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OFF-SHELL FORM FACTORS

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ABSTRACT

The use of off-shell form factors in calculating the proton-neutron mass difference is advocated. These form factors appear in a Cottingham rotated Born-like expression for the mass difference and could lead to a good value for $\Delta = M_p - M_n$.

I. INTRODUCTION

Fortunately, the proton is lighter than the neutron. Unfortunately however, nobody was able (as yet) to show why. So, after a lot of work done during quite a long time, the proton-neutron mass difference $\Delta = M_p - M_n$ remains an unsolved problem.

It is natural to think that Δ has a purely electromagnetic origin and Feynman and Speisman (FS)[1] were the first to show that even the negative sign could be accounted for in that context if the magnetic part of the self energy were larger than the coulomb part.

That was before the nucleon form factors were experimentally measured, and it might be interesting to mention that in order to cut off an otherwise divergent integral FS introduced (what amounts to) the following form factors .

$$F_1 = \left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^{1/2} \qquad , \qquad F_2 = \frac{\lambda^2}{\lambda^2 - k^2}\left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^{1/2} \qquad . \tag{1.1}$$

For $\lambda = M_p$ and $\Lambda = 4M_p$, FS obtained the experimental $\Delta = -1.29$ MeV .

Once the form factors were experimentally known, Cini, Ferrari and Gatto[2] showed that, unfortunately, the decrease with momentum was too fast (faster than FS ansatz) to allow the magnetic energy to overtake its coulomb counterpart.

Since then a concensus started growing as to the impossibility of explaining Δ as a low energy effect. Thus, Cottingham's[3] work relating Δ to the electron nucleon scattering amplitudes was much welcomed since it made possible, in principle, to take into consideration the high energy region. However, Cottingham's programme could not be implemented in practice since nobody knows how to estimate the necessary substraction terms in the dispersion relations that enter into the computation of Δ^4.

That impasse prompted us to seek a different way of incorporating (at least part of) the influence of the high energy region into the calculation of Δ. Our proposal is to calculate Δ in a Born approximation but using form factors with one of the nucleon legs off its mass shell. These form factors, hopefully, contain information on what goes on in the high energy region. The form factors employed are of the Sachs type and fit very naturally in a decomposition of the nucleon current which is discussed in section II. The use of that current parametrization give rise to the mass shift formulae of section III.

With the mass shifts in terms of on-shell form factors Δ turns out to have the wrong sign. So, section IV is devoted to a discussion of the vertex function when one of the nucleons is off-mass-shell. This vertex function leads to mass shift formulae with off-mass-shell form factors which are given in section V.

Section VI contains the results for the proton--neutron mass difference and section VII some final remarks.

II. NUCLEON CURRENT

The nucleon current is usually parametrized in terms of F_1 and F_2, the Dirac and Pauli form factors. This parametrization, which can be found in almost any book that has anything to do with nucleons, is

$$\bar{u}(p')\,\Gamma^\mu u(p) = \bar{u}(p')\left[F_1(k^2)\gamma^\mu + F_2\frac{i\sigma^{\mu\nu}}{2M}(p'_\nu - p_\nu)\right]u(p) \quad , \tag{2.1}$$

where M is the nucleon mass and $k_\mu = p_\mu - p'_\mu$. Metric and

notation is as in Bjorken and Drell book[5].

As it is well known, cross section formulae are much simpler written in terms of the so called Sachs form factors G_E and G_M, as in terms of the F's. Besides, for reasons that are not completely clear at present, the Sachs form factors also have a simpler functional form as compared to the F's; and they seem to obey a scaling law. For these reasons it is convenient to use the G's instead of the F's. This can be easily accomplished replacing in Eq. (2.1), the F's by the G's with the use of

$$F_1(k^2) = \frac{G_E(k^2) - (k^2/4M^2)G_M(k^2)}{1 - (k^2/4M^2)} \quad , \quad F_2(k^2) = \frac{G_M(k^2) - G_E(k^2)}{1 - (k^2/4M^2)} \; . \tag{2.2}$$

Instead, one can start from the beggining with a nucleon current parametrization which is specially suited for the G form factors, namely

$$\sqrt{p_o p'_o/M^2}\; \langle p',s'|J^\mu(0)|p,s\rangle =$$

$$= \bar{u}(p',s')\,\Gamma^\mu u(p,s) = (1 - k^2/4M^2)\bar{u}(p',s')\,\{G_E(k^2)\frac{P^\mu}{M} -$$

$$- \frac{G_M(k^2)}{4M^2}(\gamma^\mu \not{P}\not{k} - \not{k}\not{P}\gamma^\mu)\}\,u(p,s) \quad , \tag{2.3}$$

where $P^\mu = \frac{1}{2}(p^\mu + p'^\mu)$.

The parametrization (2.3) is hard to find in text-books[6] and is rarely used in practical applications. For k^2 time-like one has to either be carefull to cancel the pole at $k^2 = 4M^2$, or argue ones way around it. In this paper that problem will not arise since all the calculations will be done for k^2 space-like after rotating a la Cottingham the mass shift expressions[7].

III. NUCLEON ELECTROMAGNETIC SELF MASS

The nucleon electromagnetic self mass can be written as[2]

$$\delta M = \frac{e^2}{2} \int \frac{d^4k}{(2\pi)^4} D^{\mu\nu}(k) T_{\mu\nu}(p,k) \quad , \tag{3.1}$$

where $D^{\mu\nu}(k)$ is the photon propagator and

$$T_{\mu\nu}(p,k) = i \int d^4x \, e^{-ikx} \langle p | T\{J_\mu(x), J_\nu(0)\} | p \rangle_{av.} + \begin{pmatrix} \mu \leftrightarrow \nu \\ k \leftrightarrow -k \end{pmatrix} , \tag{3.2}$$

is the forward Compton amplitude for an off shell photon averaged over nucleon spins.

With the photon propagator in the Landau gauge, the Born approximation for Eq. (3.1) reads

$$\delta M = e^2 \int \frac{d^4k}{(2\pi)^4} \frac{-i(g^{\mu\nu} - k^\mu k^\nu/k^2)}{k^2 + i\epsilon} \Gamma_\mu(p',k) \frac{\not p - \not k + M}{k^2 - 2pk + i\epsilon} \Gamma_\nu(p',k), \tag{3.3}$$

where Γ_μ is the nucleon electromagnetic vertex function. The parametrization (2.3) for this vertex functions leads to

$$\delta M = - \frac{i8e^2}{M} \int \frac{d^4k}{(2\pi)^4} \frac{\left[2(M^2/k^2) G_E^2(k^2) + G_M^2(k^2)\right]\left[M^2k^2 - (pk)^2\right](2M^2 - pk)}{(k^2 + i\epsilon)(k^2 - 2pk + i\epsilon)(k^2 - 4M^2)^2} \tag{3.4}$$

In the time-like region, the inegrand in Eq. (3.4) has the nasty looking double pole at $k^2 = 4M^2$. The factors $[M^2k^2 - (pk)^2]$ and $(2M^2 - pk)$ vanish at the pair production threshold and thus cancel the pole at that point. Besides, for $k^2 = 4M^2$ one should take into account intermediate states with one nucleon plus one pair whose contribution might cancel any unwanted kinematical pole from the single nucleon intermediate state.

In the following, a Cottingham rotation will be performed on Eq. (3.4) and all computations will be done for k^2 spacelike. In this way we will not have to worry about the pole discussed above. In relation to the poles in the time-like region we have only to assume they carry an appropriate $i\epsilon$ so that all singularities in the k_o complex plane are properly located to comply with the causality principle. This, together with the convergence of the integral, are sufficient conditions to allow for a Cottingham rotation which turns Eq. (3.4) into

$$\delta M = \frac{16\alpha M^2}{\pi^2} \int_0^\infty \frac{dQ^2 \left[2M^2 G_E^2(Q^2) - Q^2 G_M^2(Q^2)\right]}{Q^4 \; (Q^2 + 4M^2)^2} \int_0^{MQ} \frac{d\omega \left[Q^2 - (\omega^2/M^2)\right]^{5/2}}{(Q^4 + 4\omega^2)} \; , \tag{3.5}$$

where $Q^2 = -k^2$ and $\omega = -i\nu$ with $\nu = -pk$. Eq. (3.5) holds when the form factors depend only on k^2 . A more general expression valid when there is also a ν dependence, will be derived in section V.

With Eq. (3.5) we can now calculate the mass shifts for the proton and neutron in terms of the form factors G_E and G_M which can be taken to be of the standard dipole type

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{\mu_p} = \frac{G_M^n(Q^2)}{\mu_n} = G_D(Q^2) \equiv \left(\frac{Q^2}{\beta M^2} + 1\right)^{-2} \quad ,$$

$$G_E^n(Q^2) = 0 \quad , \tag{3.6}$$

where μ_p and μ_n are the proton and neutron magnetic moments, and $\beta \simeq 0.81$[8].

It turns out that Eq. (3.5) with the on-shell form factors (3.6) gives the wrong sign for the mass difference $\Delta = \delta M_p - \delta M_n$. In the following we will try to remedy this situation in the simplest possible way: we will allow the form factors to go off-mass-shell.

IV. OFF-SHELL VERTEX FUNCTION AND FORM FACTORS

As it was said before, we intend to calculate the proton and neutron mass shifts in a Born like approximation as in Eq. (3.4) or Eq. (3.5) but replacing the usual G's by off-mass--shell form factors since the nucleon in the intermediate state is actually off its mass shell. Thus we need to know the nucleon electromagnetic vertex when one of the nucleon legs is off-mass--shell. The positive energy projection of that vertex is given by[9]

$$\frac{\not p' + W}{2W} \Gamma_\mu \ (p,p')\ u(p) = \frac{\not p + W}{2W} \{G_3(k^2,W)\ k_\mu +$$

$$+ \frac{1}{M(1-k^2/4M^2)} \left[G_E(k^2,W) P_\mu - \frac{G_M(k^2,W)}{4M} (\gamma_\mu \not P \not k - \not k \not P \gamma_\mu)\right]\}\ u(p) \quad , \quad (4.1)$$

with $W^2 = p'^2$.

In Eq. (4.1) we see the appearance of the new form factor $G_3(k^2,W)$ which, however, is not independent. The vertex function has to satisfy a Ward identify which leads to

$$\frac{(W^2 - M^2)\, G_E(k^2,W)}{2M(1-k^2/4M^2)} + G_3(k^2,W) = (W-M)\ q \quad . \qquad (4.2)$$

where q is the charge of the particle in units of the proton charge.

This Ward identity can be easily derived by contracting with k^μ the off-mass-shell vertex both in the form (4.1) as well as [9]

$$\frac{\not p' + W}{2W} \Gamma_\mu \ (p,p') = i(p_o/M)^{1/2} \ \frac{\not p' + W}{2W}$$

$$\int d^4x\ e^{-ipx}\ \theta(x_o)\ \langle 0|\left[J_{N'}(x), J_\mu(0)\right]|p,s\rangle \quad , \qquad (4.3)$$

where $J_{N'}$ is the source of the nucleon with momentum p' .

We see from Eqs. (4.1) and (4.2) that when one of the nucleons is off-mass-shell the vertex function is given by

$$\frac{\not p' + W}{2W} \Gamma_\mu \ (p,p')\ u(p) = \frac{\not p' + W}{2W} \left\{\frac{q(W-M)}{k_\mu} k_\mu + \right.$$

$$\left. + \frac{1}{M(1-k^2/4M^2)} \left[G_E(k^2,W)\left(p_\mu - \frac{pk}{k^2} k_\mu\right) - \frac{G_M(k^2,W)}{4M} (\gamma_\mu \not P \not k - \not k \not P \gamma_\mu)\right]\right\}. \quad (4.4)$$

Our proposal is to calculate the mass shifts of the nucleons with this vertex inserted in the equation for the Born approximation (3.3). For that we will need explicit expressions for the off--mass-shell form factors. Here we propose the simple ansatz

$$G_D(k^2,M) = (k^2/\beta M^2 - 1)^{-2} \longrightarrow G_D(k^2,W) = (k^2/\beta W^2 - 1)^{-2} . \quad (4.5)$$

That is, except for the neutron electric form factor which is assumed to be zero, all the others are proportional to

$$G_D(k^2,W) = \left[\frac{k^2}{\beta(M^2 + k^2 - 2pk)} - 1\right]^{-2} . \qquad (4.6)$$

Notice that in the Bjorken limit $(-k^2) \to \infty$, $(-pk) \equiv \nu \to \infty$, with $x = (k^2/2pk)$, G_D scales as

$$G_D \xrightarrow[B_j]{} (1-X)^2/[1+X(\beta^{-1}-1)] \quad . \tag{4.7}$$

In the rest frame of the nucleon whose mass shift we are trying to calculate, after a Cottingham rotation $(k_o \to ik_4)$ the square of the dipole form factor (4.6) turns into

$$G_D^2(Q^2,k_4) = \left[\frac{Q^2}{\beta(M^2-Q^2-2iMk_4)}\right]^{-4} \quad , \tag{4.8}$$

which can also be written as

$$G_D^2(Q^2,k_4) = 1 + \frac{\beta^4}{6}\ \frac{\partial^3}{\partial\beta^3}\ \frac{Q^2}{\beta[Q^2+\beta(M^2-Q^2-2iMk_4)]} \quad . \tag{4.9}$$

V. MASS SHIFT WITH OFF-SHELL FORM FACTOR

In section III we have obtained mass shift formulae valid in the Born approximation in terms of the on-shell electromagnetic vertices (2.3). The off-shell vertices (4.4) contain extra terms proportional k_μ . However, these extra terms will immediately drop out from the mass shift given by Eq. (3.3) when contracted with the photon propagator in the Landau gauge. So, we are lead again to Eq. (3.4), with the only change $G^2(k^2) \to G^2(k^2,W)$.

Thus, the mass shift of a nucleon with charge q and magnetic moment μ , after a Cottingham rotation and with form factors that depend both on Q^2 and k_4 (in the rest frame), is given by

$$\delta M = \frac{4\alpha M^2}{\pi^2}\int_0^\infty \frac{dQ^2\left[\frac{2M^2q^2}{Q^2}-\mu^2\right]}{Q^2(Q^2+4M^2)^2}\int_{-Q}^{Q}\frac{dk_4(Q^2-k_4^2)^{3/2}(2M-ik_4)G_D^2(Q^2,k_4)}{(Q^2+2iMk_4)} \quad . \tag{5.1}$$

Since the integral is real analytic we can also write

$$\delta M = \frac{8\alpha M^2}{\pi^2}\,R_e\int_0^\infty \frac{dQ^2\left[\frac{2M^2q^2}{Q^2}-\mu^2\right]}{Q^2(Q^2+4M^2)^2}\int_0^{Q}\frac{dk_4(Q^2-k^2)^{3/2}(2M-ik_4)G_D^2(Q^2,k_4)}{(Q^2+2iMk_4)} \quad . \tag{5.2}$$

For form factors that depend only on Q^2 [as in Eq. (3.6)] we recover Eq. (3.5). Using instead the dipole form factor of Eq. (4.9), the mass shift can be split in two parts according to

$$\delta M = \delta M_c + \delta M_v \quad , \tag{5.3}$$

where δM_c is the mass shift we would have if G_D^2 were constant (the one in the right hand side of Eq. (4.9)), while the variation of the form factor gives rise to δM_v . With $k_4 = ZQ$ and from Eqs, (5.2), (5.3) and (4.9) we have

$$\delta M_c = \frac{16\alpha M^3}{\pi^2}\,R_e\int_0^\infty dQ^2\int_0^1\frac{dZ(1-Z^2)^{5/2}(2M^2q^2-\mu^2Q^2)}{(Q^2+4M^2)^2\ (Q^2+4M^2Z^2)} \quad , \tag{5.4}$$

and

$$\delta M_v = \frac{8\alpha M^2\beta^4}{3\pi^2}\ \frac{\partial^3}{\partial\beta^3}\,R_e\int_0^\infty\frac{Q^2dQ(2M^2q^2-\mu^2Q^2)}{\beta(Q^2+4M^2)^2}\ \times$$

$$\times\int_0^1\frac{dZ(1-Z^2)^{3/2}\ (2M-iZQ)}{(Q+2iMZ)\,[Q^2+\beta(M^2-Q^2-2iMZQ)]} \quad . \tag{5.5}$$

With the change of variable $Q^2 = 4M^2 t$ the last equation can also be written

$$\delta M_v = \frac{8\alpha M\beta^4}{3\pi^2} \frac{\partial^3}{\partial\beta^3} \frac{1}{\beta} \int_0^\infty \frac{dt\; t(q^2 - 2\mu t)}{(t+1)^2} \times$$

$$\times \int_0^1 \frac{dZ(1-Z^2)^{3/2} \{(1-Z^2)[4t(1-\beta) + \beta] + 4\beta Z^2(1-t)\}}{(t+Z^2)\{16t^2(1-\beta)^2 + 8t[\beta(1-\beta) + 2\beta^2 Z^2] + \beta^2\}} \quad . \tag{5.6}$$

VI. PROTON-NEUTRON MASS DIFFERENCE

With the equation written down in the last section for the mass shifts we can calculate now the proton-neutron mass difference. This mass difference can be separated in two parts

$$\Delta = \Delta_c + \Delta_v \tag{6.1}$$

with

$$\Delta_c = (\delta M_c)_{proton} - (\delta M_c)_{neutron} \tag{6.2}$$

and

$$\Delta_v = (\delta M_v)_{proton} - (\delta M_v)_{neutron} \quad . \tag{6.3}$$

Δ_c is easy to obtain and a calculation of the integrals in Eq. (5.4) gives

$$\delta M_c = \frac{\alpha M}{2\pi} [q^2(2\,\ell n\,2 + 1/4) - \mu^2(7/4 - \ell n\,2)] \quad . \tag{6.4}$$

From this and Eq. (6.2) we get $\Delta_c = -2.99$ MeV .

We were unable to integrate Eq. (5.6) analytically in order to obtain Δ_v . So, after taking the derivatives in relation to β , the double integral was done numerically in a computer. For certain values of β (close to the experimental ~0.81) a good result for Δ can be obtained; for instance: for $\beta = 0.818 \rightarrow \Delta_v = 1.66$ MeV $\rightarrow \Delta = -1.33$ MeV. Unfortunately, Eq. (5.6) (and Δ through it) depends quite sensitively on β . Even so, we take it as a good sign that in the scheme proposed in the present work, one can obtain a good result for Δ corresponding to a β close to the experimental value.

VII. CONCLUDING REMARKS

The fact that the Sachs form factors are the ones that seem to satisfy simple relations was taken as a clue telling us that for the nucleon current, a parametrization should be used that singles out, from the beginning, that type of form factors (as in the vertex function of Eq. (2.3)). That parametrization was then used in a Born approximation for the proton-neutron mass difference.

Since the intermediate nucleon is off-mass-shell, it was proposed to replace the on-shell form factors for off-mass--shell ones. With a simple ansatz as to the possible functional dependence of these form factors on the intermediate nucleon invariant mass, we were able to calculate Δ .

Even when Δ turned out to depend in a sensitive way on the mass appearing in the form factors (i.e., on β), it is encouraging that the correct value for Δ correspond to a β of about the number given by experimental fits.

The idea is that the restriction to the Born terms, being basically a low energy approximation could, nevertheless, incorporate the information on the high energy regime that is contained in the off-mass-shell form factors. Of course the next step should be to try to estimate in a reliable way, the off-mass--shell form factors.

FOOTNOTES AND REFERENCES

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