, the Hauser-Feshbach formula can be derived under weaker conditions than have been used until now. Since this point is only of theoretical interest, we condense the proof as much as possible, using the notation of Ref. /1/ and indicating which steps in the derivation of Ref. /1/ must be altered, as shown below.

The S matrix is related to the t matrix by

$$S = 1 - 2it \tag{3}$$

and t is given by

$$t = \sum_{S=0}^{\infty} \gamma b \gamma (\hat{c} \gamma b \gamma)^S . \tag{4}$$

The propagator b is defined as

$$b_\mu = (E - \varepsilon_\mu + \frac{i}{2} \Gamma_0)^{-1} \tag{5}$$

with μ the level index, and the propagator $\hat{c}$ in channel space is given by $\hat{c} = -i$. [We suppress here the real shift parameter Δ which is due to a principal-value integral. We do this for reasons of consistency: A nonvanishing Δ would imply that the energy dependence due to penetration

*We should remark, however, that this would be true in the absence of preequilibrium or multistep compound contributions. The presence of these processes would necessarily imply the presence of several transmission coefficients, with only one of them being determined from the phenomenological optical model potential. See Ref. /3/ for a discussion of this point.



factors of the coupling matrix elements $\gamma_\mu^c$ connecting level μ and channel c is not negligible. It would then be necessary to consider Δ (and, more generally, $\hat{c}$) as a function of bombarding energy, and to take into account also the derivatives of Δ which is not done here; hence Δ = 0]. The derivation of Ref. /1/ evaluates $\overline{|S_{ab}^{f\ell}|^2}$ to lowest order in $(\Gamma/D)^{-1}$, where Γ is the correlation length, D the average level spacing. [The condition $\Gamma/D >> 1$ is equivalent to that of a large number of open channels since $\Gamma/D = (1/2\hbar)\mathrm{Tr}T$ /1/.] In terms of the contraction patterns defined in Ref. /1/ this means that only patterns with nonintersecting contraction lines are taken into account. This restriction is not lifted in the present context.

In Ref. /1/, the average of t was found to be given by the expression

$$\bar{t} = \overline{\gamma b \gamma} \sum_{S=0}^{\infty} (\hat{c} \overline{\gamma b \gamma})^S \equiv \bar{t}^{(1)} . \tag{6}$$

All the other possible ways of contracting pairs of γ,s were omitted because such contractions would force two or more propagators $b_\mu$ to have the same level index μ. Changing the sum over μ into an integral gives a pole of second or higher order. The contributions from such pole terms vanish if the statistical resonance parameters do not change with excitation energy. It is at this point that our present derivation differs from that of Ref. /1/ because we wish to include precisely such contributions. Without dropping terms containing $(b_\mu)^n$, n > 1, but still allowing only diagrams with nonintersecting contraction lines, we find for $\bar{t}$ the integral equation

$$\bar{t} = \overline{\gamma b \sum_{n=1}^{\infty} \gamma \hat{c} \bar{t} \hat{c} \gamma b \gamma} (\hat{c} \bar{t} + 1)$$
$$\equiv \bar{t}^{(2)} . \tag{7}$$

The end factors $\tilde{e}_a$ defined in Ref. /1/ are given by

$$\tilde{e} = \bar{t}^{(2)}c \;. \tag{8}$$

The modified channel propagator $\tilde{c}$ is given by

$$\tilde{c} = c(1 + \bar{t}^{(2)}c) \;. \tag{9}$$

The modified level propagator $\tilde{b}_\mu$ is given by

$$\tilde{b} = b + b\gamma c\overline{\bar{t}^{(2)}c}\gamma\tilde{b} \;. \tag{10}$$

To exhibit more clearly the differences between $\bar{t}^{(1)}$ of (6) and $\bar{t}^{(2)}$ of (7), we have worked out approximate expressions for both, following the treatment given in Ref. /1/, and have found

$$\bar{t}_a^{(2)} = \bar{t}_a^{(1)} - \frac{iD}{\pi\tau}\sum_b x_a x_b c_{bb}\bar{t}_b^{(1)} \;, \tag{11}$$

where $x^a = \frac{\pi}{D}\overline{|\gamma_\mu^a|^2}$ and $\tau$ is the compound nucleus temperature that defines the excitation energy dependence of D.

Equations (7)-(10) are formal in the sense that $\bar{t}^{(2)}$, $\tilde{e}$, $\tilde{c}$, $\tilde{b}$ can only be obtained after solving the integral equation (7). The formal structure suffices, however, to establish our claim. From (7) we see that $\bar{t}^{(2)}$ is diagonal in the channel indices. Equations (8), (9) show that the same holds true for $\tilde{e}$, $\tilde{c}$. Equation (10) shows that $\tilde{b}$ is diagonal in the level indices. Using the results (7) to (10), we can write $\overline{|S_{ab}^{f\ell}|^2}$ in the following form ($a \neq b$):

$$\overline{|S_{ab}^{f\ell}|^2} = \tilde{e}_a\gamma\tilde{b}\gamma\tilde{c}\ldots\gamma\tilde{b}\gamma\tilde{e}_b \,\vdots\, \tilde{e}_b^*\gamma\tilde{b}^*\gamma\ldots\tilde{c}^*\gamma\tilde{b}^*\gamma\tilde{e}_a^* \;. \tag{12}$$



Equation (12) and the contraction rules for pairs of $\gamma$,s show that $\overline{|S_{ab}^{f\ell}|^2}$ consists of three factors. One factor is $|\tilde{e}_a|^2\overline{|\gamma_\mu^a|^2}$, the second factor is $|\tilde{e}_b|^2\overline{|\gamma_\nu^b|}$, and the third factor is the remainder on the right-hand side of (12) which we note to be <u>independent of the level indices</u> a,b. This shows that $|S_{ab}^{f\ell}|^2$ has the form $X_a \cdot X_b \cdot Y$, typical of the Hauser-Feshbach formula. Using a symmetry argument, we can extend this statement to the elastic case, finding for $\overline{|S_{ab}^{f\ell}|^2}$ an expression $X_a \cdot X_b \cdot Y \cdot (1 + \delta_{ab})$. Defining the transmission coefficient $T_a$ by (2), we can show, using the formulae given above, that $T_a = 4x_a|\tilde{e}_a|^2$ and that the cross section takes the form (1). These statements imply a formal verification of unitarity to order $D/\Gamma$, with $\Gamma = \frac{D}{2\pi}\sum_c T_c$. The factor Y has been worked out and is given by

$$Y = \{\sum_c [|C_{cc}|^2 x_c(T_c - 4x_c|\tilde{e}_c|^2) + 4x_c|\tilde{e}_c|^2]\}^{-1} \;. \tag{13}$$

From (2) for $T_a$ and unitarity, $\sum_b|S_{ab}^{f\ell}|^2 = T_a$, one is also led to $T_a = 4x_a|\tilde{e}_a|^2$, and $Y = (\sum_c T_c)^{-1}$, thus double checking our results.

Although $\bar{t}^{(2)}$ of (7) differs from $\bar{t}^{(1)}$ of (6) by terms which reflect the excitation energy dependence of at least one of the parameters of the theory, namely the average level spacing D, the unitarity of the S matrix has preserved the Hauser-Feshbach form of $\overline{|S_{ab}^{f\ell}|^2}$. The domain of validity of the Hauser-Feshbach formula has thus been extended to include the less restrictive case where the parameters of the theory are excitation-energy dependent.

Furthermore, our results for $\bar{t}^{(2)}$ and correspondingly for the transmission coefficient, $T_a$, supply possible ways of improving upon some of the widely used approximations for calculating $T_a$, e.g., the Hill-Wheeler barrier penetration form of $T_a$ for charged particle transmission. It seems plausible to identify the origin $\bar{t}^{(1)}$ of AWM (6) with the "black-box" barrier penetration probability referred to above and the correction to $\bar{t}^{(1)}$, given approximately by the second term of (11), with some "fine structure" of the black box, namely its temperature $\tau$. In particular, recent work on heavy-ion fusion reactions /4/ has concentrated on precisely the type of correction we refer to above. It is worthwhile mentioning, though, that the "statistical yrast line" introduced in Ref. /4/ to discuss the heavy-ion fusion excitation function in the so-called energy region II is supposedly related to $\Gamma/D$, whereas, according to our (11), the correction we obtain seems to be associated with the compound nucleus temperature, $\tau$.

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